# Pearson Edexcel

# A Level Mathematics 9MA0

# Unit Test

# 11 Integration – 2

# Time allowed: 50 minutes
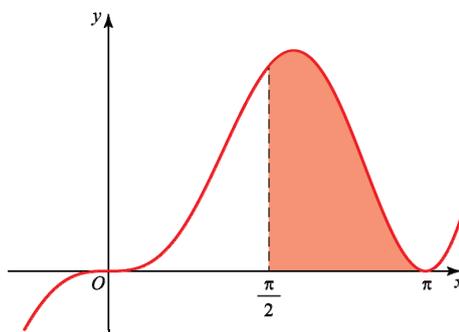
# School:
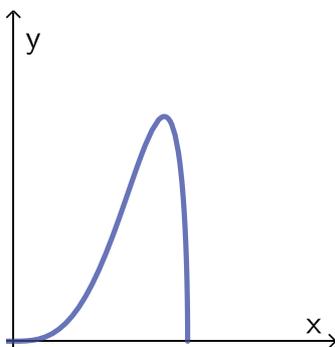
# Name:

# Teacher:

| Question | Points | Score |
|----------|--------|-------|
| 1 | 7 | |
| 2 | 11 | |
| 3 | 10 | |
| 4 | 10 | |
| 5 | 12 | |
| Total: | 50 | |

1. The diagram shows part of the curve with equation $y = x\sin^2(x)$. The finite region bounded by the line with equation $x = \frac{\pi}{2}$, the curve and the $x$-axis is shown shaded in the diagram. Find the area of the shaded region. [7]



2. The diagram shows the curve with equation $y = \frac{1}{2}x^3\sqrt{4 - x^2}$.



   (a) Complete the table with the value of $y$ corresponding to $x = 1.5$. Give your answer correct to 5 decimal places. [1]

   | $x$ | 0 | 0.5 | 1 | 1.5 | 2 |
   |---|---|---|---|---|---|
   | $y$ | 0 | 0.12103 | 0.86603 | | 0 |

   Given that

   $$I = \int_0^2 \frac{1}{2}x^3\sqrt{4 - x^2}\,\mathrm{d}x$$

   (b) Use the trapezium rule with 4 equal width strips to find an approximate value of $I$, giving your answer to 4 significant figures. [3]

   (c) By using an appropriate substitution, or otherwise, find the exact value of $I$, leaving your answer as a rational number in its simplest form. [6]

   (d) Suggest one way in which your estimate using a trapezium rule could be improved. [1]
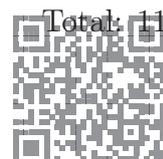
   Total: 11

3.

$$f(x) = \frac{21 - 14x}{(1 - 4x)(2x + 3)}, x \neq \frac{1}{4}, x \neq -\frac{3}{2}.$$

(a) Given that                                                                [5]

$$f(x) = \frac{A}{1 - 4x} + \frac{B}{2x + 3},$$

   find the values of the constants $A$ and $B$.

(b) Find the exact value of $\int_{-1}^{0} f(x)\,dx$.                          [5]

Total: 10

4. The value of a computer, $V$, decreases over time, $t$, measured in years. The rate of decrease of the value is proportional to the remaining value.

(a) Given that the initial value of the computer is $V_0$, show that        [4]

$$V = V_0 e^{-kt}.$$

   After 10 years the value of the computer is $\frac{1}{5}V_0$.

(b) Find the exact value of $k$.                                             [3]

(c) How old is the computer when its value is only 5% of its original value? Give your answer   [3]
   to 3 significant figures.

Total: 10

5. A large cylindrical tank has radius 40 m. Water flows into the cylinder from a pipe at a rate of $4000\pi\text{m}^3\text{min}^{-1}$. At time $t$, the depth of water in the tank is $h$m. Water leaves the bottom of the tank through another pipe at a rate of $50\pi h\text{m}^3\text{min}^{-1}$.

(a) Show that $t$ minutes after water begins to flow out of the bottom of the cylinder,    [6]

$$160\frac{dh}{dt} = 400 - 5h$$

(b) When $t = 0$ min, $h = 50$m.                                            [6]

   Find the exact value of $t$ when $h = 60$m.

Total: 12