Pearson Edexcel Level 3

GCE Mathematics 9MA0

Practice Paper B

Pure Mathematics

Time allowed: 2 **hours**

Centre:

Name:

Teacher:

Question	Points	Score
1	5	
2	5	
3	6	
4	8	
5	6	
6	8	
7	6	
8	4	
9	9	
10	10	
11	10	
12	11	
13	12	
Total:	100	

1. Use proof by contradiction to prove the statement: 'The product of two odd numbers is odd.' [5]

2. (a) Prove that the sum of the first n terms of an arithmetic series is [3]

$$S = \frac{n}{2}\left(2a + (n-1)d\right).$$

(b) Hence, or otherwise, find the sum of the first 200 odd numbers. [2]

Total: 5

3. A curve has the equation $y = \ln(3x) - e^{-2x}$. [6]

Show that the equation of the tangent at the point with an x-coordinate of 1 is

$$y = \left(\frac{e^2+2}{e^2}\right)x - \left(\frac{e^2+3}{e^2}\right) + \ln(3).$$

4. The curve C has parametric equations

$$x = 7\sin(t) - 4, \quad y = 7\cos(t) + 3, \quad -\frac{\pi}{2} \leq t \leq \frac{\pi}{3}.$$

(a) Show that the cartesian equation of C can be written as [3]

$$(x+a)^2 + (y+b)^2 = c$$

where a, b and c are integers which should be stated.

(b) Sketch the curve C on the given domain, clearly stating the endpoints of the curve. [3]

(c) Find the length of C. Leave your answer in terms of π. [2]

Total: 8

5. The coordinates of A and B are $(-1, 7, k)$ and $(4, 1, 10)$ respectively.

   Given that the distance from A to B is $5\sqrt{5}$ units,

   (a) find the possible values of the constant k. [3]

   (b) For the larger value of k, find the unit vector in the direction of $\overrightarrow{OA}$. [3]

Total: 6

6. Given that [8]

$$\int_{\ln(2)}^{\ln(b)} \frac{e^{2x}}{e^{2x}-1}\,dx = \ln(4),$$

find the value of b showing each step in your working.

7. A sequence is given by

$$x_1 = 4, \quad x_{n+1} = px_n - 9$$

where p is an integer.

(a) Show that $x_3 = 4p^2 - 9p - 9$. [2]

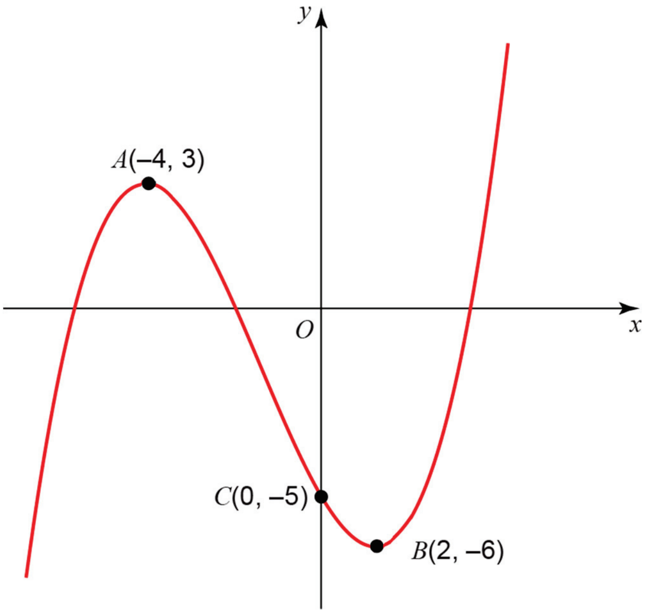
(b) Given that $x_3 = 46$, find the value of p. [3]

(c) Hence find the value of x_5. [1]

Total: 6

8. Express $\dfrac{6}{4x^2+8x-5} + \dfrac{3x+1}{2x-1}$ as a single fraction in its simplest form. [4]

9. The diagram shows the graph of $h(x)$.



The points $A(-4, 3)$ and $B(2, -6)$ are turning points on the graph and $C(0, -5)$ is the y-intercept.

Sketch on separate diagrams, the graphs of

(a) $y = |f(x)|$ [3]

(b) $y = f(|x|)$ [3]

(c) $y = 2f(x + 3)$ [3]

Where possible, label clearly the transformations of the points A, B and C on your new diagrams and give their coordinates. Total: 9

10.

$$g(x) = \frac{2}{x-1} - e^x.$$

(a) By drawing an appropriate sketch, show that there is only one solution to the equation $g(x) = 0$. [2]

(b) Show that the equation $g(x) = 0$ may be written in the form $x = 2e^{-x} + 1$. [2]

(c) Let $x_0 = 1.5$. Use the iterative formula $x_{n+1} = 2e^{-x_n} + 1$ to find to 4 decimal places the values of x_1, x_2, x_3 and x_4. [2]

(d) Using $x_0 = 1.5$ as a first approximation, apply the Newton-Raphson procedure once to $g(x)$ to find a second approximation to α, giving your answer to 4 decimal places. [4]

Total: 10

11. (a) Find the binomial expansion of $\dfrac{1+x}{\sqrt{1-2x}}$ in ascending powers of x up to and including the x^2 term, simplifying each term. [4]

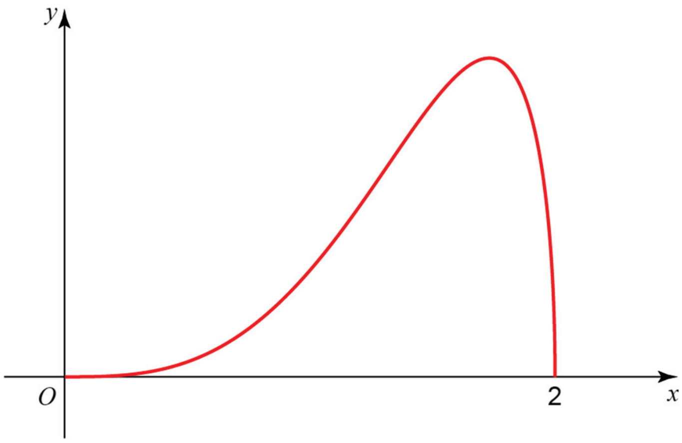
(b) State the set of values of x for which the expansion is valid. [1]

(c) Show that when $x = \dfrac{1}{100}$, the exact value of $\dfrac{1+x}{\sqrt{1-2x}}$ is $\dfrac{101\sqrt{2}}{140}$. [2]

(d) Substitute $x = \frac{1}{100}$ into the binomial expansion in part (a) and hence obtain an approximation to $\sqrt{2}$. Give your answer to 5 decimal places. [3]

Total: 10

12. The diagram shows the curve with equation $y = \frac{1}{2}x^2\sqrt{4-x^2}$.



(a) Complete the table with the value of y corresponding to $x = 1.5$. [1]

Give your answer correct to 5 decimal places.

x	0	0.5	1	1.5	2
y	0	0.12103	0.86603		0

(b) Given that [3]

$$I = \int_0^2 \frac{1}{2}x^3\sqrt{4-x^2}\,\mathrm{d}x,$$

use the trapezium rule with 4 equal width strips to find an approximate value of I, giving your answer to 4 significant figures.

(c) By using an appropriate substitution, or otherwise, find the exact value of [6]

$$\int_0^2 \frac{1}{2}x^3\sqrt{4-x^2}\,\mathrm{d}x,$$

leaving your answer as a rational number in its simplest form.

(d) Suggest one way in which your estimate using a trapezium rule could be improved. [1]

Total: 11

(Q12 continued)

13. (a) Express $5\cos(\theta) - 8\sin(\theta)$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \pi$. [4]

Write R in surd form and give the value of α correct to 4 decimal places.

(b) The temperature of a kiln, T°C, used to make pottery can be modelled by the equation [4]

$$T = 1100 + 5\cos\left(\frac{x}{3}\right) - 8\sin\left(\frac{x}{3}\right), \quad 0 \leq x \leq 72,$$

where x is the time in hours since the pottery was placed in the kiln.

Calculate the maximum value of T predicted by this model and the value of x, to 2 decimal places, when this maximum first occurs.

(c) Calculate the times during the first 24 hours when the temperature is predicted, by this model, to be exactly 1097°C. [4]

Total: 12