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Question	Answer	Marks	Guidance
1	State $1 + e^{2y} = e^x$	B1	
	Make <i>y</i> the subject	M1	Rearrange to $e^{2y} = \dots$ and use logs
	Obtain answer $y = \frac{1}{2} \ln(e^x - 1)$	A1	OE
		3	

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Question	Answer	Marks	Guidance
2	State or imply non-modular inequality $(2x-3)^2 > 4^2(x+1)^2$, or corresponding quadratic equation, or pair of linear equations $(2x-3)=\pm 4(x+1)$	B1	$12x^2 + 44x + 7 < 0$
	Make reasonable attempt at solving a 3-term quadratic, or solve two linear equations for x	M1	Correct method seen, or implied by correct answers
	Obtain critical values $x = -\frac{7}{2}$ and $x = -\frac{1}{6}$	A1	
	State final answer $-\frac{7}{2} < x < -\frac{1}{6}$	A1	
	Alternative method for question 2		
	Obtain critical value $x = -\frac{7}{2}$ from a graphical method, or by inspection, or by solving a linear equation or an inequality	B1	
	Obtain critical value $x = -\frac{1}{6}$ similarly	B2	
	State final answer $-\frac{7}{2} < x < -\frac{1}{6}$	B1	
		4	

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Question	Answer	Marks	Guidance
3	State $\frac{\mathrm{d}x}{\mathrm{d}t} = 2 + 2\cos 2t$	B1	
	Use the chain rule to find the derivative of <i>y</i>	M1	
	Obtain $\frac{dy}{dt} = \frac{2\sin 2t}{1 - \cos 2t}$	A1	OE
	Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1	
	Obtain $\frac{dy}{dx} = \csc 2t$ correctly	A1	AG
		5	

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Question	Answer	Marks	Guidance
4(i)	State $\frac{\mathrm{d}N}{\mathrm{d}t} = k\mathrm{e}^{-0.02t}N$ and show $k = -0.01$	B1	$ \begin{array}{l} \text{OE} \\ \left(-10 = k \times 1 \times 1000\right) \end{array} $
		1	
4(ii)	Separate variables correctly and integrate at least one side	B1	$\int \frac{1}{N} dN = \int -0.01 e^{-0.02t} dt$
	Obtain term ln N	B1	OE
	Obtain term $0.5e^{-0.02t}$	B1	OE
	Use $N = 1000$, $t = 0$ to evaluate a constant, or as limits, in a solution with terms $a \ln N$ and $be^{-0.02t}$, where $ab \neq 0$	M1	
	Obtain correct solution in any form e.g. $\ln N - \ln 1000 = 0.5(e^{-0.02t} - 1)$	A1	$\ln 1000 - \frac{1}{2} = 6.41$
	Substitute $N = 800$ and obtain $t = 29.6$	A1	
		6	
4(iii)	State that <i>N</i> approaches $\frac{1000}{\sqrt{e}}$	B1	Accept 606 or 607 or 606.5
		1	

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Question	Answer	Marks	Guidance
5(i)	Use correct product rule	M1	
	Obtain correct derivative in any form	A1	
	$\frac{dy}{dx} = -2e^{-2x}\ln(x-1) + \frac{e^{-2x}}{x-1}$		
	Equate derivative to zero and derive	A1	AG
	$x = 1 + e^{\frac{1}{2(x-1)}}$ or $p = 1 + \frac{1}{2(p-1)}$		
		3	
5(ii)	Calculate values of a relevant expression or pair of relevant expressions at $x = 2.2$ and $x = 2.6$	M1	
	$f(x) = \ln(x-1) - \frac{1}{2(x-1)} \Longrightarrow f(2.2) = -0.234, f(2.6) = 0.317$		
	$f(x) = 2e^{-2x}\ln(x-1) + \frac{e^{-2x}}{x-1} \Rightarrow f(2.2) = 0.005, f(2.6) = -0.0017$		
	Complete the argument correctly with correct calculated values	A1	
		2	

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Question	Answer	Marks	Guidance
5(iii)	Use the iterative process $p_{n+1} = 1 + \exp\left(\frac{1}{2(p_n - 1)}\right)$ correctly at least once	M1	
	Obtain final answer 2.42	A1	
	Show sufficient iterations to 4 d.p. to justify 2.42 to 2 d.p., or show there is a sign change in the interval (2.415, 2.425)	A1	
		3	

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Question	Answer	Marks	Guidance
6(i)	Use correct quotient rule	M1	
	Obtain $\frac{dy}{dx} = -\csc^2 x$ correctly	A1	AG
		2	
6(ii)	Integrate by parts and reach $ax \cot x + b \int \cot x dx$	*M1	
	Obtain $-x \cot x + \int \cot x dx$	A1	OE
	State $\pm \ln \sin x$ as integral of $\cot x$	M1	
	Obtain complete integral $-x \cot x + \ln \sin x$	A1	OE
	Use correct limits correctly	DM1	$0 + 0 + \frac{\pi}{4} - \ln \frac{1}{\sqrt{2}}$
	Obtain $\frac{1}{4}(\pi + \ln 4)$ following full and exact working	A1	AG
		6	

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Question	Answer	Marks	Guidance
7(i)	Express general point of <i>l</i> or <i>m</i> in component form e.g. $(a + \lambda, 2 - 2\lambda, 3 + 3\lambda)$ or $(2 + 2\mu, 1 - \mu, 2 + \mu)$	B1	
	Equate at least two pairs of corresponding components and solve for λ or for μ	M1	
	Obtain either $\lambda = -2$ or $\mu = -5$ or $\lambda = \frac{1}{3}a$ or $\mu = \frac{2}{3}a - 1$	A1	
	or $\lambda = \frac{1}{5}(a-4)$ or $\mu = \frac{1}{5}(3a-7)$		
	Obtain $a = -6$	A1	
		4	
7(ii)	Use scalar product to obtain a relevant equation in a, b and c, e.g. $a - 2b + 3c = 0$	B1	
	Obtain a second equation, e.g. $2a - b + c = 0$ and solve for one ratio	M1	
	Obtain $a : b : c = 1 : 5 : 3$	A1	OE
	Substitute a relevant point and values of <i>a</i> , <i>b</i> , <i>c</i> in general equation and find <i>d</i>	M1	
	Obtain correct answer $x + 5y + 3z = 13$	A1FT	OE. The FT is on a from part (i), if used
	Alternative method for question 7(ii)		
	Attempt to calculate vector product of relevant vectors,	M1	e.g. $(i-2j+3k).(2i-j+k)$
	Obtain two correct components	A1	
	Obtain correct answer, e.g. $\mathbf{i} + 5\mathbf{j} + 3\mathbf{k}$	A1	
	Substitute a relevant point and find <i>d</i>	M1	
	Obtain correct answer $x + 5y + 3z = 13$	A1FT	OE. The FT is on <i>a</i> from part (i), if used

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Question	Answer	Marks	Guidance
7(ii)	Alternative method for question 7(ii)		
	Using a relevant point and relevant vectors, form a 2-parameter equation for the plane	M1	
	State a correct equation, e.g. $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}) + \mu(2\mathbf{i} - \mathbf{j} + \mathbf{k})$	A1FT	
	State three correct equations in <i>x</i> , <i>y</i> , <i>z</i> , λ and μ	A1FT	
	Eliminate λ and μ	M1	
	Obtain correct answer $x + 5y + 3z = 13$	A1FT	OE. The FT is on a from part (i), if used
		5	

Question	Answer	Marks	Guidance
8(i)	State or imply the form $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+2}$	B1	
	Use a correct method for finding a constant	M1	
	Obtain one of $A = -1$, $B = 3$, $C = 2$	A1	
	Obtain a second value	A1	
	Obtain the third value	A1	Allow in the form $\frac{Ax+B}{x^2} + \frac{C}{x+2}$
		5	

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Question	Answer	Marks	Guidance
8(ii)	Integrate and obtain terms $\ln x - \frac{3}{x} + 2\ln(x+2)$	B1FT + B1FT + B1FT	The FT is on <i>A</i> , <i>B</i> , <i>C</i> ; or on <i>A</i> , <i>D</i> , <i>E</i> .
	Substitute limits correctly in an integral with terms $a \ln x$, $\frac{b}{x}$ and $c \ln (x+2)$, where $abc \neq 0$	M1	$-\ln 4 - \frac{3}{4} + 2\ln 6(+\ln 1) + 3 - 2\ln 3$
	Obtain $\frac{9}{4}$ following full and exact working	A1	AG – work to combine or simplify logs is required
		5	

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Question	Answer	Marks	Guidance
9(i)	Use $cos(A + B)$ formula to express $cos3x$ in terms of trig functions of $2x$ and x	M1	
	Use double angle formulae and Pythagoras to obtain an expression in terms of $\cos x$ only	M1	
	Obtain a correct expression in terms of cos <i>x</i> in any form	A1	
	$Obtain \ \cos 3x = 4\cos^3 x - 3\cos x$	A1	AG
		4	
9(ii)	Use identity and solve cubic $4\cos^3 x = -1$ for x	M1	$\cos x = -0.6299$
	Obtain answer 2.25 and no other in the interval	A1	Accept 0.717π M1A0 for 129.0°
		2	

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Question	Answer	Marks	Guidance
9(iii)	Obtain indefinite integral $\frac{1}{12}\sin 3x + \frac{3}{4}\sin x$	B1 + B1	
	Substitute limits in an indefinite integral of the form $a \sin 3x + b \sin x$, where $ab \neq 0$	M1	$\frac{1}{4} \left[\frac{1}{3} \sin \pi + 3 \sin \frac{\pi}{3} - \frac{1}{3} \sin \frac{\pi}{2} - 3 \sin \frac{\pi}{6} \right]$
	Obtain answer $\frac{1}{24} (9\sqrt{3} - 11)$, or exact equivalent	A1	
	Alternative method for question 9(iii)		
	$\int \cos x (1 - \sin^2 x) dx = \sin x - \frac{1}{3} \sin^3 x (+C)$	B1 + B1	
	Substitute limits in an indefinite integral of the form $a \sin x + b \sin^3 x$ where $ab \neq 0$	M1	$\left(\frac{\sqrt{3}}{2} - \frac{1}{2} - \frac{1}{4}\frac{\sqrt{3}}{2} + \frac{1}{24}\right)$
	Obtain answer $\frac{1}{24} (9\sqrt{3} - 11)$, or exact equivalent	A1	
		4	

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Question	Answer	Marks	Guidance
10(a)	Square $a + ib$ and equate real and imaginary parts to -3 and $-2\sqrt{10}$ respectively	*M1	
	Obtain $a^2 - b^2 = -3$ and $2ab = -2\sqrt{10}$	A1	
	Eliminate one unknown and find an equation in the other	DM1	
	Obtain $a^4 + 3a^2 - 10 = 0$, or $b^4 - 3b^2 - 10 = 0$, or horizontal 3-term equivalent	A1	
	Obtain answers $\pm (\sqrt{2} - \sqrt{5}i)$, or exact equivalent	A1	
		5	
10(b)	Show point representing 3 + i in relatively correct position	B1	
	Show a circle with radius 3 and centre not at the origin	B1	
	Show correct half line from the origin at $\frac{1}{4}\pi$ to the real axis	B1	
	Show horizontal line $y = 2$	B1	
	Shade the correct region	B1	Im(z) i shaded Im(z) = 2 i Re(z) Re(z)
		5	