Question	Answer	Marks	Guidance
1	$\frac{6x}{2}$ , $15 \times \frac{x^2}{4}$	B1 B1	OE In or from a correct expansion. Can be implied by correct equation.
	$\times (4 + ax) \rightarrow 3a + 15 = 3$	M1	2 terms in $x^2$ equated to 3 or $3x^2$ . Condone $x^2$ on one side only.
	a = -4	A1	CAO
		4	

Question	Answer	Marks	Guidance
2	Attempt to find the midpoint M	M1	
	(1, 4)	A1	
	Use a gradient of $\pm^{2}/_{3}$ and <i>their M</i> to find the equation of the line.	M1	
	Equation is $y - 4 = -\frac{2}{3}(x - 1)$	A1	AEF
	Alternative method for question 2		
	Attempt to find the midpoint M	M1	
	(1, 4)	A1	
	Replace 1 in the given equation by c and substitute <i>their M</i>	M1	
	Equation is $y - 4 = -\frac{2}{3}(x - 1)$	A1	AEF
		4	

Question	Answer	Marks	Guidance
3	$(y=) \frac{kx^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} \left( = \frac{k\sqrt{x}}{\frac{1}{2}} \right) (+c)$	B1	OE
	Substitutes both points into an integrated expression with a '+ $c$ ' and solve as far as a value for one variable.	M1	Expect to see $-1 = 2k + c$ and $4 = 4k + c$
	$k = 2\frac{1}{2}$ and $c = -6$	A1	WWW
	$y = 5\sqrt{x} - 6$	A1	OE From correct values of both $k \& c$ and correct integral.
		4	

Question	Answer	Marks	Guidance
4(i)	Arc length $AB = 2r\theta$	B1	
	$\operatorname{Tan} \theta = \frac{AT}{r} \text{ or } \frac{BT}{r} \to AT \text{ or } BT = r \tan \theta$	B1	Accept or $\sqrt{\left(\left(\frac{r}{\cos\theta}\right)^2 - r^2\right)}$ or $\frac{r\sin\theta}{\sin\left(\frac{\pi}{2} - \theta\right)}$ NOT (90 - $\theta$ )
	$P = 2r\theta + 2r\tan\theta$	B1FT	OE, FT for <i>their</i> arc length $+ 2 \times their AT$
		3	

Question	Answer	Marks	Guidance
4(ii)	Area $\triangle AOT = \frac{1}{2} \times 5 \times 5 \tan 1.2$ or Area $AOBT = 2 \times \frac{1}{2} \times 5 \times 5 \tan 1.2$	B1	
	Sector area = $\frac{1}{2} \times 25 \times 2.4$ (or 1.2)	*M1	Use of $\frac{1}{2}r^2\theta$ with $\theta = 1.2$ or 2.4.
	Shaded area = 2 triangles – sector	DM1	Subtraction of sector, using 2.4 where appropriate, from 2 triangles
	Area = $34.3  (\text{cm}^2)$	A1	AWRT
	Alternative method for question 4(ii)		
	Area of $\triangle ABT = \frac{1}{2} \times (5 \times \tan 1.2)^2 \times \sin(\pi - 2.4) \ (= 55.86)$	B1	
	Segment area = $\frac{1}{2} \times 25 \times (2.4 - \sin 2.4) (= 21.56)$	*M1	Use of $\frac{1}{2}r^2(\theta - \sin \theta)$ with $\theta = 1.2$ or 2.4
	Shaded area = triangle – segment	DM1	Subtraction of segment from $\triangle ABT$ , using 2.4 where appropriate.
	Area = $34.3  (\text{cm}^2)$	A1	AWRT
		4	

Question	Answer	Marks	Guidance
5(i)	Use of Pythagoras $\rightarrow r^2 = 15^2 - h^2$	M1	
	$V = \frac{1}{3}\pi(225 - h^2) \times h \to \frac{1}{3}\pi(225h - h^3)$	A1	AG WWW e.g. sight of $r = 15 - h$ gets A0.
		2	

Question	Answer	Marks	Guidance
5(ii)	$\left(\frac{\mathrm{d}v}{\mathrm{d}h}\right) = \frac{\pi}{3} \left(225 - 3h^2\right)$	B1	
	Their $\frac{\mathrm{d}v}{\mathrm{d}h} = 0$	M1	Differentiates, sets <i>their</i> differential to 0 and attempts to solve at least as far as $h^2 \neq 0$ .
	$(h =) \sqrt{75}, 5\sqrt{3} \text{ or AWRT 8.66}$	A1	Ignore $-\sqrt{75}$ OE and ISW for both A marks
	$\frac{\mathrm{d}^2 h}{\mathrm{d}h^2} = \frac{\pi}{3} \ (-6h) \ (\rightarrow -\mathrm{ve})$	M1	Differentiates for a second time and considers the sign of the second differential or any other valid complete method.
	→ Maximum	A1FT	Correct conclusion from correct 2nd differential, value for $h$ not required, or any other valid complete method. FT for <i>their</i> $h$ , if used, as long as it is positive.
			SC Omission of $\pi$ or $\frac{\pi}{3}$ throughout can score B0M1A1M1A0
		5	

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Question	Answer	Marks	Guidance
6(a)	$(2x + 1) = \tan^{-1}(\frac{1}{3}) (= 0.322 \text{ or } 18.4 \text{ OR} - 0.339 \text{ rad or } 8.7^{\circ})$	*M1	Correct order of operations. Allow degrees.
	Either their $0.322 + \pi$ or $2\pi$ Or their $-0.339 + \frac{\pi}{2}$ or $\pi$	DM1	Must be in radians
	x = 1.23 or $x = 2.80$	A1	AWRT for either correct answer, accept $0.39\pi$ or $0.89\pi$
		A1	For the second answer with no other answers between 0 and 2.8 <b>SC1</b> For both 1.2 and 2.8
		4	
6(b)(i)	$5\cos^2 x - 2$	B1	Allow $a = 5, b = -2$
		1	
6(b)(ii)	-2	B1FT	FT for sight of <i>their b</i>
	3	B1FT	FT for sight of <i>their</i> $a + b$
		2	

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Question	Answer	Marks	Guidance
7(i)	$\left(\overline{PB}\right) = 5\mathbf{i} + 8\mathbf{j} - 5\mathbf{k}$	B2,1,0	B2 all correct, B1 for two correct components.
	$\left(\overline{PQ}\right) = 4\mathbf{i} + 8\mathbf{j} + 5\mathbf{k}$	B2,1,0	B2 all correct, B1 for two correct components.
			Accept column vectors. SC B1 for each vector if all components multiplied by -1.
		4	
7(ii)	(Length of $PB =$ ) $\sqrt{(5^2 + 8^2 + 5^2)} = (\sqrt{114} \approx 10.7)$	M1	Evaluation of both lengths. Other valid complete comparisons can be accepted.
	(Length of $PQ = \sqrt{(4^2 + 8^2 + 5^2)} = (\sqrt{105} \approx 10.2)$		
	P is nearer to $Q$ .	A1	WWW
		2	
7(iii)	$\left(\overrightarrow{PB}.\overrightarrow{PQ}\right) = 20 + 64 - 25$	M1	Use of $x_1x_2 + y_1y_2 + z_1z_2$ on <i>their</i> $\overrightarrow{PB}$ and $\overrightarrow{PQ}$
	$(Their\sqrt{114})(their\sqrt{105})\cos BPQ = (their 59)$	M1	All elements present and in correct places.
	$BPQ = 57.4(^{\circ}) \text{ or } 1.00 \text{ (rad)}$	A1	AWRT Calculating the obtuse angle and then subtracting gets A0.
		3	

Question	Answer	Marks	Guidance
8(a)(i)	21st term = $13 + 20 \times 1.2 = 37$ (km)	B1	
		1	

Question	Answer	Marks	Guidance
8(a)(ii)	$S_{21} = \frac{1}{2} \times 21 \times (26 + 20 \times 1.2) \text{ or } \frac{1}{2} \times 21 \times (13 + their 37)$	M1	A correct sum formula used with correct values for $a$ , $d$ and $n$ .
	525 (km)	A1	
		2	
8(b)(i)	$\frac{x-3}{x} = \frac{x-5}{x-3}$ oe (or use of <i>a</i> , <i>ar</i> and <i>ar</i> <sup>2</sup> )	M1	Any valid method to obtain an equation in one variable.
	(a = or x =) 9	A1	
		2	
8(b)(ii)	$r = \left(\frac{x-3}{x}\right)$ or $\left(\frac{x-5}{x-3}\right)$ or $\sqrt{\frac{x-5}{x}} = \frac{2}{3}$ . Fourth term = $9 \times (\frac{2}{3})^3$	M1	Any valid method to find <i>r</i> and the fourth term with <i>their a</i> & <i>r</i> .
	2 <sup>2</sup> / <sub>3</sub> or 2.67	A1	OE, AWRT
		2	
8(b)(iii)	$S\infty = \frac{a}{1-r} = \frac{9}{1-\frac{2}{3}}$	M1	Correct formula and using <i>their</i> 'r' and 'a', with $ r  < 1$ , to obtain a numerical answer.
	27 or 27.0	A1	AWRT
		2	

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Question	Answer	Marks	Guidance
9(i)	$f(x) = g(x) \rightarrow 2x^2 + 6x + 1 + k \ (= 0)$	*M1	Forms a quadratic with all terms on same side.
	Use of $b^2 = 4ac$	DM1	Uses the discriminant $= 0$ .
	$(k =) 3\frac{1}{2}$	A1	OE, WWW
	Alternative method for question 9(i)	·	
	$4x + 8 = 2  (\rightarrow x = -1\frac{1}{2})$	*M1	Differentiating, equating gradients and solving to give $x =$
	Substitutes <i>their x</i> value into either $2x^2 + 6x + 1 + k = 0$ OR into the curve to find $y\left(=\frac{-13}{2}\right)$ then both values into the line.	DM1	Substituting appropriately for <i>their x</i> and proceeding to find a value of <i>k</i> .
	$(k =) 3^{1/2}$	A1	OE, WWW
		3	
9(ii)	$2x^2 + 6x - 8 (< 0)$	M1	Forms a quadratic with all terms on same side
	- 4 and 1	A1	
	-4 < x < 1	A1	CAO
		3	
9(iii)	$(g^{-1}(x)) = \frac{x-1}{2}$	B1	Needs to be in terms of <i>x</i> .
	$(g^{-1}f(x)) = \frac{2x^2 + 8x + 1 - 1}{2} = 0 \longrightarrow (2x^2 + 8x = 0) \longrightarrow x =$	M1	Substitutes f into $g^{-1}$ and attempts to solve it = 0 as far as $x =$
	0, -4	A1	CAO
		3	

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Question	Answer	Marks	Guidance
9(iv)	$2(x+2)^2-7$	B1 B1	or $a = +2, b = -7$
	(Least value of $f(x)$ or $y = -7$ or $\ge -7$	B1FT	FT for <i>their b</i> from a correct form of the expression.
		3	

Question	Answer	Marks	Guidance
10(i)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \left[0\right] + \left[(2x+1)^{-3}\right] \times \left[+16\right]$	B2,1,0	OE. Full marks for 3 correct components. Withhold one mark for each error or omission.
	$\int y dx = \left[x\right] + \left[(2x+1)^{-1}\right] \times \left[+2\right] (+c)$	B2,1,0	OE. Full marks for 3 correct components. Withhold one mark for each error or omission.
		4	
10(ii)	At <i>A</i> , $x = \frac{1}{2}$ .	B1	Ignore extra answer $x = -1.5$
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 2 \rightarrow \text{Gradient of normal} \left(=-\frac{1}{2}\right)$	*M1	With <i>their</i> positive value of x at A and <i>their</i> $\frac{dy}{dx}$ , uses $m_1m_2 = -1$
	Equation of normal: $y - 0 = -\frac{1}{2}(x - \frac{1}{2})$ or $y - 0 = -\frac{1}{2}(0 - \frac{1}{2})$ or $0 = -\frac{1}{2} \times \frac{1}{2} + c$	DM1	Use of <i>their x</i> at <i>A</i> and <i>their</i> normal gradient.
	<i>B</i> (0, <sup>1</sup> ⁄ <sub>4</sub> )	A1	
		4	

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Question	Answer	Marks	Guidance	
10(iii)	$\int_{0}^{\frac{1}{2}} 1 - \frac{4}{(2x+1)^{2}} (dx)$	*M1	$\int y  dx$ SOI with 0 and <i>their</i> positive <i>x</i> coordinate of <i>A</i> .	
	$[\frac{1}{2} + 1] - [0 + 2] = (-\frac{1}{2})$	DM1	Substitutes both 0 and <i>their</i> $\frac{1}{2}$ into <i>their</i> $\int y dx$ and subtracts.	
	Area of triangle above x-axis = $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{4} \left( = \frac{1}{16} \right)$	B1		
	Total area of shaded region = $\frac{9}{16}$	A1	OE (including AWRT 0.563)	
	Alternative method for question 10(iii)			
	$\int_{-3}^{0} \frac{1}{(1-y)^{\frac{1}{2}}} -\frac{1}{2} (dy)$	*M1	$\int x  dy$ SOI. Where x is of the form $k \left(1-y\right)^{-\frac{1}{2}} + c$ with 0 and <i>their</i> negative y intercept of curve.	
	$\left[-2\right] - \left[-4 + \frac{3}{2}\right] = \binom{1}{2}$	DM1	Substitutes both 0 and <i>their</i> $-3$ into <i>their</i> $\int x dy$ and subtracts.	
	Area of triangle above x-axis = $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{4} \left( = \frac{1}{16} \right)$	B1		
	Total area of shaded region = $\frac{9}{16}$	A1	OE (including AWRT 0.563)	

Question	Answer	Marks	Guidance	
	Alternative method for question 10(iii)			
	$\int_{0}^{\frac{1}{2}} -\frac{1}{2}x + \frac{1}{4} - y  \mathrm{d}x$	*M1	$\int$ ( <i>their</i> normal curve) with 0 and <i>their</i> positive <i>x</i> coordinate of A.	
	Curve $[\frac{1}{2} + 1] - [0 + 2] = (-\frac{1}{2})$	DM1	Substitutes both 0 and <i>their</i> $\frac{1}{2}$ into <i>their</i> $\int y dx$ and subtracts.	
	$\int_{0}^{\frac{1}{2}} -\frac{1}{2}x + \frac{1}{4}dx = \frac{-x^{2}}{4} + \frac{x}{4} = \left[\frac{-1}{16} + \frac{1}{8}\right] - \left[0\right] \left(=\frac{1}{16}\right)$	B1	Substitutes both 0 and 1/2 into the correct integral and subtracts.	
	Total area of shaded region = $\frac{9}{16}$	A1	OE (including AWRT 0.563)	
		4		