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1	<p>Solve for 3^x and obtain $3^x = \frac{18}{7}$</p> <p>Use correct method for solving an equation of the form $3^x = a$, where $a > 0$</p> <p>Obtain answer $x = 0.860$ 3 d.p. only</p>	<p>B1</p> <p>M1</p> <p>A1</p>	[3]
2	<p>State correct unsimplified first two terms of the expansion of $(1 + 2x)^{-\frac{3}{2}}$, e.g. $1 + (-\frac{3}{2})(2x)$</p> <p>State correct unsimplified term in x^2, e.g. $(-\frac{3}{2})(-\frac{3}{2} - 1)(2x)^2 / 2!$</p> <p>Obtain sufficient terms of the product of $(2 - x)$ and the expansion up to the term in x^2</p> <p>Obtain final answer $2 - 7x + 18x^2$ Do not ISW</p>	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p>	[4]
3	<p><i>EITHER:</i> Correctly restate the equation in terms of $\sin \theta$ and $\cos \theta$</p> <p>Correct method to obtain a horizontal equation in $\sin \theta$</p> <p>Reduce the equation to a correct quadratic in any form, e.g. $3\sin^2 \theta - \sin \theta - 2 = 0$</p> <p>Solve a three-term quadratic for $\sin \theta$</p> <p>Obtain final answer $\theta = -41.8^\circ$ only</p> <p>[Ignore answers outside the given interval.]</p> <p><i>OR 1:</i> Square both sides of the equation and use $1 + \tan^2 \theta = \sec^2 \theta$</p> <p>Correct method to obtain a horizontal equation in $\sin \theta$</p> <p>Reduce the equation to a correct quadratic in any form, e.g. $9\sin^2 \theta - 6\sin \theta - 8 = 0$</p> <p>Solve a three-term quadratic for $\sin \theta$</p> <p>Obtain final answer $\theta = -41.8^\circ$ only</p> <p><i>OR 2:</i> Multiply through by $(\sec \theta + \tan \theta)$</p> <p>Use $\sec^2 \theta - \tan^2 \theta = 1$</p> <p>Obtain $1 = 3 + 3\sin \theta$</p> <p>Solve for $\sin \theta$</p> <p>Obtain final answer $\theta = -41.8^\circ$ only</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>B1</p> <p>A1</p> <p>M1</p> <p>A1</p>	[5]

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4	<p><i>EITHER:</i> <i>EITHER:</i> State $2xy + x^2 \frac{dy}{dx}$, or equivalent, as derivative of $x^2 y$</p> <p>State $6y^2 + 12xy \frac{dy}{dx}$, or equivalent, as derivative of $6xy^2$</p> <p><i>OR:</i> Differentiating LHS using correct product rule, state term $xy(1 - 6 \frac{dy}{dx})$, or equivalent</p> <p>State term $(y + x \frac{dy}{dx})(x - 6y)$, or equivalent</p> <p>Equate attempted derivative of LHS to zero and set $\frac{dy}{dx}$ equal to zero</p> <p>Obtain a horizontal equation, e.g. $6y^2 - 2xy = 0$ (from correct work only)</p> <p>Explicitly reject $y = 0$ as a possibility $py^2 - qxy = 0$</p> <p>Obtain an equation in x or y</p> <p>Obtain answer $(-3a, -a)$</p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>M1*</p> <p>A1</p> <p>A1</p> <p>DM1</p> <p>A1</p>	[7]
	<p><i>OR:</i> Rearrange to $y = \frac{9a^3}{x(x - 6y)}$ and use correct quotient rule to obtain $-\frac{9a^3}{x^2(x - 6y)^2} \times \dots$</p> <p>State term $(x - 6y) + x(1 - 6y')$, or equivalent</p> <p>Justify division by $x(x - 6y)$</p> <p>Set $\frac{dy}{dx}$ equal to zero</p> <p>Obtain a horizontal equation, e.g. $6y^2 - 2xy = 0$ (from correct work only)</p> <p>Obtain an equation in x or y</p> <p>Obtain answer $(-3a, -a)$</p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>M1*</p> <p>A1</p> <p>DM1</p> <p>A1</p>	
5 (i)	<p><i>EITHER:</i> Use $\tan 2A$ formula to express LHS in terms of $\tan \theta$</p> <p>Express as a single fraction in any correct form</p> <p>Use Pythagoras or $\cos 2A$ formula</p> <p>Obtain the given result correctly</p> <p><i>OR:</i> Express LHS in terms of $\sin 2\theta$, $\cos 2\theta$, $\sin \theta$ and $\cos \theta$</p> <p>Express as a single fraction in any correct form</p> <p>Use Pythagoras or $\cos 2A$ formula or $\sin(A - B)$ formula</p> <p>Obtain the given result correctly</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p>	[4]
(ii)	<p>Integrate and obtain a term of the form $a \ln(\cos 2\theta)$ or $b \ln(\cos \theta)$ (or secant equivalents)</p> <p>Obtain integral $-\frac{1}{2} \ln(\cos 2\theta) + \ln(\cos \theta)$, or equivalent</p> <p>Substitute limits correctly (expect to see use of <u>both</u> limits)</p> <p>Obtain the given answer following full and correct working</p>	<p>M1*</p> <p>A1</p> <p>DM1</p> <p>A1</p>	[4]

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6	(i)	Make recognizable sketch of a relevant graph Sketch the other relevant graph and justify the given statement	B1 B1	[2]
	(ii)	Use calculations to consider the value of a relevant expression at $x = 1.4$ and $x = 1.6$, or the values of relevant expressions at $x = 1.4$ and $x = 1.6$ Complete the argument correctly with correct calculated values	M1 A1	[2]
	(iii)	State $x = 2 \sin^{-1} \left(\frac{3}{x+3} \right)$ Rearrange this in the form $\operatorname{cosec} \frac{1}{2}x = \frac{1}{3}x + 1$ If working in reverse, need $\sin \frac{x}{2} = \left(\frac{3}{x+3} \right)$ for first B1	B1 B1	[2]
	(iv)	Use the iterative formula correctly at least once Obtain final answer 1.471 Show sufficient iterations to 5 d.p. to justify 1.471 to 3 d.p., or show there is a sign change in the interval (1.4705, 1.4715)	M1 A1 A1	[3]
7	(i)	Use the correct product rule Obtain correct derivative in any form, e.g. $(2-2x)e^{\frac{1}{2}x} + \frac{1}{2}(2x-x^2)e^{\frac{1}{2}x}$ Equate derivative to zero and solve for x Obtain $x = \sqrt{5} - 1$ only	M1 A1 M1 A1	[4]
	(ii)	Integrate by parts and reach $a(2x-x^2)e^{\frac{1}{2}x} + b \int (2-2x)e^{\frac{1}{2}x} dx$ Obtain $2e^{\frac{1}{2}x}(2x-x^2) - 2 \int (2-2x)e^{\frac{1}{2}x} dx$, or equivalent Complete the integration correctly, obtaining $(12x-2x^2-24)e^{\frac{1}{2}x}$, or equivalent Use limits $x = 0, x = 2$ correctly having integrated by parts twice Obtain answer $24 - 8e$, or <u>exact</u> simplified equivalent	M1* A1 A1 DM1 A1	[5]

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8	(i)	State or imply a correct normal vector to either plane, e.g. $3\mathbf{i} + \mathbf{j} - \mathbf{k}$ or $\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ Use correct method to calculate their scalar product Show value is zero and planes are perpendicular	B1 M1 A1	[3]
	(ii)	<p><i>EITHER:</i> Carry out a complete strategy for finding a point on l the line of intersection Obtain such a point, e.g. $(0, 7, 5)$, $(1, 0, 1)$, $(5/4, -7/4, 0)$ <i>EITHER:</i> State two equations for a direction vector $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ for l, e.g. $3a + b - c = 0$ and $a - b + 2c = 0$ Solve for one ratio, e.g. $a : b$ Obtain $a : b : c = 1 : -7 : -4$, or equivalent State a correct answer, e.g. $\mathbf{r} = 7\mathbf{j} + 5\mathbf{k} + \lambda(\mathbf{i} - 7\mathbf{j} - 4\mathbf{k})$</p> <p><i>OR1:</i> Obtain a second point on l, e.g. $(1, 0, 1)$ Subtract vectors and obtain a direction vector for l Obtain $-\mathbf{i} + 7\mathbf{j} + 4\mathbf{k}$, or equivalent State a correct answer, e.g. $\mathbf{r} = \mathbf{i} + \mathbf{k} + \lambda(-\mathbf{i} + 7\mathbf{j} + 4\mathbf{k})$</p> <p><i>OR2:</i> Attempt to find the vector product of the two normal vectors Obtain two correct components of the product Obtain $\mathbf{i} - 7\mathbf{j} - 4\mathbf{k}$, or equivalent State a correct answer, e.g. $\mathbf{r} = 7\mathbf{j} + 5\mathbf{k} + \lambda(\mathbf{i} - 7\mathbf{j} - 4\mathbf{k})$</p> <p><i>OR1:</i> Express one variable in terms of a second variable Obtain a correct simplified expression, e.g. $y = 7 - 7x$ Express the third variable in terms of the second Obtain a correct simplified expression, e.g. $z = 5 - 4x$ Form a vector equation for the line Obtain a correct equation, e.g. $\mathbf{r} = 7\mathbf{j} + 5\mathbf{k} + \lambda(\mathbf{i} - 7\mathbf{j} - 4\mathbf{k})$</p> <p><i>OR2:</i> Express one variable in terms of a second variable Obtain a correct simplified expression, e.g. $z = 5 - 4x$ Express the same variable in terms of the third Obtain a correct simplified expression e.g. $z = (7 + 4y) / 7$ Form a vector equation for the line Obtain a correct equation, e.g. $\mathbf{r} = \frac{5}{4}\mathbf{i} - \frac{7}{4}\mathbf{j} + \lambda(-\frac{1}{4}\mathbf{i} + \frac{7}{4}\mathbf{j} + \mathbf{k})$</p>	M1 A1 B1 M1 A1 A1✓ B1 M1 A1 A1✓ M1 A1 A1 A1✓ M1 A1 M1 A1 M1 A1✓ M1 A1 M1 A1 M1 A1✓	[6]

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9	(a)	<p>EITHER: Use quadratic formula to solve for w Use $i^2 = -1$ Obtain one of the answers $w = \frac{1}{2i+1}$ and $w = -\frac{5}{2i+1}$ Multiply numerator and denominator of an answer by $-2i + 1$, or equivalent Obtain final answers $\frac{1}{5} - \frac{2}{5}i$ and $-1 + 2i$</p> <p>OR1: Multiply the equation by $1 - 2i$ Use $i^2 = -1$ Obtain $5w^2 + 4w(1 - 2i) - (1 - 2i)^2 = 0$, or equivalent Use quadratic formula or factorise to solve for w Obtain final answers $\frac{1}{5} - \frac{2}{5}i$ and $-1 + 2i$</p> <p>OR2: Substitute $w = x + iy$ and form equations for real and imaginary parts Use $i^2 = -1$ Obtain $(x^2 - y^2) - 4xy + 4x - 1 = 0$ and $2(x^2 - y^2) + 2xy + 4y + 2 = 0$ o.e. Form equation in x only or y only and solve Obtain final answers $\frac{1}{5} - \frac{2}{5}i$ and $-1 + 2i$</p>	<p>M1 M1 A1 M1 A1</p> <p>M1 M1 A1 M1 A1</p> <p>M1 M1 A1 M1 A1</p>	[5]
	(b)	<p>Show a circle with centre $1 + i$ Show a circle with radius 2 Show half-line $\arg z = \frac{1}{4}\pi$ Show half-line $\arg z = -\frac{1}{4}\pi$ Shade the correct region</p>	<p>B1 B1 B1 B1 B1</p>	[5]

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10	(i)	<p>Separate variables correctly and integrate at least one side Integrate and obtain term kt, or equivalent</p> <p>Carry out a relevant method to obtain A and B such that $\frac{1}{x(4-x)} \equiv \frac{A}{x} + \frac{B}{4-x}$, or equivalent</p> <p>Obtain $A = B = \frac{1}{4}$, or equivalent</p> <p>Integrate and obtain terms $\frac{1}{4} \ln x - \frac{1}{4} \ln(4-x)$, or equivalent</p> <p>EITHER: Use a pair of limits in an expression containing $p \ln x$, $q \ln(4-x)$ and rt and evaluate a constant Obtain correct answer in any form, e.g. $\ln x - \ln(4-x) = 4kt - \ln 9$, or $\ln\left(\frac{x}{4-x}\right) = 4kt - 8k$ Use a second pair of limits and determine k Obtain the given exact answer correctly</p> <p>OR: Use both pairs of limits in a definite integral Obtain the given exact answer correctly Substitute k and either pair of limits in an expression containing $p \ln x$, $q \ln(4-x)$ and rt and evaluate a constant Obtain $\ln \frac{x}{4-x} = t \ln 3 - \ln 9$ or equivalent</p>	<p>M1 A1 M1* A1 A1^{1/2} DM1 A1 DM1 A1 M1* A1 DM1 A1</p>	[9]
	(ii)	<p>Substitute $x = 3.6$ and solve for t Obtain answer $t = 4$</p>	<p>M1 A1</p>	[2]