1	$kx^2 - 3x = x - k \Rightarrow kx^2 - 4x + k(=0)$ $(-4)^2 - 4(k)(k)$ soi $k > 2$, $k < -2$ cao Allow $(2, \infty)$ etc. Allow $2<k<-k$	**M1** **M1** **A1**	[3]	Eliminate y and rearrange into 3-term quad $b^2 - 4ac$.
2	$(+/-)20 \times 3^3(x^3)$, $10a^3(x^3)$ soi $-540 + 10a^3 = 100$ oe $a = 4$	**B1B1** **M1** **A1**	[4]	Each term can include x^3 Must have 3 terms and include a^3 and 100
3	$4\sin^2 x = 6\cos^2 x \Rightarrow \tan^2 x = \frac{6}{4}$ or $4\sin^2 x = 6(1 - \sin^2 x)$ [$\tan x = (\pm)1.225$ or $\sin x = (\pm)0.7746$ or $\cos x = (\pm)0.6325$] $x = 50.8$ (Allow 0.886 (rad)) Another angle correct $x = 50.8°, 129.2°, 230.8°, 309.2°$ [0.886, 2.25/6, 4.03, 5.40 (rad)]	**M1** **A1** **A1**√ **A1**	[4]	Or $4(1 - \cos^2 x) = 6\cos^2 x$ Or any other angle correct Ft from 1st angle (Allow radians) All 4 angles correct in degrees
4	$f'(x) = 3x^2 - 6x - 9$ soi Attempt to solve $f'(x) = 0$ or $f'(x) > 0$ or $f'(x) \geqslant 0$ soi $(3)(x-3)(x+1)$ or 3,−1 seen or 3 only seen Least possible value of n is 3. Accept $n = 3$. Accept $n \geqslant 3$	**B1** **M1** **A1** **A1**	[4]	With or without equality/inequality signs Must be in terms of n
5 (i)	$\cos 0.9 = OE / 6$ or = $\sin\left(\frac{\pi}{2} - 0.9\right)$ oe $OE = 6\cos 0.9 = 3.73$ oe AG	**M1** **A1**	[2]	Other methods possible
(ii)	Use of $(2\pi - 1.8)$ or equivalent method Area of large sector $= ½ \times 6^2 \times (2\pi - 1.8)$ oe Area of small sector $½ \times 3.73^2 \times 1.8$ Total area = 80.7(0) + 12.5(2) = 93.2	**M1** **M1** **M1** **A1**	[4]	Expect 4.48 Or $\pi 6^2 - ½ 6^2 1.8$. Expect 80.70 Expect 12.52 Other methods possible
6 (i)	$\frac{2+x}{2} = n \Rightarrow x = 2n - 2$ $\frac{m+y}{2} = -6 \Rightarrow y = -12 - m$	**B1** **B1**	[2]	No MR for (½(2+n), ½(m – 6)) Expect $(2n-2, -12-m)$

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(ii)	Sub *their* x, y into $y = x + 1 \rightarrow -12 - m = 2n - 2 + 1$ $\frac{m+6}{2-n} = -1$ oe Not nested in an equation Eliminate a variable $m = -9,\ n = -1$	**M1*** **B1** **DM1** **A1A1**	[5]	Expect $m + 2n = -11$ Expect $m - n = -8$ Note: other methods possible
7 (i)	**AB.AC** $= 3 - 2 - 1 = 0$ hence perpendicular or 90° **AB.AD** $= 3 + 4 - 7 = 0$ hence perpendicular or 90° **AC.AD** $= 1 - 8 + 7 = 0$ hence perpendicular or 90° AG	**B1** **B1** **B1**	[3]	3 − 2 − 1 or sum of prods etc must be seen Or single statement: mutually perpendicular or 90° seen at least once .
(ii)	Area $ABC = (\frac{1}{2})\sqrt{3^2 + 1^2 + 1^2} \times \sqrt{1^2 + (-2)^2 + (-1)^2}$ $= \frac{1}{2}\sqrt{11} \times \sqrt{6}$ Vol. $= \frac{1}{3} \times$ *their* $\Delta ABC \times \sqrt{1^2 + 4^2 + (-7)^2}$ $= \frac{1}{6}\sqrt{66} \times \sqrt{66} = 11$	**M1** **A1** **M1** **A1**	[4]	Expect $\frac{1}{2}\sqrt{66}$ Not 11.0
8 (i)	$(2x+3)^2 + 1$ Cannot score retrospectively in **(iii)**	**B1B1B1**	[3]	For $a = 2,\ b = 3,\ c = 1$
(ii)	$g(x) = 2x + 3$ cao	**B1**	[1]	In **(ii)**,**(iii)** Allow if from $4\left(x + \frac{3}{2}\right)^2 + 1$
(iii)	$y = (2x+3)^2 + 1 \Rightarrow 2x + 3 = (\pm)\sqrt{y-1}$ or ft from **(i)** $x = (\pm)\frac{1}{2}\sqrt{y-1} - \frac{3}{2}$ or ft from **(i)** $(\text{fg})^{-1}(x) = \frac{1}{2}\sqrt{x-1} - \frac{3}{2}$ cao Note alt. method $\text{g}^{-1}\text{f}^{-1}$ Domain is $(x) > 10$ ALT. method for first 3 marks: Trying to obtain $g^{-1}\left[f^{-1}(x)\right]$ $\text{g}^{-1} = \frac{1}{2}(x-3),\ \text{f}^{-1} = \sqrt{x-1}$ A1 for $\frac{1}{2}\sqrt{x-1} - \frac{3}{2}$	**M1** **M1** **A1** **B1** ***M1** **DM1** **A1**	[4]	Or with *x*/*y* transposed. Or with *x*/*y* transposed Allow sign errors. Must be a function of *x*. Allow *y* = ... Allow (10, ∞), $10 < x < \infty$ etc. but not with *y* or f or g involved. Not $\geqslant 10$ Both required

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9 **(a)**	$\frac{6}{1-r}=\frac{12}{1+r}$ $r=\frac{1}{3}$ $S=9$	**M1** **A1** **A1**	[3]	
(b)	$\frac{13}{2}\left[2\cos\theta+12\sin^2\theta\right]=52$ $2\cos\theta+12(1-\cos^2\theta)=8\rightarrow 6\cos^2\theta-\cos\theta-2(=0)$ $\cos\theta=2/3$ or $-1/2$ soi $\theta=0.841$, 2.09 Dep on previous A1	**M1*** **DM1** **A1** **A1A1**	[5]	Use of correct formula for sum of AP Use $s^2=1-c^2$ & simplify to 3-term quad Accept 0.268π, 2π/3. SRA1 for 48.2°, 120° Extra solutions in range –1
10 **(i)**	at $x=a^2$, $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2}{a^2}+\frac{1}{a^2}$ or $2a^{-2}+a^{-2}\left(=\frac{3}{a^2}\text{ or }3a^{-2}\right)$ $y-3=\frac{3}{a^2}\left(x-a^2\right)$ or $y=\frac{3}{a^2}x+c\rightarrow 3=\frac{3}{a^2}a^2+c$ $y=\frac{3}{a^2}x$ or $3a^{-2}x$ cao	**B1** **M1** **A1**	[3]	$\frac{2}{a^2}+\frac{1}{a^2}$ or $2a^{-2}+a^{-2}$ seen anywhere in (**i**) Through $(a^2,3)$ & with *their* grad as f(a)
(ii)	$(y)=\frac{2}{a}\frac{x^{½}}{½}+\frac{ax^{-½}}{-½}$ $(+c)$ sub $x=a^2$, $y=3$ into $\int \mathrm{d}y/\mathrm{d}x$ $c=1$ $\left(y=\frac{4x^{½}}{a}-2ax^{-½}+1\right)$	**B1B1** **M1** **A1**	[4]	*c* must be present. Expect 3 = 4 – 2 + *c*
(iii)	sub $x=16$, $y=8\rightarrow 8=\frac{4}{a}\times 4-2a\times\frac{1}{4}+1$ $a^2+14a-32(=0)$ $a=2$ $A=(4,3)$, $B=(16,8)$ $AB^2=12^2+5^2\rightarrow AB=13$	***M1** **A1** **A1** **DM1A1**	[5]	Sub into *their y* Allow −16 in addition

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11 (i)	Attempt diffn. and equate to 0 $\frac{dy}{dx} = -k(kx-3)^{-2} + k = 0$	***M1**		Must contain $(kx-3)^{-2}$ + other term(s)
	$(kx-3)^2 = 1$ or $k^3x^2 - 6k^2x + 8k\,(=0)$	**DM1**		Simplify to a quadratic
	$x = \frac{2}{k}$ or $\frac{4}{k}$	***A1*A1**		Legitimately obtained
	$\frac{d^2y}{dx^2} = 2k^2(kx-3)^{-3}$	**B1**√		Ft must contain $Ak^2(kx-3)^{-3}$ where $A>0$
	When $x = \frac{2}{k}$, $\frac{d^2y}{dx^2} = (-2k^2) < 0$ MAX All previous	**DB1**		Convincing alt. methods (values either side) must show which values used & cannot use $x = 3/k$
	When $x = \frac{4}{k}$, $\frac{d^2y}{dx^2} = (2k^2) > 0$ MIN working correct	**DB1**	[7]	
(ii)	$V = (\pi)\int\left[(x-3)^{-1} + (x-3)\right]^2 dx$	***M1**		Attempt to expand y^2 and then integrate
	$= (\pi)\int[(x-3)^{-2} + (x-3)^2 + 2\,]dx$	**A1**		
	$= (\pi)\left[-(x-3)^{-1} + \frac{(x-3)^3}{3}(+2x)\right]$ Condone missing $2x$	**A1**		Or $\left[-(x-3)^{-1} + \frac{x^3}{3} - 3x^2 + 9x + 2x\right]$
	$= (\pi)\left[1 - \frac{1}{3} + 4 - \left(\frac{1}{3} - 9 + 0\right)\right]$	**DM1**		Apply limits 0→2
	$= 40\pi/3$ oe or 41.9	**A1**	[5]	2 missing → $28\pi/3$ scores M1A0A1M1A0

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