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Question	Answer	Marks	Guidance
1	Use law of the logarithm of a product or quotient	M1	
	Use law of the logarithm of power <b>twice</b>	M1	
	Obtain a correct linear equation in x, e.g. $(3-2x)\ln 5 = \ln 4 + x\ln 7$	A1	
	Obtain answer $x = 0.666$	A1	
		4	

Question	Answer	Marks	Guidance
2	Commence integration and reach $ax^2 \sin 2x + b \int x \sin 2x  dx$	M1*	
	Obtain $\frac{1}{2}x^2 \sin 2x - \int x \sin 2x  dx$ , or equivalent	A1	
	Complete the integration and obtain $\frac{1}{2}x^{2}\sin 2x + \frac{1}{2}x\cos 2x - \frac{1}{4}\sin 2x$ , or equivalent	A1	
	Use limits correctly, having integrated twice	DM1	
	Obtain given answer correctly	A1	
		5	

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Question	Answer	Marks	Guidance
3(i)	Use double angle formulae and express entire fraction in terms of $\sin\theta$ and $\cos\theta$	M1	
	Obtain a correct expression	A1	
	Obtain the given answer	A1	
		3	
3(ii)	State integral of the form $\pm \ln \cos \theta$	M1*	
	Use correct limits correctly and insert exact values for the trig ratios	DM1	
	Obtain a correct expression, e.g. $-\ln \frac{1}{\sqrt{2}} + \ln \frac{\sqrt{3}}{2}$	A1	
	Obtain the given answer following full and exact working	A1	
		4	

Question	Answer	Marks	Guidance
4(i)	Use the quotient or product rule	M1	
	Obtain correct derivative in any form	A1	
	Reduce to $-\frac{2e^{-x}}{(1-e^{-x})^2}$ , or equivalent, and explain why this is always negative	A1	
		3	

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Question	Answer	Marks	Guidance
4(ii)	Equate derivative to $-1$ and obtain the given equation	B1	
	State or imply $u^2 - 4u + 1 = 0$ , or equivalent in $e^a$	B1	
	Solve for <i>a</i>	M1	
	Obtain answer $a = \ln(2 + \sqrt{3})$ and no other	A1	
		4	

Question	Answer	Marks	Guidance
5	Separate variables correctly and integrate at least one side	B1	
	Obtain term $\ln(x+1)$	B1	
	Obtain term of the form $a \ln(y^2 + 5)$	M1	
	Obtain term $\frac{1}{2}\ln(y^2+5)$	A1	
	Use $y = 2$ , $x = 0$ to determine a constant, or as limits, in a solution containing terms $a\ln(y^2+5)$ and $b\ln(x+1)$ , where $ab \neq 0$	M1	
	Obtain correct solution in any form	A1	
	Obtain final answer $y^2 = 9(x+1)^2 - 5$	A1	
		7	

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Question	Answer	Marks	Guidance
6(i)	State $b = 3$	B1	
		1	
6(ii)	Commence division by $x - b$ and reach partial quotient $x^3 + kx^2$	M1	
	Obtain quotient $x^3 + x^2 + 3x + 2$	A1	There being no remainder
	Equate quotient to zero and rearrange to make the subject <i>a</i>	M1	
	Obtain the given equation	A1	
		4	
6(iii)	Use the iterative formula $a_{n+1} = -\frac{1}{3}(2 + a_n^2 + a_n^3)$ correctly at least once	M1	
	Obtain final answer –0.715	A1	
	Show sufficient iterations to 5 d.p. to justify -0.715 to 3 d.p., or show there is a sign change in the interval (-0.7145, -0.7155)	A1	
		3	

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Question	Answer	Marks	Guidance
7(i)	Use product rule	M1	
	Obtain correct derivative in any form	A1	
		2	
7(ii)	Equate derivative to zero and use correct $cos(A + B)$ formula	M1	
	Obtain the given equation	A1	
		2	
7(iii)	Use correct method to solve for <i>x</i>	M1	
	Obtain answer, e.g. $x = \frac{1}{12}\pi$	A1	
	Obtain second answer, e.g. $\frac{7}{12}\pi$ , and no other	A1	
		3	

Question	Answer	Marks	Guidance
8(i)	Multiply numerator and denominator by $1 + \sqrt{3}i$ , or equivalent	M1	
	$4i - 4\sqrt{3}$ and $3 + 1$	A1	
	Obtain final answer $-\sqrt{3} + i$	A1	
		3	

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Question	Answer	Marks	Guidance
8(ii)	State that the modulus of $u$ is 2	B1	
	State that the argument of <i>u</i> is $\frac{5}{6}\pi$ (or 150°)	B1	
		2	
8(iii)	Show a circle with centre the origin and radius 2	B1	
	Show <i>u</i> in a relatively correct position	B1	FT
	Show the perpendicular bisector of the line joining <i>u</i> and the origin	B1	FT
	Shade the correct region	B1	
		4	

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Question	Answer	Marks	Guidance
9(i)	State or imply the form $\frac{A}{3+x} + \frac{B}{1-x} + \frac{C}{(1-x)^2}$	B1	
	Use a correct method for finding a constant	M1	
	Obtain one of $A = -3$ , $B = -1$ , $C = 2$	A1	
	Obtain a second value	A1	
	Obtain the third value	A1	Mark the form $\frac{A}{3+x} + \frac{Dx+E}{(1-x)^2}$ , where $A = -3$ ,
			D = 1 and $E = 1$ , B1M1A1A1A1 as above.
		5	
9(ii)	Use a correct method to find the first two terms of the expansion of $(3 + x)^{-1}$ , $(1 + \frac{1}{3}x)^{-1}$ , $(1 - x)^{-1}$ or $(1 - x)^{-2}$	M1	
	Obtain correct unsimplified expansions up to the term in $x^3$ of each partial fraction	A1	FT on A
		A1	FT on B
		A1	FT on C
	Obtain final answer $\frac{10}{3}x + \frac{44}{9}x^2 + \frac{190}{27}x^3$	A1	For the A, D, E form of fractions give M1A1ftA1ft for the expanded partial fractions, then, if $D \neq 0$ , M1 for multiplying out fully, and A1 for the final answer.
		5	

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Question	Answer	Marks	Guidance
10(i)	Find $\overrightarrow{PQ}$ for a general point Q on l, e.g. $-3\mathbf{i}+6\mathbf{k}+\mu(2\mathbf{i}-\mathbf{j}-2\mathbf{k})$	B1	
	Calculate scalar product of $\overrightarrow{PQ}$ and a direction vector for <i>l</i> and equate the result to zero	M1	
	Solve for $\mu$ and obtain $\mu = 2$	A1	
	Carry out a complete method for finding the length of $\overrightarrow{PQ}$	M1	
	Obtain answer 3	A1	
	Alternative method for question 10(i)		
	Calling the point (1, 2, 3) A, state $\overrightarrow{AP}$ (or $\overrightarrow{PA}$ ) in component form, e.g. $3\mathbf{i} - 6\mathbf{k}$	B1	
	Use a scalar product with a direction vector for <i>l</i> to find the projection of $\overrightarrow{AP}$ (or $\overrightarrow{PA}$ ) on <i>l</i>	M1	
	Obtain correct answer in any form, e.g. $\frac{18}{\sqrt{9}}$	A1	
	Use Pythagoras to find the perpendicular	M1	
	Obtain answer 3	A1	

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Question	Answer	Marks	Guidance
10(i)	Alternative method for question 10(i)		
	State $\overline{AP}$ (or $\overrightarrow{PA}$ ) in component form	B1	
	Calculate a vector product with a direction vector for <i>l</i>	M1	
	Obtain correct answer, e.g. $6\mathbf{i} - 6\mathbf{j} - 3\mathbf{k}$	A1	
	Divide modulus of the product by that of the direction vector	M1	
	Obtain answer 3	A1	
		5	

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Question	Answer	Marks	Guidance	
10(ii)	Substitute coordinates of a general point of <i>l</i> in the plane equation and equate constant terms	M1		
	Obtain a correct equation, e.g. $a + 2b + 6 = 13$	A1		
	Equate the coefficient of $\mu$ to zero	M1		
	Obtain a correct equation, e.g. $2a - b - 4 = 0$	A1		
	Substitute (1, 2, 3) in the plane equation	M1		
	Obtain a correct equation, e.g. $a + 2b + 6 = 13$	A1		
	Alternative method for question 10(ii)			
	Find a second point on <i>l</i> and obtain an equation in <i>a</i> and/or <i>b</i>	M1		
	Obtain a correct equation, e.g. $5a - 2 = 13$	A1		
	Equate scalar product of a direction vector for $l$ and a vector normal for the plane to zero	M1		
	Obtain a correct equation, e.g. $2a - b - 4 = 0$	A1		
	Solve for <i>a</i> or for <i>b</i>	M1		
	Obtain $a = 3$ and $b = 2$	A1		
		6		