Question	Answer	Marks	Guidance
1	State unsimplified term in x^2, or its coefficient in the expansion of $(1+3x)^{\frac{1}{3}}$ $\left(\dfrac{\frac{1}{3}\times\frac{-2}{3}}{2}(3x)^2\right)$	**B1**	Symbolic binomial coefficients are not sufficient for the B marks
	State unsimplified term in x^3, or its coefficient in the expansion of $(1+3x)^{\frac{1}{3}}$ $\left(\dfrac{\frac{1}{3}\times\frac{-2}{3}\times\frac{-5}{3}}{6}(3x)^3\right)$	**B1**	
	Multiply by $(3-x)$ to give 2 terms in x^3, or their coefficients	**M1**	$\left(3\times\frac{10}{6}+1\right)$ Ignore errors in terms other than x^3 $3\times x^3\text{coeff}-x^2\text{coeff}$ and no other term in x^3
	Obtain answer 6	**A1**	Not $6x^3$
		4	

Question	**Answer**	**Marks**	**Guidance**
2	State or imply $u^2 - u - 12 (= 0)$, or equivalent in 3^x	**B1**	Need to be convinced they know $3^{2x} = \left(3^x\right)^2$
	Solve for *u*, or for 3^x, and obtain root 4	**B1**	
	Use a correct method to solve an equation of the form $3^x = a$ where a >0	**M1**	Need to see evidence of method. Do not penalise an attempt to use the negative root as well. e.g. $x \ln 3 = \ln a, \quad x = \log_3 a$ If seen, accept solution of straight forward cases such as $3^x = 3$, $x = 1$ without working
	Obtain final answer $x = 1.26$ **only**	**A1**	The Q asks for 2 dp
		4	

Question	Answer	Marks	Guidance
3	Use correct trig formulae to obtain an equation in tan θ or equivalent (e.g all in sin θ or all in cos θ)	***M1**	$\frac{1-\tan^2\theta}{2\tan\theta}=2\tan\theta$. Allow $\frac{\cot^2\theta-1}{2\cot\theta}=\frac{2}{\cot\theta}$
	Obtain a correct simplified equation	**A1**	$5\tan^2\theta=1$ or $\sin^2\theta=\frac{1}{6}$ or $\cos^2\theta=\frac{5}{6}$
	Solve for θ	**DM1**	Dependent on the first M1
	Obtain answer 24.1° (or 155.9°)	**A1**	One correct in range to at least 3 sf
	Obtain second answer	**A1**	**FT** $180°-$*their* $24.1°$ and no others in range. Correct to at least 3 sf. Accept $156°$ but not 156.0 Ignore values outside range If working in $\tan\theta$ or $\cos\theta$ need to be considering both square roots to score the second A1 Mark 0.421, 2.72 as a MR, so A0A1
		5	

Question	Answer	Marks	Guidance
4	Use correct quotient rule	**M1**	Allow use of correct product rule on $x\times(1+\ln x)^{-1}$
	Obtain correct derivative in any form	**A1**	$\frac{dy}{dx}=\frac{(1+\ln x)-x\times\frac{1}{x}}{(1+\ln x)^2}=\left(\frac{1}{1+\ln x}-\frac{1}{(1+\ln x)^2}\right)$
	Equate derivative to $\frac{1}{4}$ and obtain a quadratic in $\ln x$ or $(1+\ln x)$	**M1**	Horizontal form. Accept $\ln x=\frac{1}{4}(1+\ln x)^2$
	Reduce to $(\ln x)^2-2\ln x+1=0$	**A1**	or 3-term equivalent. Condone $\ln x^2$ if later used correctly
	Solve a 3-term quadratic in $\ln x$ for x	**M1**	Must see working if solving incorrect quadratic
	Obtain answer $x = \mathrm{e}$	**A1**	Accept e^1
	Obtain answer $y=\frac{1}{2}\mathrm{e}$	**A1**	Exact only with no decimals seen before the exact value. Accept $\frac{\mathrm{e}^1}{2}$ but not $\frac{\mathrm{e}}{1+\ln \mathrm{e}}$
		7	

Question	Answer	Marks	Guidance
5(i)	State answer $-1-\sqrt{3}\mathrm{i}$	**B1**	If $-\frac{1}{2}$ given as well at this point, still just B1
		1	

Question	Answer	Marks	Guidance
5(ii)	Substitute $x = -1+\sqrt{3}\text{i}$ in the equation and attempt expansions of x^2 and x^3	M1	Need to see sufficient working to be convinced that a calculator has not been used.
	Use $\text{i}^2 = -1$ correctly at least once	M1	Allow for relevant use at any point in the solution
	Obtain $k = 2$	A1	
	Carry out a complete method for finding a quadratic factor with zeros $-1+\sqrt{3}\text{i}$ and $-1-\sqrt{3}\text{i}$	M1	Could use factor theorem from this point. Need to see working. M1 for correct testing of correct root or allow M1 for three unsuccessful valid attempts.
	Obtain $x^2 + 2x + 4$	A1	Using factor theorem, obtain $\text{f}\left(-\frac{1}{2}\right) = 0$
	Obtain root $x = -\frac{1}{2}$, or equivalent, *via* division or inspection	A1	Final answer

Question	Answer	Marks	Guidance
5(ii)	**Alternative method 1**		
	Carry out a complete method for finding a quadratic factor with zeros $-1+\sqrt{3}\mathrm{i}$ and $-1-\sqrt{3}\mathrm{i}$ (multiplying two linear factors or using sum and product of roots)	**M1**	Need to see sufficient working to be convinced that a calculator has not been used.
	Use $\mathrm{i}^2=-1$ correctly at least once	**M1**	Allow for relevant use at any point in the solution
	Obtain x^2+2x+4	**A1**	Allow M1A0 for x^2+2x+3
	Obtain linear factor $kx+1$ and compare coefficients of x or x^2 and solve for k	**M1**	Can find the factor by inspection or by long division Must get to zero remainder
	Obtain $k=2$	**A1**	
	Obtain root $x=-\frac{1}{2}$	**A1**	Final answer
			Note: Verification that $x=-\frac{1}{2}$ is a root is worth no marks without a clear demonstration of how the root was obtained

Question	Answer	Marks	Guidance
5(ii)	**Alternative method 2**		
	Use equation for sum of roots of cubic and use equation for product of roots of cubic	**M1**	
	Use $\mathrm{i}^2 = -1$ correctly at least once	**M1**	Allow for relevant use at any point in the solution
	Obtain $-\frac{5}{k} = -2 + \gamma, \quad -\frac{4}{k} = 4\gamma$	**A1**	
	Solve simultaneous equations for k and γ	**M1**	
	Obtain $k = 2$	**A1**	
	Obtain root $\gamma = -\frac{1}{2}$	**A1**	Final answer
		6	

Question	Answer	Marks	Guidance
6(i)	Correct use of trigonometry to obtain $AB = 2r\cos x$	**B1**	**AG**
		1	

Question	Answer	Marks	Guidance
6(ii)	Use correct method for finding the area of the sector and the semicircle and form an equation in x	M1	$\frac{1}{2}\times\frac{1}{2}\pi r^2=\frac{1}{2}(2r\cos x)^2 2x$
	Obtain $x=\cos^{-1}\sqrt{\frac{\pi}{16x}}$ correctly **AG**	A1	Via correct simplification e.g. from $\cos^2 x=\frac{\pi}{16x}$
		2	
6(iii)	Calculate values of a relevant expression or pair of expressions at x = 1 and x = 1.5 Must be working in radians	M1	e.g. $\begin{matrix} x=1 & 1\to 1.11 \\ x=1.5 & 1.5\to 1.20 \end{matrix}$ Accept $\begin{matrix} \mathrm{f}(1)=1.11 \\ \mathrm{f}(1.5)=1.20 \end{matrix}$ $\mathrm{f}(x)=x-\cos^{-1}\sqrt{\frac{\pi}{16x}}$: $\mathrm{f}(1)=-0.111.,\mathrm{f}(1.5)=0.3..$ $\mathrm{f}(x)=\cos x-\sqrt{\frac{\pi}{16x}}$: $\mathrm{f}(1)=0.097.,\mathrm{f}(1.5)=-0.291.$ For $16x\cos^2 x-\pi$ $\mathrm{f}(1)=1.529..,\mathrm{f}(1.5)=-3.02..$ Must find values. M1 if at least one value correct
	Correct values and complete the argument correctly	A1	
		2	

Question	Answer	Marks	Guidance
6(iv)	Use $x_{n+1} = \cos^{-1}\sqrt{\left(\frac{\pi}{16x_n}\right)}$ correctly at least twice Must be working in radians	**M1**	1, 1.11173, 1.13707, 1.14225, 1.14329, 1.14349, 1.14354, 1.14354 1.25, 1.16328, 1.14742, 1.14432, 1.14370 1.5, 1.20060, 1.15447, 1.14570, 1.14397, 1.14363
	Obtain final answer 1.144	**A1**	
	Show sufficient iterations to at least 5 d.p. to justify 1.144 to 3 d.p. or show there is a sign change in the interval (1.1435, 1.1445)	**A1**	
		3	

Question	Answer	Marks	Guidance
7(i)	Separate variables correctly and attempt integration of at least one side	**B1**	$\int e^{-y}\,dy = \int xe^{x}\,dx$
	Obtain term $-e^{-y}$	**B1**	B0B1 is possible
	Commence integration by parts and reach $xe^{x} \pm \int e^{x}\,dx$	**M1**	B0B0M1A1 is possible
	Obtain $xe^{x} - e^{x}$	**A1**	or equivalent
			B1B1M1A1 is available if there is no constant of integration
	Use $x = 0$, $y = 0$ to evaluate a constant, or as limits in a definite integral, in a solution with terms ae^{-y}, bxe^{x} and ce^{x}, where $abc \neq 0$	**M1**	Must see this step
	Obtain correct solution in any form	**A1**	e.g. $e^{-y} = e^{x} - xe^{x}$
	Rearrange as $y = -\ln(1-x) - x$	**A1**	or equivalent e.g. $y = \ln\dfrac{1}{e^{x}(1-x)}$ ISW
		7	
7(ii)	Justify the given statement	**B1**	e.g. require $1-x>0$ for the ln term to exist, hence $x<1$ Must be considering the range of values of *x*, and must be relevant to *their y* involving $\ln(1-x)$
		1	

Question	Answer	Marks	Guidance
8(i)	State or imply the form $\dfrac{A}{2x+1}+\dfrac{B}{2x+3}+\dfrac{C}{(2x+3)^2}$	**B1**	
	Use a correct method to find a constant	**M1**	
	Obtain the values $A = 1$, $B = -1$, $C = 3$	**A1 A1 A1**	
	[Mark the form $\dfrac{A}{2x+1}+\dfrac{Dx+E}{(2x+3)^2}$, where $A = 1$, $D = -2$ and $E = 0$, B1M1A1A1A1 as above.]		Full marks for the three correct constants – do not actually need to see the partial fractions
		5	
8(ii)	Integrate and obtain terms $\frac{1}{2}\ln(2x+1)-\frac{1}{2}\ln(2x+3)-\frac{3}{2(2x+3)}$ [Correct integration of the A, D, E form of fractions gives $\frac{1}{2}\ln(2x+1)+\frac{x}{2x+3}-\frac{1}{2}\ln(2x+3)$ if integration by parts is used for the second partial fraction.]	**B1 B1 B1**	**FT** on A, B and C.
	Substitute limits correctly in an integral with terms $a\ln(2x+1)$, $b\ln(2x+3)$ and $c/(2x+3)$, where $abc \neq 0$ If using alternative form: $cx/(2x+3)$	**M1**	value for upper limit – value for lower limit 1 slip in substituting can still score M1 Condone omission of $\ln(1)$
	Obtain the **given answer** following full and correct working	**A1**	Need to see at least one interim step of valid log work. **AG**
		5	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td>9(i)</td><td>Carry out correct method for finding a vector equation for <i>AB</i></td><td>M1</td><td></td></tr>
<tr><td></td><td>Obtain $(\mathbf{r} =)\mathbf{i}+2\mathbf{j}-\mathbf{k}+\lambda(2\mathbf{i}-\mathbf{j}+2\mathbf{k})$, or equivalent</td><td>A1</td><td></td></tr>
<tr><td></td><td>Equate two pairs of components of general points on <i>their AB</i> and <i>l</i> and solve for λ or for μ</td><td>M1</td><td>$\begin{pmatrix}1+2\lambda\\2-\lambda\\-1+2\lambda\end{pmatrix}=\begin{pmatrix}2+\mu\\1+\mu\\1+2\mu\end{pmatrix}$</td></tr>
<tr><td></td><td>Obtain correct answer for λ or μ, e.g. $\lambda = 0$, $\mu = -1$</td><td>A1</td><td></td></tr>
<tr><td></td><td>Verify that all three equations are not satisfied and the lines fail to intersect ($\neq$ is sufficient justification e.g. $2 \neq 0$)
Conclusion needs to follow correct values</td><td>A1</td><td>Alternatives
<table>
<tr><th><i>A</i></th><th>λ</th><th>μ</th><th></th><th><i>B</i></th><th>λ</th><th>μ</th><th></th></tr>
<tr><td>ij</td><td>$2/3$</td><td>$1/3$</td><td>$\frac{1}{3}\neq\frac{5}{3}$</td><td></td><td>$-1/3$</td><td>$1/3$</td><td>$\frac{1}{3}\neq\frac{5}{3}$</td></tr>
<tr><td>ik</td><td>0</td><td>–1</td><td>$2\neq0$</td><td></td><td>–1</td><td>–1</td><td>$2\neq0$</td></tr>
<tr><td>jk</td><td>1</td><td>0</td><td>$3\neq2$</td><td></td><td>0</td><td>0</td><td>$3\neq2$</td></tr>
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Question	Answer	Marks	Guidance
9(ii)	State or imply midpoint has position vector $2\mathbf{i}+\frac{3}{2}\mathbf{j}$	**B1**	
	Substitute in $2x-y+2z=d$ and find d	**M1**	Correct use of *their* direction for *AB* and *their* midpoint
	Obtain plane equation $4x-2y+4z=5$	**A1**	or equivalent e.g. $\mathbf{r}.\begin{pmatrix}2\\-1\\2\end{pmatrix}=\frac{5}{2}$
	Substitute components of l in plane equation and solve for μ	**M1**	Correct use of their plane equation.
	Obtain $\mu=-\frac{1}{2}$ and position vector $\frac{3}{2}\mathbf{i}+\frac{1}{2}\mathbf{j}$ for the point P	**A1**	Final answer Accept coordinates in place of position vector
		5	

Question	Answer	Marks	Guidance
10(i)	State correct expansion of $\sin\left(3x+x\right)$ or $\sin\left(3x-x\right)$	**B1**	B0 If their formula retains ± in the middle
	Substitute expansions in $\frac{1}{2}(\sin 4x+\sin 2x)$	**M1**	
	Obtain $\sin 3x\cos x=\frac{1}{2}\left(\sin 4x+\sin 2x\right)$ correctly	**A1**	Must see the $\sin 4x$ and $\sin 2x$ or reference to LHS and RHS for A1 **AG**
		3	
10(ii)	Integrate and obtain $-\frac{1}{8}\cos 4x-\frac{1}{4}\cos 2x$	**B1 B1**	
	Substitute limits $x = 0$ and $x = \frac{1}{3}\pi$ correctly	**M1**	In their expression
	Obtain answer $\frac{9}{16}$	**A1**	From correct working seen.
		4	

Question	Answer	Marks	Guidance
10(iii)	State correct derivative $2\cos 4x + \cos 2x$	**B1**	
	Using correct double angle formula, express derivative in terms of $\cos 2x$ and equate the result to zero	**M1**	
	Obtain $4\cos^2 2x + \cos 2x - 2 = 0$	**A1**	
	Solve for x or $2x$ (could be labelled x) $\left(\cos 2x = \dfrac{-1 \pm \sqrt{33}}{8}\right)$	**M1**	Must see working if solving an incorrect quadratic The roots of the correct quadratic are -0.843 and 0.593 Need to get as far as $x = \ldots$ The wrong value of x is 0.468 and can imply M1 if correct quadratic seen Could be working from a quartic in $\cos x$: $16\cos^4 x - 14\cos^2 x + 1 = 0$
	Obtain answer $x = 1.29$ only	**A1**	
		5	