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1	State	or imply ordinates 0, 0.405465, 0.623810, 0.693147		B1	
	Use	correct formula, or equivalent, with $h = \frac{1}{6}\pi$ and four ordinates		M1	
	Obta	in answer 0.72		A1	[3]
2	Use	laws of indices correctly and solve for <i>u</i>		M1	
	Obta	in <i>u</i> in any correct form, e.g. $u = \frac{16}{16-1}$		A1	
	Use Obta	correct method for solving an equation of the form $4^x = a$, where $a > 0$ in answer $x = 0.0466$		M1 A1	[4]
3	EITI	HER: Use correct product rule Obtain correct derivative in any form, e.g. $-\sin x \cos 2x - 2\cos x \sin 2x$ Use the correct double angle formulae to express derivative in cos x an	d sin <i>x</i> .	M1 A1	
	OR1	 or cos 2x and sin x Use correct double angle formula to express y in terms of cos x and atte differentiation 	empt	M1 M1	
		Use chain rule correctly Obtain correct derivative in any form, e.g. $-6\cos^2 x \sin x + \sin x$		M1 A1	
	OR2	Use correct factor formula and attempt differentiation Obtain correct derivative in any form $a = a^{3} \sin 3x = \frac{1}{2} \sin x$		M1	
		Use correct trig formulae to express derivative in terms of $\cos x$ and $\sin x$	x, or sin x	M1	
	Equa	the derivative to zero and obtain an equation in one trig function in $6 \cos^2 n = 1 - 6 \sin^2 n = 5 - \tan^2 n = 5 \cos^2 n = 2$,	M1	
	Obta Obta [Igno	in answer $x = 1, 0 \sin^2 x = 5, \tan^2 x = 5$ of $5\cos 2x = -2$ in answer $x = 1.15$ (or 65.9°) and no other in the given interval pre answers outside the given interval.]	·	A1 A1	[6]
	[SR:	Solution attempts following the <i>EITHER</i> scheme for the first two marks can second and third method marks as follows:	earn the		
	Equa Use	Ite derivative to zero and obtain an equation in tan $2x$ and tan x correct double angle formula to obtain an equation in tan x		M1 M1]	
4	(i)	State $R = \sqrt{13}$		B1	
		Use trig formula to find α		M1	[2]
		Obtain $\alpha = 55.09$ with no errors seen		AI	[3]
	(ii)	Evaluate $\sin^{-1}(1/\sqrt{13})$ to at least 1 d.p. (16.10° to 2 d.p.)	<u>8</u> 00	B1√ M1	
		Obtain answer $\theta = 130.2^{\circ}$ and no other in the given interval	80	A1	[3]
		[Ignore answers outside the given interval.] [Treat answers in radians as a misread and deduct A1 from the marks for the	angles.]		
5	(i)	State or imply $AT = r \tan x$ or $BT = r \tan x$		B1	
		Use correct arc formula and form an equation in r and x Rearrange in the given form		MI A1	[3]
	(ii)	Calculate values of a relevant expression or expressions at $x = 1$ and $x = 1.3$ Complete the argument correctly with correct calculated values		M1 A1	[2]

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	(iii)	Use the iterative formula correctly at least once		M1	
		Obtain final answer 1.11		A1	
		Show sufficient iterations to 4 d.p. to justify 1.11 to 2 d.p., or show there is a st	ign change ii	1	
		the interval (1.105, 1.115)		Al	[3]
6		State or imply $dy = \frac{1}{dx}$ or equivalent		D 1	
U	(1)	State of imply $du = -\frac{1}{2\sqrt{x}}dx$, of equivalent		DI	
		Substitute for <i>x</i> and d <i>x</i> throughout		M1	
		Obtain integrand $\frac{\pm 2(2-u)^2}{2}$, or equivalent		A1	
			•.4		
		Show correct working to justify the change in limits and obtain the given answ	er with	Δ1	[4]
				111	[ד]
	(ii)	Integrate and obtain at least two terms of the form $a \ln u$, bu , and cu^2		M1*	
		Obtain indefinite integral $8 \ln u - 8u + u^2$, or equivalent		A1	
		Substitute limits correctly	M1(dep*)	
		Obtain the given answer correctly having shown sufficient working		A1	[4]
7	(i)	Square $x + iv$ and equate real and imaginary parts to -1 and $4\sqrt{3}$		M1	
		Obtain $x^2 - y^2 = -1$ and $2xy = 4\sqrt{3}$		A1	
		Eliminate one unknown and find an equation in the other		M1	
		Obtain $x^4 + x^2 - 12 = 0$ or $y^4 - y^2 - 12 = 0$, or three term equivalent		A1	
		Obtain answers $\pm (\sqrt{3} \pm 2i)$		A1	[5]
		If the equations are solved by inspection, give B2 for the answers and B1 for i	ustifving the	ml	[0]
		[]		1	
	(ii)	Show a circle with centre $-1 + 4\sqrt{3}$ in a relatively correct position		B1	
		Show a circle with radius 1 and centre not at the origin		B1	
		Carry out a complete method for calculating the greatest value of $\arg z$		Ml	[4]
		Obtain answer 1.80 of 100.4		AI	[4]
8	(i)	State or imply the form $\frac{A}{A} + \frac{Bx+C}{C}$		B1	
5	(-)	$3-2x + x^2 + 4$		1	
		Use a relevant method to determine a constant Obtain any of the malance $(1 - 2) = 1 - (1 - 2)$		M1	
		Obtain one of the values $A = 5$, $B = -1$, $C = -2$ Obtain a second value		AI A1	
		Obtain the third value		Al	[5]

(ii) Use correct method to find the first two terms of the expansion of $(3-2x)^{-1}$, $(1-\frac{2}{3}x)^{-1}$,

$$(4+x^2)^{-1}$$
 or $(1+\frac{1}{4}x^2)^{-1}$ M1

Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction $A1\sqrt[4]{+}A1\sqrt[4]{}$ Multiply out up to the term in x^2 by Bx + C, where $BC \neq 0$ M1

Obtain final answer
$$\frac{1}{2} + \frac{5}{12}x + \frac{41}{72}x^2$$
, or equivalent A1 [5]

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[Symbolic coefficients, e.g. $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ are not sufficient for the first M1.The f.t. is on *A*, *B*, *C*.] [In the case of an attempt to expand $(5x^2 + x + 6)(3 - 2x)^{-1}(x^2 + 4)^{-1}$, give

[In the case of an attempt to expand $(3x^2 + x + 0)(3 - 2x)^2$ $(x^2 + 4)^2$, give M1A1A1for the expansions, M1 for multiplying out fully, and A1 for the final answer.]

9	(i)	Separate variables correctly and attempt integration of one side Obtain term $\ln x$	B1 B1	
		Obtain term of the form $a \ln(k + e^{-t})$	M1	
		Obtain term $-\ln(k + e^{-t})$	A1	
		Evaluate a constant or use limits $x = 10$, $t = 0$ in a solution containing terms $a \ln(k + e^{-t})$ and $b \ln x$) M1*	
		Obtain correct solution in any form, e.g. $\ln x - \ln 10 = -\ln(k + e^{-t}) + \ln(k + 1)$	A1	[6]
(ii)	Substitute $x = 20$, $t = 1$ and solve for k Obtain the given answer	A1(dep*) A1	[2]
(i	ii)	Using $e^{-t} \rightarrow 0$ and the given value of k, find the limiting value of x Justify the given answer	M1 A1	[2]

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10	(i)	Carry out Obtain \mathbf{r} :	a correct method for finding a vector equation for AB = $2\mathbf{i} - \mathbf{j} + 3\mathbf{k} + \lambda(-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$, or equivalent		M1 A1			
		Equate at for μ	least two pairs of components of general points on AB and l and solve	e for λ or	M1			
		Obtain co	rrect answer for λ or μ , e.g. $\lambda = 1$ or $\mu = 0$; $\lambda = -\frac{4}{5}$ or $\mu = \frac{3}{5}$;					
		or λ	$=\frac{1}{4}$ or $\mu = -\frac{3}{2}$		A1			
		Verify that	at not all three pairs of equations are satisfied and that the lines fail to	intersect	A1	[5]		
	(ii)	EITHER:	Obtain a vector parallel to the plane and not parallel to l , e.g. $\mathbf{i} - 2\mathbf{j} + \mathbf{j}$	k	B1			
			Use scalar product to obtain an equation in a, b and c, e.g. $3a + b - a$	c = 0	B1			
			Form a second relevant equation, e.g. $a - 2b + c = 0$ and solve for on e.g. $a : b$	ne ratio,	M1			
			Obtain final answer $a: b: c = 1:4:7$ Af Use coordinates of a relevant point and values of a b and c in gener	al equation				
			and find d	ur oquunon	M1			
			Obtain answer $x + 4y + 7z = 19$, or equivalent		A1			
		<i>OR</i> 1:	Obtain a vector parallel to the plane and not parallel to l , e.g. $\mathbf{i} - 2\mathbf{j} + \mathbf{j}$	k	B1			
			Obtain a second relevant vector parallel to the plane and attempt to c their vector product $a = (i - 2i + k) \times (2i + i - k)$	calculate	M1			
			their vector product, e.g. $(I - 2J + K) \times (3I + J - K)$					
			Obtain correct answer, e.g. $\mathbf{i} + 4\mathbf{i} + 7\mathbf{k}$		A1 A1			
			Substitute coordinates of a relevant point in $x + 4v + 7z = d$, or equi	valent.				
			and find <i>d</i>		M1			
			Obtain answer $x + 4y + 7z = 19$, or equivalent		A1			
		OR2:	Obtain a vector parallel to the plane and not parallel to <i>l</i> , e.g. $\mathbf{i} - 2\mathbf{j} + \mathbf{j}$	· k	B1			
			Using a relevant point and second relevant vector, form a 2-paramet	er equation	M1			
			State a correct equation, e.g. $\mathbf{r} = 2\mathbf{i} - \mathbf{i} + 3\mathbf{k} + s(\mathbf{i} - 2\mathbf{i} + \mathbf{k}) + t(3\mathbf{i} + \mathbf{i} - \mathbf{k})$	k)	A1			
			State 3 correct equations in x, y, z, s and t		A1			
			Eliminate <i>s</i> and <i>t</i>		M1			
			Obtain answer $x + 4y + 7z = 19$, or equivalent		A1			
		OR3:	Using the coordinates of A and two points on l , state three simultane	ous	1 D 1			
			equations in a, b, c and d, e.g. $a + b + 2c = d$, $2a - b + 3c = d$ and $4a$ Solve and find one ratio e.g. $a : b$	i + 2b + c = a	M1			
			State one correct ratio		A1			
			Obtain a correct ratio of three of the unknowns, e.g. $a:b:c=1:4:$: 7,				
			or equivalent		A1			
			Either use coordinates of a relevant point and the found ratio to find	the fourth	M 1			
			unknown, e.g. <i>d</i> , or find the ratio $a : b : c : d$ Obtain answer $x + 4y + 7z = 19$ or equivalent					
		<i>OR</i> 4:	Obtain a vector parallel to the plane and not parallel to l , e.g. $\mathbf{i} - 2\mathbf{j} + \mathbf{j}$	k	B1			
			Using a relevant point and second relevant vector, form a determination	nt equation				
			for the plane		M1			
			$\begin{vmatrix} x-2 & y+1 & z-3 \end{vmatrix}$					
			State a correct equation, e.g. $\begin{vmatrix} 1 & -2 & 1 \end{vmatrix} = 0$		A1			
			Attempt to expand the determinant		M1			
			Obtain or imply two correct cofactors		A1			
			Obtain answer $x + 4y + 7z = 19$, or equivalent		A1	[6]		