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<p>1 <math>\left(x^2 - \frac{2}{x}\right)^5</math></p> <p>Term in <math>x</math> is <math>10 \times (x^2)^2 \times \left(\frac{-2}{x}\right)^3</math></p> <p>Coefficient = <math>-80(x)</math></p>	B1 B1 B1 [3]	B1 10 or ${}^5C_2$ or ${}^5C_3$ , B1 $\left(\frac{-2}{x}\right)^3$ co Must be identified
<p>2 36, 32, ...</p> <p>(i) <math>r = \frac{8}{9}</math> <math>S_\infty = (\text{their } a) \div (1 - \text{their } r)</math></p> $S_\infty = 36 \div \frac{1}{9} = 324$ <p>(ii) <math>d = -4</math></p> $0 = \frac{n}{2} (72 + (n-1)(-4))$ $\rightarrow n = 19$	M1 A1  B1 M1 A1 [3]	Method for $r$ and $S_\infty$ ok. ( $ r  < 1$ ) co  co $S_n$ formula ok and a value for $d$ ( $\neq \frac{8}{9}$ ) Condone $n = 0$ but no other soln
<p>3 (i) <math>s = r\theta</math> Angle of major arc = <math>2\pi - 2.2 = (4.083)</math> Perimeter = <math>12 + 24.5 = 36.5</math> or <math>12\pi - 1.2</math> (or full circle – minor arc B1)</p> <p>(ii) Area of major sector = <math>\frac{1}{2}r^2\theta = (73.49)</math> Area of triangle = <math>\frac{1}{2} \cdot 6^2 \sin 2.2 = (14.55)</math> Ratio = <math>5.05 : 1</math> (Allow <math>5.03 \rightarrow 5.06</math>)</p>	M1 B1 A1  M1 M1 A1 [3]	Used with major or minor arc Could be gained in (ii). co  Used with major/minor sector. Correct formula or method. $(2\pi - 2.2)/\sin 2.2$ gets M1M1 co
<p>4 <math>\frac{\tan x + 1}{\sin x \tan x + \cos x} \equiv \sin x + \cos x</math></p> <p>(i) LHS <math>\frac{\left(\frac{s}{c}\right) + 1}{\left(\frac{s^2}{c^2} + c\right)} = \frac{s+c}{s^2+c^2}</math> = RHS</p> <p>(ii) <math>s + c = 3s - 2c</math> <math>\rightarrow \tan x = \frac{3}{2}</math> Allow <math>\cos^2 = \frac{4}{13}</math>, <math>\sin^2 = \frac{9}{13}</math> <math>\rightarrow x = 0.983</math> and <math>4.12</math> or <math>4.13</math></p>	M1 M1  A1  M1 A1 A1 [3]	Use of $t = s/c$ twice Correct algebra and use of $s^2 + c^2 = 1$ AG all ok  Uses (i) and $t = \frac{s}{c}$ $t = \frac{2}{3}$ or 0 is M0 co. $\frac{1}{3}1st + \pi$ , providing no excess solns in range. Allow $0.313\pi$ , $1.31\pi$

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<p>5 <math>f(x) = \frac{15}{2x+3}</math></p> <p>(i) <math>f'(x) = \frac{-15}{(2x+3)^2} \times 2</math>  <math>( )^2</math> always +ve <math>\rightarrow f'(x) &lt; 0</math>          (No turning points) – therefore an inverse</p> <p>(ii) <math>y = \frac{15}{2x+3} \rightarrow 2x+3 = \frac{15}{y}</math>  <math>\rightarrow x = \frac{\frac{15}{y}-3}{2} \rightarrow \frac{15-3x}{2x}</math>          (Range) <math>0 \leq f^{-1}(x) \leq 6</math>.          Allow <math>0 \leq y \leq 6</math>, <math>[0,6]</math>          (Domain) <math>1 \leq x \leq 5</math>. Allow <math>[1, 5]</math></p>	B1 B1 B1 [3] M1 A1 B1 B1 [4]	Without the “ $\times 2$ ”. For “ $\times 2$ ” (indep of 1 <sup>st</sup> B1). $\sqrt{ }$ providing $( )^2$ in $f'(x)$ . 1–1 insuff. Order of ops – allow sign error co as function of $x$ . Allow $y = \dots$ For range/domain ignore letters unless range/domain not identified
<p>6 <math>\frac{dy}{dx} = \frac{12}{\sqrt{4x+a}}</math> P(2, 14) Normal <math>3y+x=44</math></p> <p>(i) m of normal <math>= -\frac{1}{3}</math>  <math>\frac{dy}{dx} = 3 = \frac{12}{\sqrt{4x+a}} \rightarrow a = 8</math></p> <p>(ii) <math>\int y = 12(4x+a)^{\frac{1}{2}} \div \frac{1}{2} \div 4 (+c)</math>          Uses (2, 14)  <math>c = -10</math></p>	B1 M1 A1 [3] B1 B1 M1 A1 [4]	co Use of $m_1m_2 = -1$ . AG. Correct without “ $\div 4$ ”. for “ $\div 4$ ”. Uses in an integral only. Dep ‘c’. co All 4 marks can be given in (i)

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7	(i) Angle $BAC$ needs sides $AB, AC$ or $BA, CA$ $\mathbf{AB} \cdot \mathbf{AC} = (\mathbf{b} - \mathbf{a}) \cdot (\mathbf{c} - \mathbf{a})$	B1 M1  M1M1  A1  [5]	Ignore <i>their</i> labels:  One of <b>AB</b> , <b>BA</b> , <b>AC</b> , <b>CA</b> correct Use of $x_1x_2 + y_1y_2$ , etc.
			M1 prod of moduli. M1 all linked  If e.g. <b>BA</b> . <b>OC</b> max B1M1M1. If both vectors wrong 0/5. If e.g. <b>BA</b> . <b>AC</b> used $\rightarrow \cos^{-1}\left(-\frac{1}{3}\right)$ final mark A0
8	(ii) $\sin BAC = \sqrt{1 - \frac{1}{9}}$  $\text{Area} = \frac{1}{2} \times 6 \times 5 \times \sqrt{\frac{8}{9}} = 5\sqrt{8}$ oe	B1  M1 A1  [3]	Use of $s^2 + c^2 = 1$ – not decimals  Correct formula for area. Decimals seen A0
9	(i) $2x^2 - 10x + 8 \rightarrow a(x + b)^2 + c$  (i) $a = 2, b = -2\frac{1}{2}, c = -4\frac{1}{2}$  $\rightarrow$ min value is $-4\frac{1}{2}$ Allow $(2\frac{1}{2}, -4\frac{1}{2})$	3 × B1  B1  [4]	Or $2\left(x - 2\frac{1}{2}\right)^2 - 4\frac{1}{2}$  Can score by sub $x = 2\frac{1}{2}$ into original but not by differentiation
	(ii) $2x^2 - 10x + 8 - kx = 0$ Use of “ $b^2 - 4ac$ ” $(-10 - k)^2 - 64 < 0$ or $k^2 + 20k + 36 < 0$ $\rightarrow k = -18$ or $-2$ $-18 < k < -2$	M1 M1 A1 A1  [4]	Sets equation to 0 and uses discriminant correctly  Realises discriminant $< 0$ . Allow $\leq$ co Dep on 1 <sup>st</sup> M1 only co
	(i) $3x^2y = 288$ $y$ is the height  $A = 2(3x^2 + xy + 3xy)$  Sub for $y \rightarrow A = 6x^2 + \frac{768}{x}$	B1 M1 A1  [3]	co  Considers at least 5 faces ( $y \neq x$ )  co answer given
	(ii) $\frac{dA}{dx} = 12x - \frac{768}{x^2}$  $= 0$ when $x = 4 \rightarrow A = 288$ . Allow (4, 288)  $\frac{d^2A}{dx^2} = 12 + \frac{1536}{x^3}$  $(= 36) > 0$ Minimum	B1 M1 A1 M1 A1  [5]	co  Sets differential to 0 + solution. co  Any valid method  co www dep on correct f'' and $x = 4$

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<p><b>10</b> pts of intersection <math>2x + 1 = -x^2 + 12x - 20</math>  <math>\rightarrow x = 3, 7</math></p> <p>Area of trapezium = <math>\frac{1}{2}(4)(7 + 15) = 44</math>  (or <math>\int (2x+1) dx</math> from 3 to 7 = 44)</p>	M1A1  M1A1	Attempt at soln of sim eqns. co  Either method ok. co
<p>Area under curve = <math>-\frac{1}{3}x^3 + 6x^2 - 20x</math></p> <p>Uses 3 to 7 <math>\rightarrow (54\frac{2}{3})</math></p>	B2,1  DM1	-1 each term incorrect  Correct use of limits (Dep 1 <sup>st</sup> M1)
<p>Shaded area = <math>10\frac{2}{3}</math></p>	A1  [8]	co
<b>OR</b>  $\int_3^7 \left( -x^2 + 10x - 21 \right) = -\frac{x^3}{3} + 5x^2 - 21x$ <p>M1 subtraction, A1A1A1 for integrated terms,  DM1 correct use of limits, A1</p>		Functions subtracted before integration  Subtraction reversed allow A3A0. Limits reversed allow DM1A0
<p><b>11</b> Sim eqns <math>\rightarrow A(1, 3)</math>  Vectors or mid-point <math>\rightarrow C(12, 14)</math></p>	M1 A1  M1 A1	co Allow answer only B2  Allow answer only B2
<p>Eqn of BC <math>4y = x + 44</math> or CD <math>y = 3x - 22</math></p> <p>Sim eqns <math>\rightarrow B(4, 12)</math> or D <math>(9, 5)</math></p> <p>Vectors or mid-point <math>\rightarrow B(4, 12)</math> or D <math>(9, 5)</math></p>	M1  DM1A1  DM1A1	equation ok – unsimplified  Sim eqns. co  Valid method (or sim eqns) co
		[9]