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1 *EITHER*: State or imply non-modular inequality $x^2 < (5+2x)^2$, or corresponding equation, or pair of linear equations $x = \pm(5+2x)$ M1

Obtain critical values –5 and $-\frac{5}{3}$ only A1

Obtain final answer $x < -5$, $x > -\frac{5}{3}$ A1

OR: State one critical value e.g. –5, by solving a linear equation or inequality, or from a graphical method, or by inspection B1

State the other critical value, e.g. $-\frac{5}{3}$, and no other B1

Obtain final answer $x < -5$, $x > -\frac{5}{3}$ B1 [3]

[Do not condone ≤ or ≥.]

2 **(i)** Use law for the logarithm of a product or quotient M1

Use $\log_2 32 = 5$ or $2^5 = 32$ M1

Obtain $x^2 + 5x - 32 = 0$, or horizontal equivalent A1 [3]

(ii) Solve a 3-term quadratic equation M1

Obtain answer $x = 3.68$ only, or exact equivalent, e.g. $\frac{\sqrt{153}-5}{2}$ A1 [2]

3 Use correct trig formula (or formulae) and obtain an equation in $\cos\theta$ M1

Obtain $8\cos^2\theta + \cos\theta - 7 = 0$, or equivalent A1

Solve a 3-term quadratic in $\cos\theta$ and reach $\theta = \cos^{-1}(a)$ M1

Obtain answer 29.0° A1

Obtain answer 180° and no others A1 [5]

[Ignore answers outside the given interval. Treat answers in radians (0.505 and 3.14 or π) as a misread.]

[SR: The answer 180° found by inspection can earn B1.]

4 **(i)** State or imply $CT = r\tan x$ or $OT = r\sec x$, or equivalent B1

Using correct area formulae, form an equation in r and x M1

Obtain the given answer correctly A1 [3]

(ii) Use the iterative formula correctly at least once M1

Obtain the final answer 1.35 A1

Show sufficient iterations to 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change in the interval (1.345, 1.355) A1 [3]

5 **(i)** *EITHER*: State $\frac{dx}{dt} = \sec^2 t / \tan t$, or equivalent — B1

State $\frac{dy}{dt} = 2\sin t \cos t$, or equivalent — B1

Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ — M1

Obtain correct answer in any form, e.g. $2\sin^2 t \cos^2 t$ — A1

OR: Obtain $y = e^{2x} / (1 + e^{2x})$, or equivalent — B1

Use correct quotient or product rule — M1

Obtain correct derivative in any form, e.g. $2e^{2x} / (1 + e^{2x})^2$ — A1

Obtain correct derivative in terms of t in any form, e.g. $(2\tan^2 t) / (1 + \tan^2 t)^2$ — A1 [4]

(ii) State or imply $t = \frac{1}{4}\pi$ when $x = 0$ — B1

Form the equation of the tangent at $x = 0$ — M1

Obtain correct answer in any horizontal form, e.g. $y = \frac{1}{2}x + \frac{1}{2}$ — A1 [3]

[SR: If the *OR* method is used in part **(i)**, give B1 for stating or implying $y = \frac{1}{2}$ or $\frac{dy}{dx} = \frac{1}{2}$ when $x = 0$.]

6 **(i)** Show that the differential equation is $\frac{dy}{dx} = 2xy$ — B1

Separate variables correctly and attempt integration of both sides — M1

Obtain term $\ln y$, or equivalent — A1

Obtain term x^2, or equivalent — A1

Evaluate a constant, or use limits $x = 1$, $y = 2$, in a solution containing terms $a\ln y$ and bx^2 — M1

Obtain correct solution in any form — A1

Obtain the given answer correctly — A1 [7]

(ii) State that the gradient at (–1, 2) is –4 — B1

Show the sketch of curve with correct concavity, positive y-intercept and axis of symmetry $x = 0$ — B1 [2]

[SR: A solution with $k \neq 2$, or not evaluated, can earn B0M1A1A1M1A1A0 in part **(i)**.]

[SR: If given answer is assumed valid, give B1 if $\frac{dy}{dx}$ is shown correctly to be equal to $2xy$, is stated to be proportional to xy, and shown to be equal to 4 at (1, 2).]

7 **(a)** **(i)** *EITHER*: Multiply numerator and denominator by $a - 2\mathrm{i}$, or equivalent M1

Obtain final answer $\frac{5a}{a^2+4} - \frac{10\mathrm{i}}{a^2+4}$, or equivalent A1

OR: Obtain two equations in x and y, solve for x or for y M1

Obtain final answer $x = \frac{5a}{a^2+4}$ and $y = \frac{10}{a^2+4}$, or equivalent A1 [2]

(ii) Either state $\arg(u) = -\frac{3}{4}\pi$, or express u^* in terms of a (f.t. on u) B1√

Use correct method to form an equation in a, e.g. $5a = -10$ M1

Obtain $a = -2$ correctly A1 [3]

(b) Show a point representing $2 + 2\mathrm{i}$ in relatively correct position in an Argand diagram B1

Show the circle with centre at the origin and radius 2 B1

Show the perpendicular bisector of the line segment from the origin to the point representing $2 + 2\mathrm{i}$ B1√

Shade the correct region B1 [4]

[SR: Give the first B1 and the B1√ for obtaining $y = 2 - x$, or equivalent, and sketching the attempt.]

8 **(i)** State or imply partial fractions are of the form $\frac{A}{1+x} + \frac{Bx+C}{2+x^2}$ B1

Use a relevant method to determine a constant M1

Obtain one of the values $A = -2$, $B = 1$, $C = 4$ A1

Obtain a second value A1

Obtain the third value A1 [5]

(ii) Use correct method to obtain the first two terms of the expansion of $(1+x)^{-1}$, $\left(1+\frac{1}{2}x^2\right)^{-1}$ or $(2+x^2)^{-1}$ in ascending powers of x M1

Obtain correct unsimplified expansion up to the term in x^3 of each partial fraction A1√ + A1√

Multiply out fully by $Bx + C$, where $BC \neq 0$ M1

Obtain final answer $\frac{5}{2}x - 3x^2 + \frac{7}{4}x^3$, or equivalent A1 [5]

[Symbolic binomial coefficients, e.g. $\binom{-1}{1}$, are not sufficient for the first M1. The f.t. is on A, B, C.]

[If B or C omitted from the form of fractions, give B0M1A0A0A0 in **(i)**; M1A1√A1√ in **(ii)**, max 4/10.]

[In the case of an attempt to expand $(5x - x^2)(1 + x)^{-1}(2 + x^2)^{-1}$, give M1A1A1 for the expansions, M1 for the multiplying out fully, and A1 for the final answer.]

[Allow use of Maclaurin, giving M1A1√A1√ for differentiating and obtaining f(0) = 0 and f '(0) = $\frac{5}{2}$, A1√ for f ''(0) = –6, and A1 for f '''(0) = $\frac{21}{2}$ and the final answer (the f.t. is on A, B, C if used).]

[For the identity $5x - x^2 \equiv \left(2 + 2x + x^2 + x^3\right)\left(a + bx + cx^2 + dx^3\right)$ give M1A1; then M1A1 for using a relevant method to obtain two of $a = 0$, $b = \frac{5}{2}$, $c = -3$ and $d = \frac{7}{4}$; then A1 for the final answer in series form.]

9 **(i)** State or imply a correct normal vector to either plane, e.g. $\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ or $2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ B1
Carry out correct process for evaluating the scalar product of the two normals M1
Using the correct process for the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result M1
Obtain the final answer 79.7° (or 1.39 radians) A1 [4]

(ii) *EITHER*: Carry out a method for finding a point on the line M1
Obtain such a point, e.g. (1, 3, 0) A1
EITHER: State two correct equations for the direction vector (a, b, c) of the line, e.g. $a + 2b - 2c = 0$ and $2a + b + 3c = 0$ B1
Solve for one ratio, e.g. $a : b$ M1
Obtain $a : b : c = 8 : -7 : -3$, or equivalent A1
State a correct final answer, e.g. $\mathbf{r} = \mathbf{i} + 3\mathbf{j} + \lambda(8\mathbf{i} - 7\mathbf{j} - 3\mathbf{k})$ A1√

OR1: Obtain a second point on the line, e.g. $\left(0, \frac{31}{8}, \frac{3}{8}\right)$ A1
Subtract position vectors to find a direction vector M1
Obtain $\mathbf{i} - \frac{7}{8}\mathbf{j} - \frac{3}{8}\mathbf{k}$, or equivalent A1
State a correct final answer, e.g. $\mathbf{r} = \mathbf{i} + 3\mathbf{j} + \lambda(\mathbf{i} - \frac{7}{8}\mathbf{j} - \frac{3}{8}\mathbf{k})$ A1√

OR2: Attempt to calculate the vector product of two normals M1
Obtain two correct components A1
Obtain $8\mathbf{i} - 7\mathbf{j} - 3\mathbf{k}$, or equivalent A1
State a correct final answer, e.g. $\mathbf{r} = \mathbf{i} + 3\mathbf{j} + \lambda(8\mathbf{i} - 7\mathbf{j} - 3\mathbf{k})$ A1√

OR3: Express one variable in terms of a second M1
Obtain a correct simplified expression, e.g. $x = (31 - 8y) / 7$ A1
Express the first variable in terms of a third M1
Obtain a correct simplified expression, e.g. $x = (3 - 8z) / 3$ A1
Form a vector equation of the line M1
State a correct final answer, e.g. $\mathbf{r} = \frac{31}{8}\mathbf{j} + \frac{3}{8}\mathbf{k} + \lambda(8\mathbf{i} - 7\mathbf{j} - 3\mathbf{k})$ A1√

OR4: Express one variable in terms of a second M1
Obtain a correct simplified expression, e.g. $y = (31 - 7x) / 7$ A1
Express the third variable in terms of the second M1
Obtain a correct simplified expression, e.g. $z = (3 - 3x) / 8$ A1
Form a vector equation of the line M1
State a correct final answer, e.g. $\mathbf{r} = \frac{31}{8}\mathbf{j} + \frac{3}{8}\mathbf{k} + \lambda(-8\mathbf{i} + 7\mathbf{j} + 3\mathbf{k})$ A1√ [6]

[The f.t. is dependent on all M marks having been earned.]

10 **(i)** Attempt integration by parts and reach $\pm x^2e^{-x} \pm \int 2xe^{-x}dx$ M1*

Obtain $-x^2e^{-x} + \int 2xe^{-x}dx$, or equivalent A1

Integrate and obtain $-x^2e^{-x} - 2xe^{-x} - 2e^{-x}$, or equivalent A1

Use limits $x = 0$ and $x = 3$, having integrated by parts twice M1(dep*)

Obtain the given answer correctly A1 [5]

(ii) Use correct product or quotient rule M1

Obtain correct derivative in any form A1

Equate derivative to zero and solve for non-zero x M1

Obtain $x = 2$ with no errors send A1 [4]

(iii) Carry out a complete method for finding the x-coordinate of P M1

Obtain answer $x = 1$ A1 [2]