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1	$(a+x)^5 + (1-2x)^6$ <p>Coeff of <math>x^3</math> in 1<sup>st</sup> = <math>10 \times a^2</math>          Coeff of <math>x^3</math> in 2<sup>nd</sup> = <math>20 \times (-2)^3</math>  <math>\rightarrow 10a^2 - 160 = 90</math>  <math>\rightarrow a = 5</math></p>	B1 B1 + B1 M1 A1 [5]	co co Forming an equation for $a$ + solution co (condone $\pm$ )
2	$y = mx + 4$ $y = 3x^2 - 4x + 7$ <p>Equate <math>\rightarrow 3x^2 - (4+m)x + 3 = 0</math>          Uses <math>b^2 - 4ac \rightarrow (4+m)^2 - 36</math>          Solution of quadratic <math>m = 2</math> or <math>-10</math>          Set of values <math>m &gt; 2</math> or <math>m &lt; -10</math></p>	M1 M1 DM1 A1 A1 [5]	Eliminates $y$ (or $x$ ) completely Any use of $b^2 - 4ac$ Method shown. Correct end-values co
3	$\frac{x}{a} + \frac{y}{b} = 1$ $P(a, 0)$ and $Q(0, b)$ <p>Distance <math>\rightarrow \sqrt{(a^2 + b^2)} = \sqrt{45}</math>          Gradients <math>\rightarrow \frac{-a}{b} = \frac{-1}{2}</math>          Solution of sim eqns <math>\rightarrow a = 6, b = 3</math></p>	M1 A1 M1 A1 A1 [5]	M1 even if sign(s) incorrect. Correct values $a$ and $b$ (both)
4	<p>(a) <math>y = \frac{2x^3 + 5}{x} = 2x^2 + \frac{5}{x}</math>  <math>d/dx = 4x - \frac{5}{x^2}</math> or <math>4x - 5x^{-2}</math></p> <p>(b) <math>\int (3x-2)^5 dx = \frac{(3x-2)^6}{6} \div 3 (+c)</math>  <math>\int_0^1 (3x-2)^5 dx = \left[ \frac{(3x-2)^6}{18} \right]_0^1</math>          Limits used correctly <math>\rightarrow -3\frac{1}{2}</math></p>	M1 A1 + A1 [3]  B1 B1 M1 A1 [4]	Knows to divide numerator by $x$ co  B1 without “ $\div 3$ ”. B1 for “ $\div 3$ ”. (ignore $(+c)$ ) Uses limits after integration. co
5	<p>(i) <math>\vec{PQ} = 3\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}</math>  <math>\vec{RQ} = -3\mathbf{i} + 8\mathbf{j} + 3\mathbf{k}</math></p> <p>(ii) <math>\vec{PQ} \cdot \vec{RQ} = -9 + 48 - 9 = 30</math>  <math>= \sqrt{54} \sqrt{82} \cos RQP</math>  <math>\rightarrow RQP = 63.2^\circ</math></p>	B2,1 B1 [3]  M1 M1 M1 A1 [4]	Allow B2,1 for either one, B1 for the other.  Use of $x_1x_2 + y_1y_2 + z_1z_2$ Correct use of modulus All linked correctly co

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<p>6 (a) <math>ar^2 = 20</math></p> $\frac{a}{1-r} = 3a$ <p>Soln of equations <math>\rightarrow (r = \frac{2}{3}) a = 45</math></p> <p>(b) <math>a + 7d = 3(a + 2d)</math>  <math>\rightarrow 2a = d</math>  <math>S_8 = 4(2a + 7d) = 32d</math> or <math>64a</math>  <math>S_4 = 2(2a + 3d) = 8d</math> or <math>16a</math></p>	B1 B1 M1 A1 [4]	co co Complete method to find $a$ . co M1 A1 M1 A1 [4]
7 (i) $AX = 6\tan\frac{\pi}{3} = 6\sqrt{3}$  (ii) Area of triangle $= \frac{1}{2} \times 6 \times 6\sqrt{3}$ Area of sector $= \frac{1}{2} 6^2 \times \frac{\pi}{3}$ Area shaded $= 18\sqrt{3} - 6\pi$  (iii) Arc $AB = 6 \times \frac{\pi}{3} = 2\pi$ $OX = 6 \div \cos\frac{\pi}{3} = 12$ , $BX = 6$ Perimeter $= 6\sqrt{3} + 2\pi + 6$	B1 [1] M1 M1 A1 [3] M1 B1 M1 A1 [4]	ag Use of $\frac{1}{2}bh$ Use of $\frac{1}{2}r^2\theta$ co Use of $r\theta$ Use of trig to find ( $OX$ and then) $BX$ . [4]
8 (i) $\left(\frac{1}{\sin\theta} - \frac{1}{\tan\theta}\right)^2 = \frac{1-\cos\theta}{1+\cos\theta}$ $\left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2 = \frac{(1-\cos\theta)^2}{\sin^2\theta}$ $= \frac{(1-\cos\theta)(1-\cos\theta)}{1-\cos^2\theta} = \frac{1-\cos\theta}{1+\cos\theta}$  (ii) $\left(\frac{1}{\sin\theta} - \frac{1}{\tan\theta}\right)^2 = \frac{2}{5}$ $\frac{1-\cos\theta}{1+\cos\theta} = \frac{2}{5}$ $\cos\theta = \frac{3}{7}$ $\theta = 64.6^\circ \text{ or } 295.4^\circ$	M1 M1 A1 [3] M1 A1 A1 A1 ✓ [4]	Use of $\tan = \sin/\cos$ Use of $\sin^2 + \cos^2 = 1$ . All correct. (NB ag. – ensure cancelling has been done) Uses part (i) to obtain an eqn in $\cos\theta$ co co. ✓ for $360 - "1^{\text{st}} \text{ answer}"$ .

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9	$\frac{dy}{dx} = \frac{2}{\sqrt{x}} - 1 \quad P(9, 5)$		
(i)	$y = 4\sqrt{x} - x (+ c)$ Uses (9, 5) in an integrated expression $\rightarrow c = 2$	B1 B1 M1 A1 [4]	Ignore $+ c$ . Substitution of point after integration. co.
(ii)	$\frac{dy}{dx} = 0 \rightarrow x = 4, y = 6$	M1 A1 A1 [3]	Attempt to solve $dy/dx = 0$ . $x$ correct. $y$ correct.
(iii)	$\frac{d^2y}{dx^2} = -x^{-\frac{3}{2}} \rightarrow -ve \rightarrow \text{Max}$	B1 B1 ✓ [2]	co. ✓ for correct deduction.
(iv)	$\frac{dy}{dx} = -\frac{1}{3}$ Perpendicular $m = 3$ $\tan\theta = 3$ Angle is $\tan^{-1}3$ $k = 3$	M1 A1 [2]	Use of $m_1m_2 = -1$ Needs $k = 3$

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10	$f : x \mapsto 3x - 4$ $g : x \mapsto 2(x - 1)^3 + 8$		
(i)	$fg(2) = f(10) = 26$ $f^{-1}(x)$	M1 A1 [2]	Must use g first, then f. co
(ii)		B1 B1 B1 [3]	$y = f(x)$ correct in 1 <sup>st</sup> , 4 <sup>th</sup> quadrants. $y = f^{-1}(x)$ correct in 1 <sup>st</sup> , 2 <sup>nd</sup> quadrants. $y = x$ marked, or quoted.
(iii)	$g'(x) = 6(x - 1)^2$ $g'(x) > 0 \rightarrow$ no turning points $\rightarrow g$ is 1 : 1, $g$ has an inverse.	B1 B1✓ B1✓ [3]	co allow only for incorrect “6” following from incorrect “6”
(iv)	$f^{-1}(x) = \frac{x+4}{3}$ Attempt at making $x$ Order correct. $-8, \div 2, \sqrt[3]{\phantom{x}}, +1$ $g^{-1}(x) = \sqrt[3]{\frac{x-8}{2}} + 1$	B1 M1 M1 A1 [4]	co May change $x$ and $y$ first. Must all be correct, but allow for $+8, -1$ co as function of $x$ , not $y$ .