

2 An organisation runs courses to train students to become engineers. These students are taught in groups of 8. The director of the organisation claims that on average 60% of the students in a group achieve a pass. A random sample of 150 groups of 8 students is chosen. The following table shows the observed frequencies together with some of the expected frequencies using the appropriate binomial distribution.

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|----------------------------|-----|-------|-------|--------|--------|-----|-----|--------|-------|
| Number of passes per group | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Observed frequency | 0 | 0 | 8 | 24 | 45 | 36 | 26 | 10 | 1 |
| Expected frequency | p | 1.180 | 6.193 | 18.579 | 34.836 | q | r | 13.437 | 2.519 |

(a) Find the values of p , q and r giving your answers correct to 3 decimal places. [2]

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(b) Carry out a goodness of fit test, at the 10% significance level, to test whether there is evidence to reject the director’s claim. [6]

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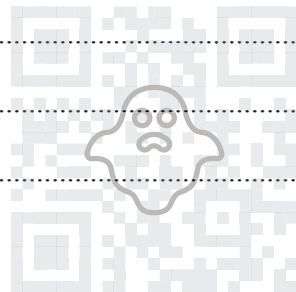
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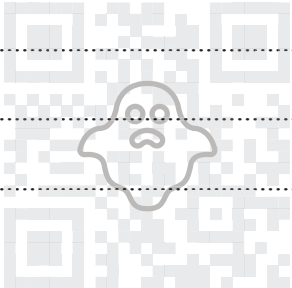
Handwriting practice area consisting of 25 horizontal dotted lines.



- 3 A large college is holding a piano competition. Each student has played a particular piece of music and two judges have each awarded a mark out of 80. The marks awarded to a random sample of 14 students are shown in the following table.

| Student | A | B | C | D | E | F | G | H | I | J | K | L | M | N |
|---------|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| Judge 1 | 79 | 54 | 63 | 74 | 69 | 52 | 50 | 57 | 55 | 42 | 63 | 55 | 56 | 48 |
| Judge 2 | 75 | 62 | 60 | 73 | 76 | 41 | 31 | 51 | 45 | 55 | 49 | 50 | 65 | 36 |

- (a) One of the students claims that on average Judge 1 is awarding higher marks than Judge 2. Carry out a Wilcoxon matched-pairs signed-rank test at the 5% significance level to test whether the data supports the student's claim. [7]



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(b) Give a reason why it is preferable to use a Wilcoxon matched-pairs signed-rank test in this situation rather than a paired sample t -test. [1]

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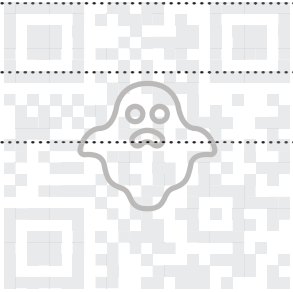
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4 Jason has three biased coins. For each coin the probability of obtaining a head when it is thrown is $\frac{2}{3}$. Jason throws all three coins. The number of heads obtained is denoted by X .

(a) Find the probability generating function $G_X(t)$ of X . [3]

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Jason also has two unbiased coins. He throws all five coins. The number of heads obtained from the two unbiased coins is denoted by Y . It is given that $G_Y(t) = \frac{1}{4} + \frac{1}{2}t + \frac{1}{4}t^2$. The random variable Z is the total number of heads obtained when Jason throws all five coins.

(b) Find the probability generating function of Z , expressing your answer as a polynomial. [3]

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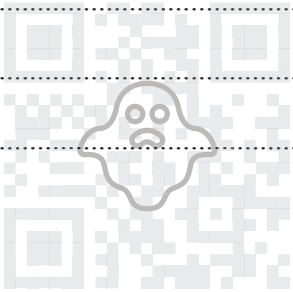
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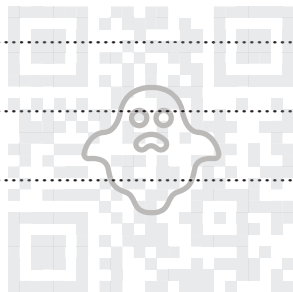
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Ruled lines for writing the answer to part (c).

(c) Find $E(Z)$. [2]

Ruled lines for writing the answer to part (c).



5 The continuous random variable X has cumulative distribution function F given by

$$F(x) = \begin{cases} 0 & x < 0, \\ 1 - \frac{1}{144}(12-x)^2 & 0 \leq x \leq 12, \\ 1 & x > 12. \end{cases}$$

(a) Find the upper quartile of X . [2]

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(b) Find $\text{Var}(X^2)$. [5]

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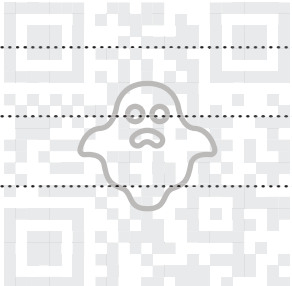
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The random variable Y is given by $Y = \sqrt{X}$.

(c) Find the probability density function of Y . [3]

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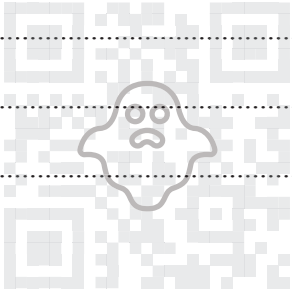
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