1 The cubic equation $x^3 + bx^2 + d = 0$ has roots α, β, γ, where $\alpha = \beta$ and $d \neq 0$.

(a) Show that $4b^3 + 27d = 0$. [5]

(b) Given that $2\alpha^2 + \gamma^2 = 3b$, find the values of b and d. [3]

2 Prove by mathematical induction that, for all positive integers n, $7^{2n} + 97^n - 50$ is divisible by 48. [6]

3 (a) By considering $(2r+1)^3 - (2r-1)^3$, use the method of differences to prove that

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1).$$

[5]

Let $S_n = 1^2 + 3\times 2^2 + 3^2 + 3\times 4^2 + 5^2 + 3\times 6^2 + \ldots + \left(2+(-1)^n\right)n^2$.

(b) Show that $S_{2n} = \frac{1}{3}n(2n+1)(an+b)$, where a and b are integers to be determined. [3]

(c) State the value of $\lim\limits_{n\to\infty} \dfrac{S_{2n}}{n^3}$. [1]

4 The plane Π contains the lines $\mathbf{r} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k} + \lambda(-\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ and $\mathbf{r} = 4\mathbf{i} + 4\mathbf{j} + 2\mathbf{k} + \mu(3\mathbf{i} + 2\mathbf{j} - \mathbf{k})$.

(a) Find a Cartesian equation of Π, giving your answer in the form $ax + by + cz = d$. [4]

The line l passes through the point P with position vector $2\mathbf{i}+3\mathbf{j}+\mathbf{k}$ and is parallel to the vector $\mathbf{j}+\mathbf{k}$.

(b) Find the acute angle between l and Π. [3]

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(c) Find the position vector of the foot of the perpendicular from P to Π. [4]

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5 The matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} \frac{1}{2}\sqrt{2} & -\frac{1}{2}\sqrt{2} \\ \frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \end{pmatrix}\begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}$, where k is a constant.

(a) The matrix $\mathbf{M}$ represents a sequence of two geometrical transformations.

State the type of each transformation, and make clear the order in which they are applied. [2]

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(b) The triangle ABC in the x–y plane is transformed by $\mathbf{M}$ onto triangle DEF.

Find, in terms of k, the single matrix which transforms triangle DEF onto triangle ABC. [2]

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(c) Find the set of values of k for which the transformation represented by $\mathbf{M}$ has no invariant lines through the origin. [7]

6 **(a)** Show that the curve with Cartesian equation

$$(x^2+y^2)^2 = 36(x^2-y^2)$$

has polar equation $r^2 = 36\cos 2\theta$. [3]

The curve C has polar equation $r^2 = 36\cos 2\theta$, for $-\frac{1}{4}\pi \leqslant \theta \leqslant \frac{1}{4}\pi$.

(b) Sketch C and state the maximum distance of a point on C from the pole. [3]

(c) Find the area of the region enclosed by C. [2]

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(d) Find the maximum distance of a point on C from the initial line, giving the answer in exact form. [6]

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7 The curve C has equation $y = \frac{5x^2}{5x-2}$.

(a) Find the equations of the asymptotes of C. [3]

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(b) Find the coordinates of the stationary points on C. [4]

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(c) Sketch C. [3]

(d) Sketch the curve with equation $y = \left|\frac{5x^2}{5x-2}\right|$ and find in exact form the set of values of x for which $\left|\frac{5x^2}{5x-2}\right| < 2$. [6]