

1 The cubic equation $x^3 + bx^2 + d = 0$ has roots α, β, γ , where $\alpha = \beta$ and $d \neq 0$.

(a) Show that $4b^3 + 27d = 0$. [5]

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(b) Given that $2\alpha^2 + \gamma^2 = 3b$, find the values of b and d . [3]

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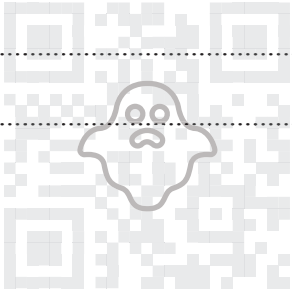
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3 (a) By considering $(2r + 1)^3 - (2r - 1)^3$, use the method of differences to prove that

$$\sum_{r=1}^n r^2 = \frac{1}{6}n(n + 1)(2n + 1). \quad [5]$$

A series of horizontal dotted lines provided for the student to write their proof.



The line l passes through the point P with position vector $2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ and is parallel to the vector $\mathbf{j} + \mathbf{k}$.

(b) Find the acute angle between l and Π . [3]

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(c) Find the position vector of the foot of the perpendicular from P to Π . [4]

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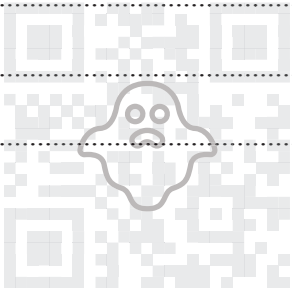
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5 The matrix \mathbf{M} is given by $\mathbf{M} = \begin{pmatrix} \frac{1}{2}\sqrt{2} & -\frac{1}{2}\sqrt{2} \\ \frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \end{pmatrix} \begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}$, where k is a constant.

(a) The matrix \mathbf{M} represents a sequence of two geometrical transformations.

State the type of each transformation, and make clear the order in which they are applied. [2]

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(b) The triangle ABC in the x - y plane is transformed by \mathbf{M} onto triangle DEF .

Find, in terms of k , the single matrix which transforms triangle DEF onto triangle ABC . [2]

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6 (a) Show that the curve with Cartesian equation

$$(x^2 + y^2)^2 = 36(x^2 - y^2)$$

has polar equation $r^2 = 36 \cos 2\theta$. [3]

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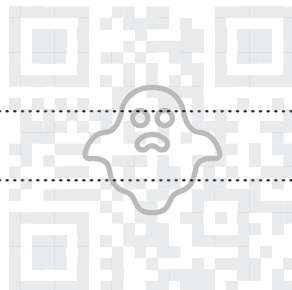
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The curve C has polar equation $r^2 = 36 \cos 2\theta$, for $-\frac{1}{4}\pi \leq \theta \leq \frac{1}{4}\pi$.

(b) Sketch C and state the maximum distance of a point on C from the pole. [3]



7 The curve C has equation $y = \frac{5x^2}{5x-2}$.

(a) Find the equations of the asymptotes of C . [3]

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(b) Find the coordinates of the stationary points on C . [4]

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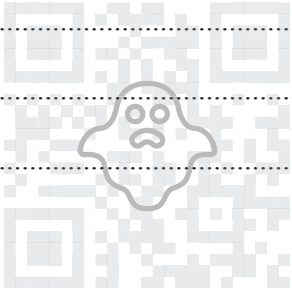
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(c) Sketch C .

[3]

(d) Sketch the curve with equation $y = \left| \frac{5x^2}{5x-2} \right|$ and find in exact form the set of values of x for which $\left| \frac{5x^2}{5x-2} \right| < 2$. [6]



A series of horizontal dotted lines for writing.

