1 Find the roots of the equation $z^3 = 7\sqrt{3} - 7i$, giving your answers in the form $re^{i\theta}$, where $r > 0$ and $-\pi \leqslant \theta < \pi$. [5]

2 **(a)** Find the coefficient of x^2 in the Maclaurin's series for $-\ln\cos x$. [4]

(b) Find the length of the arc of the curve with equation $y = -\ln\cos x$ from the point where $x = 0$ to the point where $x = \frac{1}{4}\pi$. [4]

3   The matrix **A** is given by

$$\mathbf{A} = \begin{pmatrix} 6 & -9 & 5 \\ 5 & -8 & 5 \\ 1 & -1 & 2 \end{pmatrix}.$$

(a) Find the eigenvalues of **A**.   [4]
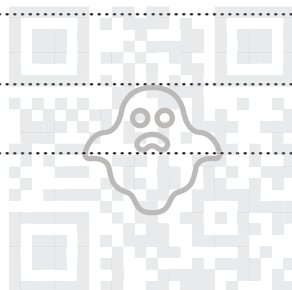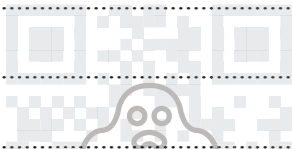
(b) Use the characteristic equation of **A** to show that $\mathbf{A}^{-1} = p\mathbf{A}^2 + q\mathbf{I}$, where p and q are constants to be determined. [3]

4 It is given that

$$x = -t + \tan^{-1} t \qquad \text{and} \qquad y = t + \sinh^{-1} t.$$

(a) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{t^2 + 1 + \sqrt{t^2 + 1}}{t^2}$. [4]
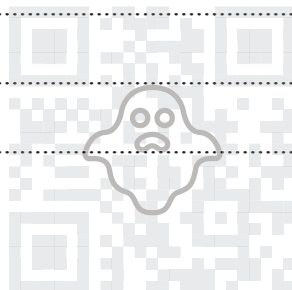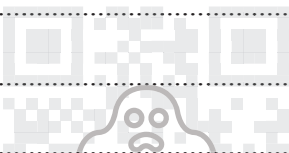
(b) Find the value of $\dfrac{d^2y}{dx^2}$ when $t = \frac{3}{4}$. [5]

5 Find the solution of the differential equation
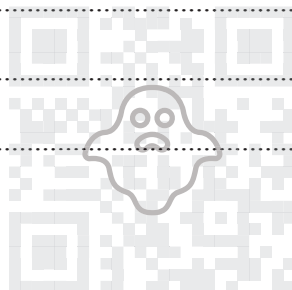
$$x(x+7)\frac{dy}{dx}+7y=x$$

for which $y = 7$ when $x = 1$. Give your answer in the form $y = f(x)$. [9]

6

The diagram shows the curve with equation $y = \ln(1+x)$ for $0 \leqslant x \leqslant 1$, together with a set of n rectangles each of width $\frac{1}{n}$.
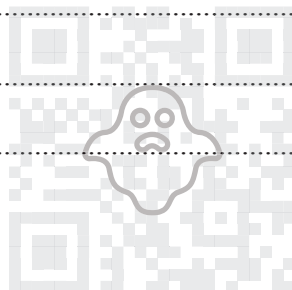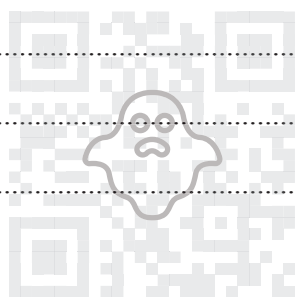
(a) By considering the sum of the areas of these rectangles, show that $\int_0^1 \ln(1+x)\,dx < U_n$, where
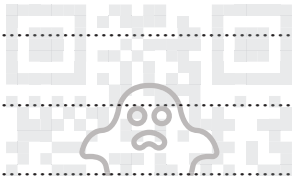
$$U_n = \frac{1}{n}\ln\frac{(2n)!}{n!} - \ln n.$$ [4]

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(b) Use a similar method to find, in terms of n, a lower bound L_n for $\int_0^1 \ln(1+x)\,dx$. [4]

(c) By simplifying $U_n - L_n$, show that $\lim_{n \to \infty}(U_n - L_n) = 0$. [2]

7 The variables x and y are related by the differential equation

$$4\frac{d^2y}{dx^2} - y = 3.$$

It is given that, when $x = 0$, $y = -3$ and $\frac{dy}{dx} = 2$.

(a) Find y in terms of x. [8]

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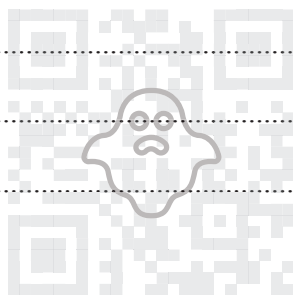
(b) Deduce the exact value of x for which $y = 0$. Give your answer in logarithmic form. [3]

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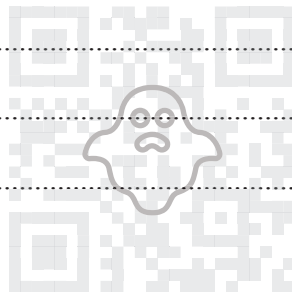
8 **(a)** Find $\int \sin\theta \cos^n\theta \, d\theta$, where $n \neq -1$. [2]

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Let $I_{m,n} = \displaystyle\int_0^{\frac{1}{2}\pi} \sin^m\theta \cos^n\theta \, d\theta$.

(b) Show that, for $m \geqslant 2$ and $n \geqslant 0$,

$$I_{m,n} = \frac{m-1}{m+n} I_{m-2,n}.$$

[5]

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(c) By considering the binomial expansion of $\left(z+\dfrac{1}{z}\right)^5$, where $z = \cos\theta + i\sin\theta$, use de Moivre's theorem to show that

$$\cos^5\theta = a\cos 5\theta + b\cos 3\theta + c\cos\theta,$$

where a, b and c are constants to be determined. [5]

(d) Using the results given in parts **(b)** and **(c)**, find the exact value of $I_{2,5}$. [4]