

- 1 Find the roots of the equation  $z^3 = 7\sqrt{3} - 7i$ , giving your answers in the form  $re^{i\theta}$ , where  $r > 0$  and  $-\pi \leq \theta < \pi$ . [5]

Handwriting practice paper with 20 horizontal lines and a QR code at the bottom center.



- 2 (a)** Find the coefficient of  $x^2$  in the Maclaurin's series for  $-\ln \cos x$ . [4]

[illegible]

- (b)** Find the length of the arc of the curve with equation  $y = -\ln \cos x$  from the point where  $x = 0$  to the point where  $x = \frac{1}{4}\pi$ . [4]

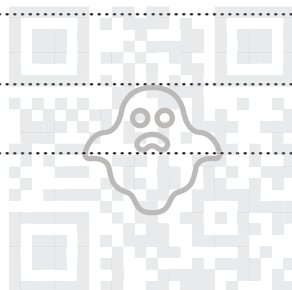


**3** The matrix  $\mathbf{A}$  is given by

$$\mathbf{A} = \begin{pmatrix} 6 & -9 & 5 \\ 5 & -8 & 5 \\ 1 & -1 & 2 \end{pmatrix}.$$

(a) Find the eigenvalues of  $\mathbf{A}$ . [4]


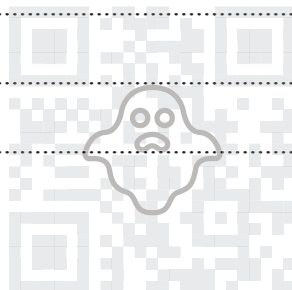
[4]

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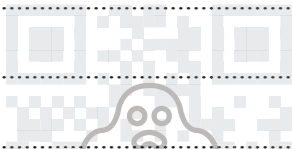
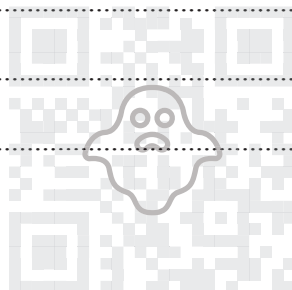
- (b)** Use the characteristic equation of  $\mathbf{A}$  to show that  $\mathbf{A}^{-1} = p\mathbf{A}^2 + q\mathbf{I}$ , where  $p$  and  $q$  are constants to be determined. [3]

This image shows a full page of white paper with horizontal blue or grey ruling lines, typical of notebook paper. The lines are evenly spaced and run across the width of the page. In the bottom right corner, there is a small, faint watermark consisting of a QR code and a cartoon ghost icon.



$$x = -t + \tan^{-1} t \quad \text{and} \quad y = t + \sinh^{-1} t.$$
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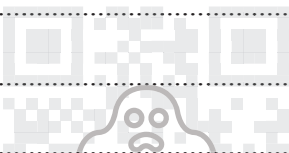
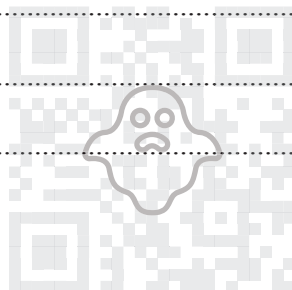
**(b)** Find the value of  $\frac{d^2y}{dx^2}$  when  $t = \frac{3}{4}$ . [5]

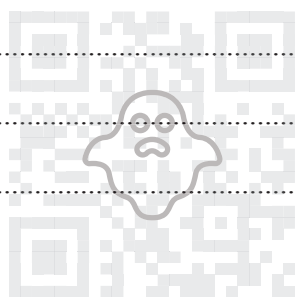
A QR code is located at the bottom center of the page, below the main body of horizontal lines. It is a standard black and white matrix code used for digital linking.

$$x(x+7)\frac{dy}{dx} + 7y = x$$

for which  $y = 7$  when  $x = 1$ . Give your answer in the form  $y = f(x)$ .

[9]

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(a) By considering the sum of the areas of these rectangles, show that  $\int_0^1 \ln(1+x) dx < U_n$ , where

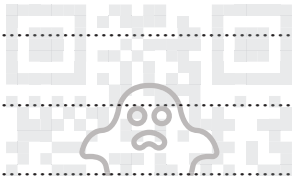
$$U_n = \frac{1}{n} \ln \frac{(2n)!}{n!} - \ln n. \quad [4]$$

A QR code is located at the bottom center of the page. Overlaid on the bottom center of the QR code is a small, stylized ghost icon with large eyes and a small mouth.

(b) Use a similar method to find, in terms of  $n$ , a lower bound  $L_n$  for  $\int_0^1 \ln(1+x) dx$ . [4]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page, providing a template for handwriting practice or general writing. There are no margins, text, or other markings on the page.

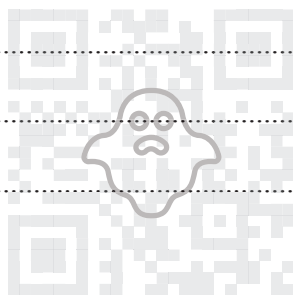
(c) By simplifying  $U_n - L_n$ , show that  $\lim_{n \rightarrow \infty} (U_n - L_n) = 0$ . [2]

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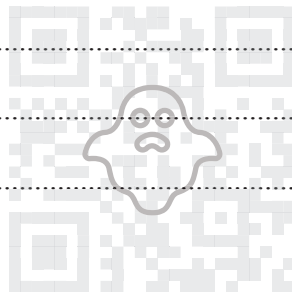
$$4\frac{d^2y}{dx^2} - y = 3.$$

(a) Find  $y$  in terms of  $x$ .

[8]



- (b) Deduce the exact value of  $x$  for which  $y = 0$ . Give your answer in logarithmic form. [3]

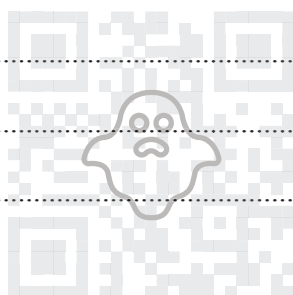


- 8 (a)** Find  $\int \sin \theta \cos^n \theta \, d\theta$ , where  $n \neq -1$ . [2]

Let  $I_{m,n} = \int_0^{\frac{1}{2}\pi} \sin^m \theta \cos^n \theta d\theta$ .

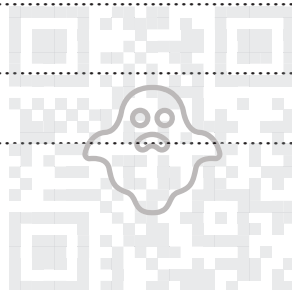
- (b) Show that, for  $m \geq 2$  and  $n \geq 0$ ,

$$I_{m,n} = \frac{m-1}{m+n} I_{m-2,n}. \quad [5]$$



- $$\cos^5 \theta = a \cos 5\theta + b \cos 3\theta + c \cos \theta,$$

[5]

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[4]

Blank lined paper for writing.

