Assessment overview

Paper 1

Further Pure Mathematics 1 — 2 hours

75 marks

6 to 8 structured questions based on the Further Pure Mathematics 1 subject content

Answer all questions

Written examination

Externally assessed

60% of the AS Level

30% of the A Level

Compulsory for AS Level and A Level

Paper 2

Further Pure Mathematics 2 — 2 hours

75 marks

7 to 9 structured questions based on the Further Pure Mathematics 2 subject content

Answer all questions

Written examination

Externally assessed

30% of the A Level only

Compulsory for A Level

Paper 3

Further Mechanics — 1 hour 30 minutes

50 marks

5 to 7 structured questions based on the Further Mechanics subject content

Answer all questions

Written examination

Externally assessed

40% of the AS Level

20% of the A Level

Offered as part of AS Level or A Level

Paper 4

Further Probability & Statistics — 1 hour 30 minutes

50 marks

5 to 7 structured questions based on the Further Probability & Statistics subject content

Answer all questions

Written examination

Externally assessed

40% of the AS Level

20% of the A Level

Offered as part of AS Level or A Level

1 Further Pure Mathematics 1 (for Paper 1)

1.1 Roots of polynomial equations

Candidates should be able to:	Notes and examples
• recall and use the relations between the roots and coefficients of polynomial equations	e.g. to evaluate symmetric functions of the roots or to solve problems involving unknown coefficients in equations; restricted to equations of degree 2, 3 or 4 only.
• use a substitution to obtain an equation whose roots are related in a simple way to those of the original equation.	Substitutions will not be given for the easiest cases, e.g. where the new roots are reciprocals or squares or a simple linear function of the old roots.

1.2 Rational functions and graphs

Candidates should be able to:	Notes and examples
• sketch graphs of simple rational functions, including the determination of oblique asymptotes, in cases where the degree of the numerator and the denominator are at most 2	Including determination of the set of values taken by the function, e.g. by the use of a discriminant. Detailed plotting of curves will not be required, but sketches will generally be expected to show significant features, such as turning points, asymptotes and intersections with the axes.
• understand and use relationships between the graphs of $y = \mathrm{f}(x)$, $y^2 = \mathrm{f}(x)$, $y = \dfrac{1}{\mathrm{f}(x)}$, $y = \lvert \mathrm{f}(x) \rvert$ and $y = \mathrm{f}(\lvert x \rvert)$.	Including use of such sketch graphs in the course of solving equations or inequalities.

1.3 Summation of series

Candidates should be able to:	Notes and examples
• use the standard results for $\sum r$, $\sum r^2$, $\sum r^3$ to find related sums	
• use the method of differences to obtain the sum of a finite series	Use of partial fractions to express a general term in a suitable form may be required.
• recognise, by direct consideration of a sum to n terms, when a series is convergent, and find the sum to infinity in such cases.	

1 Further Pure Mathematics 1

1.4 Matrices

Candidates should be able to:	Notes and examples
• carry out operations of matrix addition, subtraction and multiplication, and recognise the terms zero matrix and identity (or unit) matrix	Including non-square matrices. Matrices will have at most 3 rows and columns.
• recall the meaning of the terms 'singular' and 'non-singular' as applied to square matrices and, for 2×2 and 3×3 matrices, evaluate determinants and find inverses of non-singular matrices	The notations $\det \mathbf{M}$ for the determinant of a matrix $\mathbf{M}$, and $\mathbf{I}$ for the identity matrix, will be used.
• understand and use the result, for non-singular matrices, $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$	Extension to the product of more than two matrices may be required.
• understand the use of 2×2 matrices to represent certain geometric transformations in the x-y plane, in particular – understand the relationship between the transformations represented by $\mathbf{A}$ and $\mathbf{A}^{-1}$ – recognise that the matrix product $\mathbf{AB}$ represents the transformation that results from the transformation represented by $\mathbf{B}$ followed by the transformation represented by $\mathbf{A}$ – recall how the area scale factor of a transformation is related to the determinant of the corresponding matrix – find the matrix that represents a given transformation or sequence of transformations	Understanding of the terms 'rotation', 'reflection', 'enlargement', 'stretch' and 'shear' for 2D transformations will be required. Other 2D transformations may be included, but no particular knowledge of them is expected.
• understand the meaning of 'invariant' as applied to points and lines in the context of transformations represented by matrices, and solve simple problems involving invariant points and invariant lines.	e.g. to locate the invariant points of the transformation represented by $\begin{pmatrix} 6 & 5 \\ 2 & 3 \end{pmatrix}$, or to find the invariant lines through the origin for $\begin{pmatrix} 4 & -1 \\ 2 & 1 \end{pmatrix}$, or to show that any line with gradient 1 is invariant for $\begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}$.

1 Further Pure Mathematics 1

1.5 Polar coordinates

Candidates should be able to:	Notes and examples
• understand the relations between Cartesian and polar coordinates, and convert equations of curves from Cartesian to polar form and vice versa	The convention $r \geqslant 0$ will be used.
• sketch simple polar curves, for $0 \leqslant \theta < 2\pi$ or $-\pi < \theta \leqslant \pi$ or a subset of either of these intervals	Detailed plotting of curves will not be required, but sketches will generally be expected to show significant features, such as symmetry, coordinates of intersections with the initial line, the form of the curve at the pole and least/greatest values of r.
• recall the formula $\frac{1}{2}\int r^2 \mathrm{d}\theta$ for the area of a sector, and use this formula in simple cases.	

1.6 Vectors

Candidates should be able to:	Notes and examples
• use the equation of a plane in any of the forms $ax + by + cz = d$ or $\mathbf{r.n} = p$ or $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$ and convert equations of planes from one form to another as necessary in solving problems	
• recall that the vector product $\mathbf{a} \times \mathbf{b}$ of two vectors can be expressed either as $\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\sin\theta\,\hat{\mathbf{n}}$, where $\hat{\mathbf{n}}$ is a unit vector, or in component form as $(a_2b_3 - a_3b_2)\mathbf{i} + (a_3b_1 - a_1b_3)\mathbf{j} + (a_1b_2 - a_2b_1)\mathbf{k}$	
• use equations of lines and planes, together with scalar and vector products where appropriate, to solve problems concerning distances, angles and intersections, including – determining whether a line lies in a plane, is parallel to a plane or intersects a plane, and finding the point of intersection of a line and a plane when it exists – finding the foot of the perpendicular from a point to a plane – finding the angle between a line and a plane, and the angle between two planes – finding an equation for the line of intersection of two planes – calculating the shortest distance between two skew lines – finding an equation for the common perpendicular to two skew lines.	

1 Further Pure Mathematics 1

1.7 Proof by induction

Candidates should be able to:	Notes and examples
• use the method of mathematical induction to establish a given result	e.g. $\sum_{r=1}^{n} r^4 = \frac{1}{4}n^2(n+1)^2$, $u_n = \frac{1}{2}(1+3^{n-1})$ for the sequence given by $u_{n+1} = 3u_n - 1$ and $u_1 = 1$, $\begin{pmatrix} 4 & -1 \\ 6 & -1 \end{pmatrix}^n = \begin{pmatrix} 3\times 2^n - 2 & 1-2^n \\ 3\times 2^{n+1} - 6 & 3-2^{n+1} \end{pmatrix}$, $3^{2n} + 2\times 5^n - 3$ is divisible by 8.
• recognise situations where conjecture based on a limited trial followed by inductive proof is a useful strategy, and carry this out in simple cases.	e.g. find the nth derivative of $x\,\mathrm{e}^x$, find $\sum_{r=1}^{n} r \times r!$.

2 Further Pure Mathematics 2 (for Paper 2)

Knowledge of Paper 1: Further Pure Mathematics 1 subject content from this syllabus is assumed for this component.

2.1 Hyperbolic functions

Candidates should be able to:	Notes and examples
• understand the definitions of the hyperbolic functions $\sinh x$, $\cosh x$, $\tanh x$, $\operatorname{sech} x$, $\operatorname{cosech} x$, $\coth x$ in terms of the exponential function	
• sketch the graphs of hyperbolic functions	
• prove and use identities involving hyperbolic functions	e.g. $\cosh^2 x - \sinh^2 x \equiv 1$, $\sinh 2x \equiv 2\sinh x \cosh x$, and similar results corresponding to the standard trigonometric identities.
• understand and use the definitions of the inverse hyperbolic functions and derive and use the logarithmic forms.	

2.2 Matrices

Candidates should be able to:	Notes and examples
• formulate a problem involving the solution of 3 linear simultaneous equations in 3 unknowns as a problem involving the solution of a matrix equation, or vice versa	
• understand the cases that may arise concerning the consistency or inconsistency of 3 linear simultaneous equations, relate them to the singularity or otherwise of the corresponding matrix, solve consistent systems, and interpret geometrically in terms of lines and planes	e.g. three planes meeting in a common point, or in a common line, or having no common points.
• understand the terms 'characteristic equation', 'eigenvalue' and 'eigenvector', as applied to square matrices	Including use of the definition $\mathbf{Ae} = \lambda\mathbf{e}$ to prove simple properties, e.g. that λ^n is an eigenvalue of $\mathbf{A}^n$.
• find eigenvalues and eigenvectors of 2×2 and 3×3 matrices	Restricted to cases where the eigenvalues are real and distinct.
• express a square matrix in the form $\mathbf{QDQ}^{-1}$, where $\mathbf{D}$ is a diagonal matrix of eigenvalues and $\mathbf{Q}$ is a matrix whose columns are eigenvectors, and use this expression	e.g. in calculating powers of 2×2 or 3×3 matrices.
• use the fact that a square matrix satisfies its own characteristic equation.	e.g. in finding successive powers of a matrix or finding an inverse matrix; restricted to 2×2 or 3×3 matrices only.

2 Further Pure Mathematics 2

2.3 Differentiation

Candidates should be able to:	Notes and examples
• differentiate hyperbolic functions and differentiate $\sin^{-1}x$, $\cos^{-1}x$, $\sinh^{-1}x$, $\cosh^{-1}x$ and $\tanh^{-1}x$	
• obtain an expression for $\frac{d^2y}{dx^2}$ in cases where the relation between x and y is defined implicitly or parametrically	
• derive and use the first few terms of a Maclaurin's series for a function.	Derivation of a general term is not included, but successive 'implicit' differentiation steps may be required, e.g. for $y = \tan x$ following an initial differentiation rearranged as $y' = 1 + y^2$.

2.4 Integration

Candidates should be able to:	Notes and examples
• integrate hyperbolic functions and recognise integrals of functions of the form $\frac{1}{\sqrt{a^2 - x^2}}$, $\frac{1}{\sqrt{x^2 + a^2}}$ and $\frac{1}{\sqrt{x^2 - a^2}}$, and integrate associated functions using trigonometric or hyperbolic substitutions as appropriate	Including use of completing the square where necessary, e.g. to integrate $\frac{1}{\sqrt{x^2 + x}}$.
• derive and use reduction formulae for the evaluation of definite integrals	e.g. $\int_0^{\frac{1}{2}\pi} \sin^n x \, dx$, $\int_0^1 e^{-x}(1 - x)^n \, dx$. In harder cases hints may be given, e.g. $\int_0^{\frac{1}{4}\pi} \sec^n x \, dx$ by considering $\frac{d}{dx}(\tan x \sec^n x)$.
• understand how the area under a curve may be approximated by areas of rectangles, and use rectangles to estimate or set bounds for the area under a curve or to derive inequalities or limits concerning sums	Questions may involve either rectangles of unit width or rectangles whose width can tend to zero, e.g. $1 + \ln n > \sum_{r=1}^{n} \frac{1}{r} > \ln(n + 1)$, $\sum_{r=1}^{n} \frac{1}{n}\left(1 + \frac{r}{n}\right)^{-1} \approx \int_0^1 (1 + x)^{-1} \, dx$.
• use integration to find – arc lengths for curves with equations in Cartesian coordinates, including the use of a parameter, or in polar coordinates – surface areas of revolution about one of the axes for curves with equations in Cartesian coordinates, including the use of a parameter.	Any questions involving integration may require techniques from Cambridge International A Level Mathematics (9709) applied to more difficult cases, e.g. integration by parts for $\int e^x \sin x \, dx$, or use of the substitution $t = \tan\frac{1}{2}x$. Surface areas of revolution for curves with equations in polar coordinates will not be required.

2 Further Pure Mathematics 2

2.5 Complex numbers

Candidates should be able to:	Notes and examples
• understand de Moivre's theorem, for a positive or negative integer exponent, in terms of the geometrical effect of multiplication and division of complex numbers	
• prove de Moivre's theorem for a positive integer exponent	e.g. by induction.
• use de Moivre's theorem for a positive or negative rational exponent	e.g. expressing $\cos 5\theta$ in terms of $\cos\theta$ or $\tan 5\theta$ in terms of $\tan\theta$.
– to express trigonometrical ratios of multiple angles in terms of powers of trigonometrical ratios of the fundamental angle	e.g. expressing $\sin^6\theta$ in terms of $\cos 2\theta$, $\cos 4\theta$ and $\cos 6\theta$.
– to express powers of $\sin\theta$ and $\cos\theta$ in terms of multiple angles	
– in the summation of series	e.g. using the '$C + \mathrm{i}S$' method to sum series such as $\sum_{r=1}^{n}\binom{n}{r}\sin r\theta$.
– in finding and using the nth roots of unity.	

2.6 Differential equations

Candidates should be able to:	Notes and examples
• find an integrating factor for a first order linear differential equation, and use an integrating factor to find the general solution	e.g. $\frac{\mathrm{d}y}{\mathrm{d}x} - 2y = x^2$, $x\frac{\mathrm{d}y}{\mathrm{d}x} - y = x^4$, $\frac{\mathrm{d}y}{\mathrm{d}x} + y\coth x = \cosh x$.
• recall the meaning of the terms 'complementary function' and 'particular integral' in the context of linear differential equations, and recall that the general solution is the sum of the complementary function and a particular integral	
• find the complementary function for a first or second order linear differential equation with constant coefficients	For second order equations, including the cases where the auxiliary equation has distinct real roots, a repeated real root or conjugate complex roots.
• recall the form of, and find, a particular integral for a first or second order linear differential equation in the cases where a polynomial or $a\mathrm{e}^{bx}$ or $a\cos px + b\sin px$ is a suitable form, and in other simple cases find the appropriate coefficient(s) given a suitable form of particular integral	e.g. evaluate k given that $kx\cos 2x$ is a particular integral of $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 4y = \sin 2x$.

continued

2 Further Pure Mathematics 2

2.6 Differential equations continued

Candidates should be able to:	Notes and examples
• use a given substitution to reduce a differential equation to a first or second order linear equation with constant coefficients or to a first order equation with separable variables	e.g. the substitution $x = \mathrm{e}^t$ to reduce to linear form a differential equation with terms of the form $ax^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + bx\frac{\mathrm{d}y}{\mathrm{d}x} + cy$, or the substitution $y = ux$ to reduce $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x+y}{x-y}$ to separable form.
• use initial conditions to find a particular solution to a differential equation, and interpret a solution in terms of a problem modelled by a differential equation.	

3 Further Mechanics (for Paper 3)

Knowledge of Cambridge International AS & A Level Mathematics (9709) Paper 4: Mechanics subject content is assumed for this component.

3.1 Motion of a projectile

Candidates should be able to:	Notes and examples
• model the motion of a projectile as a particle moving with constant acceleration and understand any limitations of the model	Vector methods are not required.
• use horizontal and vertical equations of motion to solve problems on the motion of projectiles, including finding the magnitude and direction of the velocity at a given time or position, the range on a horizontal plane and the greatest height reached	
• derive and use the Cartesian equation of the trajectory of a projectile, including problems in which the initial speed and/or angle of projection may be unknown.	Knowledge of the 'bounding parabola' for accessible points is not included.

3.2 Equilibrium of a rigid body

Candidates should be able to:	Notes and examples
• calculate the moment of a force about a point	For questions involving coplanar forces only; understanding of the vector nature of moments is not required.
• use the result that the effect of gravity on a rigid body is equivalent to a single force acting at the centre of mass of the body, and identify the position of the centre of mass of a uniform body using considerations of symmetry	
• use given information about the position of the centre of mass of a triangular lamina and other simple shapes	Proofs of results given in the MF19 List of formulae are not required.
• determine the position of the centre of mass of a composite body by considering an equivalent system of particles	Simple cases only, e.g. a uniform L-shaped lamina, or a uniform cone joined at its base to a uniform hemisphere of the same radius.
• use the principle that if a rigid body is in equilibrium under the action of coplanar forces then the vector sum of the forces is zero and the sum of the moments of the forces about any point is zero, and the converse of this	
• solve problems involving the equilibrium of a single rigid body under the action of coplanar forces, including those involving toppling or sliding.	

3 Further Mechanics

3.3 Circular motion

Candidates should be able to:	Notes and examples
• understand the concept of angular speed for a particle moving in a circle, and use the relation $v = r\omega$	
• understand that the acceleration of a particle moving in a circle with constant speed is directed towards the centre of the circle, and use the formulae $r\omega^2$ and $\frac{v^2}{r}$.	Proof of the acceleration formulae is not required.
• solve problems which can be modelled by the motion of a particle moving in a horizontal circle with constant speed	
• solve problems which can be modelled by the motion of a particle in a vertical circle without loss of energy.	Including finding a normal contact force or the tension in a string, locating points at which these are zero, and conditions for complete circular motion.

3.4 Hooke's law

Candidates should be able to:	Notes and examples
• use Hooke's law as a model relating the force in an elastic string or spring to the extension or compression, and understand the term modulus of elasticity	
• use the formula for the elastic potential energy stored in a string or spring	Proof of the formula is not required.
• solve problems involving forces due to elastic strings or springs, including those where considerations of work and energy are needed.	e.g. a particle moving horizontally or vertically or on an inclined plane while attached to one or more strings or springs, or a particle attached to an elastic string acting as a 'conical pendulum'.

3.5 Linear motion under a variable force

Candidates should be able to:	Notes and examples
• solve problems which can be modelled as the linear motion of a particle under the action of a variable force, by setting up and solving an appropriate differential equation.	Including use of $v\frac{\mathrm{d}v}{\mathrm{d}x}$ for acceleration, where appropriate. Calculus required is restricted to content from Pure Mathematics 3 in Cambridge International A Level Mathematics (9709). Only differential equations in which the variables are separable are included.

3 Further Mechanics

3.6 Momentum

Candidates should be able to:	Notes and examples
• recall Newton's experimental law and the definition of the coefficient of restitution, the property $0 \leqslant e \leqslant 1$, and the meaning of the terms 'perfectly elastic' $(e = 1)$ and 'inelastic' $(e = 0)$ • use conservation of linear momentum and/or Newton's experimental law to solve problems that may be modelled as the direct or oblique impact of two smooth spheres, or the direct or oblique impact of a smooth sphere with a fixed surface.	

4 Further Probability & Statistics (for Paper 4)

Knowledge of Cambridge International AS & A Level Mathematics (9709) Papers 5 and 6: Probability & Statistics subject content is assumed for this component.

Please see the support document *Guide to prior learning for Paper 4 Further Probability & Statistics* on the Cambridge website for recommended prior knowledge for this paper.

4.1 Continuous random variables

Candidates should be able to:	Notes and examples
• use a probability density function which may be defined piecewise	
• use the general result $\mathrm{E}(\mathrm{g}(X)) = \int \mathrm{f}(x)\mathrm{g}(x)\,\mathrm{d}x$ where $\mathrm{f}(x)$ is the probability density function of the continuous random variable X and $\mathrm{g}(X)$ is a function of X	
• understand and use the relationship between the probability density function (PDF) and the cumulative distribution function (CDF), and use either to evaluate probabilities or percentiles	
• use cumulative distribution functions (CDFs) of related variables in simple cases.	e.g. given the CDF of a variable X, find the CDF of a related variable Y, and hence its PDF, e.g. where $Y = X^3$.

4.2 Inference using normal and *t*-distributions

Candidates should be able to:	Notes and examples
• formulate hypotheses and apply a hypothesis test concerning the population mean using a small sample drawn from a normal population of unknown variance, using a *t*-test	
• calculate a pooled estimate of a population variance from two samples	Calculations based on either raw or summarised data may be required.
• formulate hypotheses concerning the difference of population means, and apply, as appropriate – a 2-sample *t*-test – a paired sample *t*-test – a test using a normal distribution	The ability to select the test appropriate to the circumstances of a problem is expected.
• determine a confidence interval for a population mean, based on a small sample from a normal population with unknown variance, using a *t*-distribution	
• determine a confidence interval for a difference of population means, using a *t*-distribution or a normal distribution, as appropriate.	

4 Further Probability & Statistics

4.3 χ^2-tests

Candidates should be able to:	Notes and examples
• fit a theoretical distribution, as prescribed by a given hypothesis, to given data	Questions will not involve lengthy calculations.
• use a χ^2-test, with the appropriate number of degrees of freedom, to carry out the corresponding goodness of fit analysis	Classes should be combined so that each expected frequency is at least 5.
• use a χ^2-test, with the appropriate number of degrees of freedom, for independence in a contingency table.	Yates' correction is not required. Where appropriate, either rows or columns should be combined so that the expected frequency in each cell is at least 5.

4.4 Non-parametric tests

Candidates should be able to:	Notes and examples
• understand the idea of a non-parametric test and appreciate situations in which such a test might be useful	e.g. when sampling from a population which cannot be assumed to be normally distributed.
• understand the basis of the sign test, the Wilcoxon signed-rank test and the Wilcoxon rank-sum test	Including knowledge that Wilcoxon tests are valid only for symmetrical distributions.
• use a single-sample sign test and a single-sample Wilcoxon signed-rank test to test a hypothesis concerning a population median	Including the use of normal approximations where appropriate. Questions will not involve tied ranks or observations equal to the population median value being tested.
• use a paired-sample sign test, a Wilcoxon matched-pairs signed-rank test and a Wilcoxon rank-sum test, as appropriate, to test for identity of populations.	Including the use of normal approximations where appropriate. Questions will not involve tied ranks or zero-difference pairs.

4.5 Probability generating functions

Candidates should be able to:	Notes and examples
• understand the concept of a probability generating function (PGF) and construct and use the PGF for given distributions	Including the discrete uniform, binomial, geometric and Poisson distributions.
• use formulae for the mean and variance of a discrete random variable in terms of its PGF, and use these formulae to calculate the mean and variance of a given probability distribution	
• use the result that the PGF of the sum of independent random variables is the product of the PGFs of those random variables.	