Please check the examination details below before entering your candidate information

| Candidate surname | Other names |
|---|---|
| | |

Centre Number | Candidate Number

# Pearson Edexcel Level 3 GCE

## Monday 13 May 2024

Afternoon (Time: 1 hour 40 minutes)

Paper reference **8FM0/01**

# Further Mathematics

## Advanced Subsidiary

## PAPER 1: Core Pure Mathematics

**You must have:**
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

**Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 80.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

*Turn over* ▶

**1.** The cubic equation

$$2x^3 - 3x^2 + 5x + 7 = 0$$

has roots $\alpha$, $\beta$ and $\gamma$.

Without solving the equation, determine the exact value of

(i) $\alpha^2 + \beta^2 + \gamma^2$

**(3)**

(ii) $\dfrac{3}{\alpha} + \dfrac{3}{\beta} + \dfrac{3}{\gamma}$

**(3)**

(iii) $(5 - \alpha)(5 - \beta)(5 - \gamma)$

**(3)**

**Question 1 continued**

**Question 1 continued**

**Question 1 continued**

**(Total for Question 1 is 9 marks)**

**2.** With respect to the **right-hand rule**, a rotation through $\theta°$ anticlockwise about the $z$-axis is represented by the matrix

$$\begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Given that the matrix $\mathbf{M}$, where

$$\mathbf{M} = \begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

represents a rotation through $\alpha°$ anticlockwise about the $z$-axis with respect to the **right-hand rule**,

(a) determine the value of $\alpha$.

**(1)**

(b) Hence determine the smallest possible positive integer value of $k$ for which $\mathbf{M}^k = \mathbf{I}$

**(2)**

The $3 \times 3$ matrix $\mathbf{N}$ represents a reflection in the plane with equation $y = 0$

(c) Write down the matrix $\mathbf{N}$.

**(1)**

The point $A$ has coordinates $(-2, 4, 3)$

The point $B$ is the image of the point $A$ under the transformation represented by matrix $\mathbf{M}$ followed by the transformation represented by matrix $\mathbf{N}$.

(d) Show that the coordinates of $B$ are $\left(2 + \sqrt{3}, 2\sqrt{3} - 1, 3\right)$

**(2)**

Given that $O$ is the origin,

(e) show that, to 3 significant figures, the size of angle $AOB$ is $66.9°$

**(2)**

(f) Hence determine the area of triangle $AOB$, giving your answer to 3 significant figures.

**(2)**

**Question 2 continued**

**Question 2 continued**

**Question 2 continued**

**(Total for Question 2 is 10 marks)**

**3.** (a) Use the standard results for summations to show that, for all positive integers $n$,

$$\sum_{r=1}^{n} r^2(r+1) = \frac{1}{12}n(n+1)(n+2)(an+b)$$

where $a$ and $b$ are integers to be determined.

**(4)**

(b) Hence show that, for all positive integers $k$,

$$\sum_{r=k+1}^{3k} r^2(r+1) = \frac{1}{3}k(3k+1)(Ak^2+Bk+C)$$

where $A$, $B$ and $C$ are integers to be determined.

**(3)**

(c) Hence, using algebra and making your method clear, determine the value of $k$ for which

$$25\sum_{r=k+1}^{3k} r^2(r+1) = 192k^3(3k+1)$$

**(3)**

**Question 3 continued**

**Question 3 continued**

**Question 3 continued**

**(Total for Question 3 is 10 marks)**

**4.**

$$\mathbf{A} = \begin{pmatrix} -1 & -2 & -7 \\ 3 & k & 2 \\ 1 & 1 & 4 \end{pmatrix} \qquad \mathbf{B} = \begin{pmatrix} 4k-2 & 1 & 7k-4 \\ -10 & 3 & -19 \\ 3-k & -1 & 6-k \end{pmatrix}$$

where $k$ is a constant.

(a) Determine the value of the constant $c$ for which

$$\mathbf{AB} = (3k + c)\mathbf{I}$$

**(2)**

(b) Hence determine the value of $k$ for which $\mathbf{A}^{-1}$ does not exist.

**(2)**

Given that $\mathbf{A}^{-1}$ does exist,

(c) write down $\mathbf{A}^{-1}$ in terms of $k$.

**(1)**

(d) Use the answer to part (c) to solve the simultaneous equations

$$\begin{aligned} -x - 2y - 7z &= 10 \\ 3x + ky + 2z &= 3 \\ x + y + 4z &= 1 \end{aligned}$$

giving the values of $x$, $y$ and $z$ in simplest form in terms of $k$.

**(3)**

**Question 4 continued**

**Question 4 continued**

**Question 4 continued**

**(Total for Question 4 is 8 marks)**

**5.** Given that on an Argand diagram the locus of points defined by $|z + 5 - 12i| = 10$ is a circle,

(a) write down,

(i) the coordinates of the centre of this circle,

(ii) the radius of this circle.

**(2)**

(b) Show, by shading on an Argand diagram, the set of points defined by

$$|z + 5 - 12i| \leqslant 10$$

**(1)**

(c) For the set of points defined in part (b), determine the maximum value of $|z|$

**(3)**

The set of points $A$ is defined by

$$A = \{z : 0 \leqslant \arg(z + 5 - 20i) \leqslant \pi\} \cap \{z : |z + 5 - 12i| \leqslant 10\}$$

(d) Determine the area of the region defined by $A$, giving your answer to 3 significant figures.

**(4)**

**Question 5 continued**

**Question 5 continued**

**Question 5 continued**

**(Total for Question 5 is 10 marks)**

**6.** The drainage system for a sports field consists of underground pipes.

This situation is modelled with respect to a fixed origin $O$.

According to the model,

- the surface of the sports field is a plane with equation $z = 0$
- the pipes are straight lines
- one of the pipes, $P_1$, passes through the points $A(3, 4, -2)$ and $B(-2, -8, -3)$
- a different pipe, $P_2$, has equation $\frac{x-1}{2} = \frac{y-3}{4} = \frac{z+1}{-2}$
- the units are metres

(a) Determine a vector equation of the line representing the pipe $P_1$

**(2)**

(b) Determine the coordinates of the point at which the pipe $P_1$ meets the surface of the playing field, according to the model.

**(2)**

Determine, according to the model,

(c) the acute angle between pipes $P_1$ and $P_2$, giving your answer in degrees to 3 significant figures,

**(3)**

(d) the shortest distance between pipes $P_1$ and $P_2$

**(5)**

**Question 6 continued**

**Question 6 continued**

**Question 6 continued**

**(Total for Question 6 is 12 marks)**

**7.** (i) Prove by induction that, for all positive integers $n$,

$$\sum_{r=1}^{n} \frac{1}{r(r+1)} = \frac{n}{n+1}$$

**(5)**

(ii) Prove by induction that, for all positive integers $n$,

$$f(n) = 3^{2n+4} - 2^{2n}$$

is divisible by 5

**(5)**

**Question 7 continued**

**Question 7 continued**

**Question 7 continued**

**(Total for Question 7 is 10 marks)**

**8.**

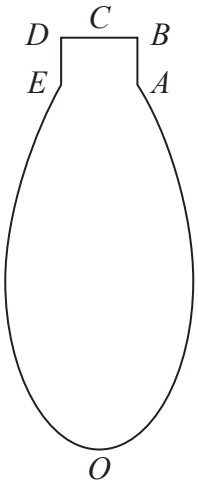


**Figure 1**

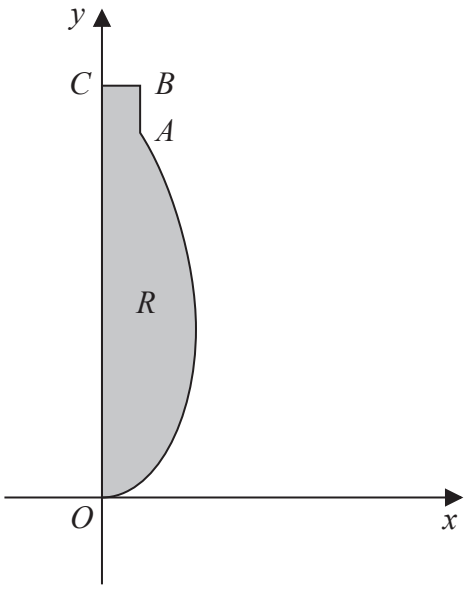


**Figure 2**

Figure 1 shows the central vertical cross-section, *OABCDEO*, of the design for a solid glass ornament.

Figure 2 shows the finite region, *R*, which is bounded by the *y*-axis, the horizontal line *CB*, the vertical line *BA*, and the curve *AO*.

The ornament is formed by rotating the region *R* through 360° about the *y*-axis.

The curve *AO* is modelled by the equation

$$x = ky^2 + \sqrt{y} \qquad 0 \leqslant y \leqslant 4$$

where $k$ is a constant.

The point *A* has coordinates (0.4, 4) and the point *B* has coordinates (0.4, 4.5)

The units are centimetres.

(a) Determine the value of $k$ according to this model.

**(2)**

(b) Use algebraic integration to determine the exact volume of glass that would be required to make the ornament, according to the model.

**(7)**

(c) State a limitation of the model.

**(1)**

When the ornament was manufactured, 9 $cm^3$ of glass was required.

(d) Use this information and your answer to part (b) to evaluate the model, explaining your reasoning.

**(1)**

**Question 8 continued**

**Question 8 continued**

**(Total for Question 8 is 11 marks)**

**TOTAL FOR CORE PURE MATHEMATICS IS 80 MARKS**