

VECTORS

[ESTIMATED TIME: 70 minutes]



52

GCSE

(+ IGCSE) EXAM QUESTION PRACTICE

1.

[5 marks]

$$\mathbf{a} = \begin{pmatrix} 5 \\ -2 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} 1 \\ 7 \end{pmatrix} \quad \mathbf{c} = \begin{pmatrix} -7 \\ 0 \end{pmatrix}$$

(a) Write, as a column vector, $2\mathbf{a}$

$\begin{pmatrix} \\ \end{pmatrix}$
.....
(1)

(b) Write, as a column vector, $3\mathbf{b} - \mathbf{c}$

$\begin{pmatrix} \\ \end{pmatrix}$
.....
(2)

(c) Work out the magnitude of \mathbf{a}
Give your answer as a surd.

.....
(2)

2.

[2 marks]

$ABCD$ is a parallelogram.

$$\vec{BC} = \begin{pmatrix} 5 \\ -1 \end{pmatrix} \quad \vec{DC} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$$

Find \vec{BD} as a column vector.

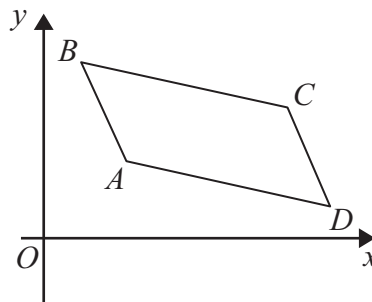


Diagram NOT
accurately drawn



3.

[2 marks]

A is the point with coordinates $(2, 3)$.

$$\overrightarrow{AB} = \begin{pmatrix} 5 \\ -4 \end{pmatrix}.$$

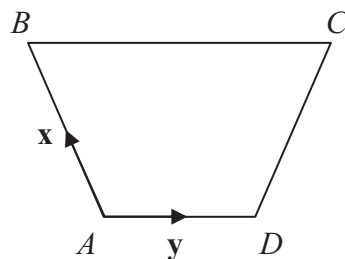
Find the coordinates of B .

(.....,)

4.

[4 marks]

The diagram shows a trapezium $ABCD$.



$$\overrightarrow{BC} = 2\overrightarrow{AD}.$$

$$\overrightarrow{AB} = \mathbf{x}, \quad \overrightarrow{AD} = \mathbf{y}.$$

(a) Find, in terms of \mathbf{x} and \mathbf{y} ,

(i) \overrightarrow{AC}

.....

(ii) \overrightarrow{DC}

.....

(2)

(b) The point E is such that $\overrightarrow{AE} = \mathbf{x} + \mathbf{y}$.

Use your answer to part (a)(ii) to explain why $AECD$ is a parallelogram.

.....

.....

(2)



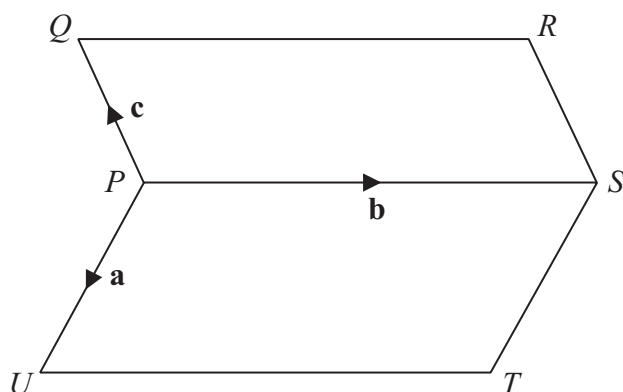


Diagram **NOT**
accurately drawn

$PQRS$ and $PSTU$ are parallelograms.

$$\vec{PU} = \mathbf{a} \quad \vec{PS} = \mathbf{b} \quad \vec{PQ} = \mathbf{c}$$

Find, in terms of \mathbf{a} , \mathbf{b} and \mathbf{c}

(i) \vec{TQ}

(ii) \vec{PX} where X is the midpoint of TQ .

Simplify your answer as much as possible.



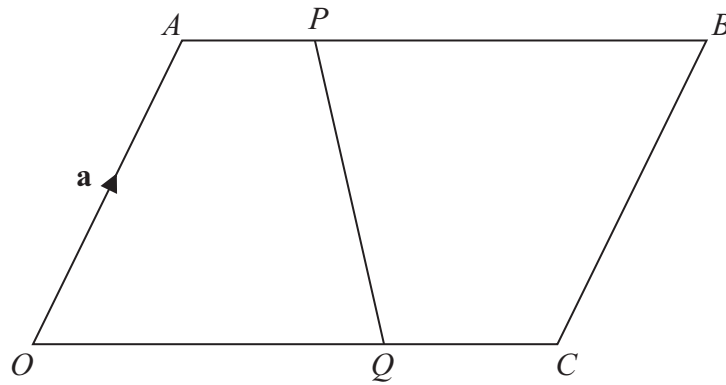


Diagram **NOT**
accurately drawn

$OABC$ is a parallelogram.

$\vec{OA} = \mathbf{a}$ and $\vec{OC} = \mathbf{c}$

P is the point on AB such that $AP = \frac{1}{4}AB$.

Q is the point on OC such that $OQ = \frac{2}{3}OC$.

Find, in terms of \mathbf{a} and \mathbf{c} , \vec{PQ} .

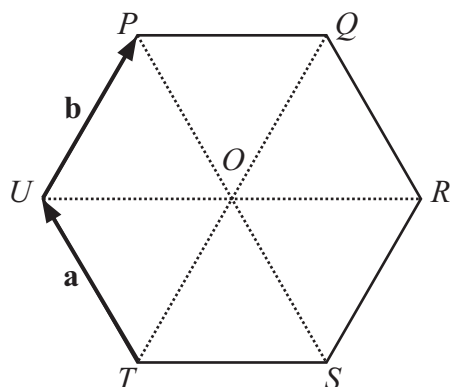
Give your answer in its simplest form.

.....



$PQRSTU$ is a regular hexagon, centre O .

The hexagon is made from six equilateral triangles of side 2.5 cm.



$$\overrightarrow{TU} = \mathbf{a}, \overrightarrow{UP} = \mathbf{b}.$$

(a) Find, in terms of \mathbf{a} and/or \mathbf{b} , the vectors

(i) \overrightarrow{TP}

.....
(1)

(ii) \overrightarrow{PO}

.....
(1)

(iii) \overrightarrow{UO}

.....
(1)

(b) Find the modulus (magnitude) of \overrightarrow{UR} .

..... cm
(1)



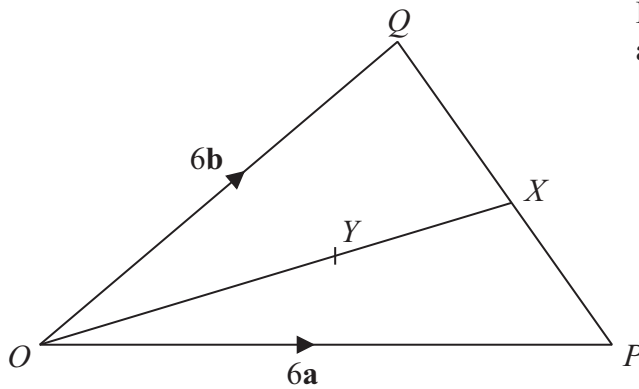


Diagram **NOT**
accurately drawn

In triangle OPQ , $\vec{OP} = 6\mathbf{a}$ and $\vec{OQ} = 6\mathbf{b}$

X is the midpoint of PQ .

- (a) Find, in terms of \mathbf{a} and \mathbf{b} , the vector \vec{OX}
Give your answer in its simplest form.

.....
(2)

Y is the point on OX such that $OY:YX = 2:1$

- (b) Find, in terms of \mathbf{a} and \mathbf{b} , the vector \vec{QY}
Give your answer in its simplest form.

.....
(2)



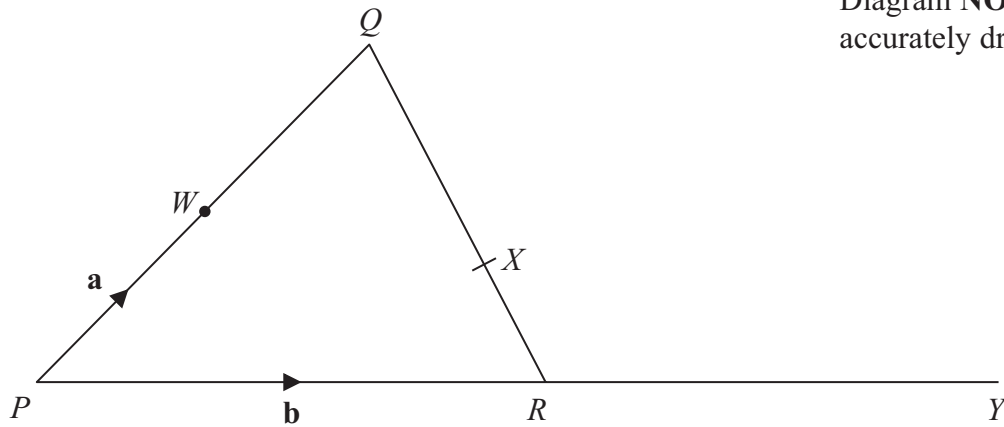


Diagram **NOT**
accurately drawn

PQR is a triangle.

The midpoint of PQ is W .

X is the point on QR such that $QX : XR = 2 : 1$

PRY is a straight line.

$$\overrightarrow{PW} = \mathbf{a} \quad \overrightarrow{PR} = \mathbf{b}$$

(a) Find, in terms of \mathbf{a} and \mathbf{b} ,

(i) \overrightarrow{QR}

(ii) \overrightarrow{QX}

(iii) \overrightarrow{WX}

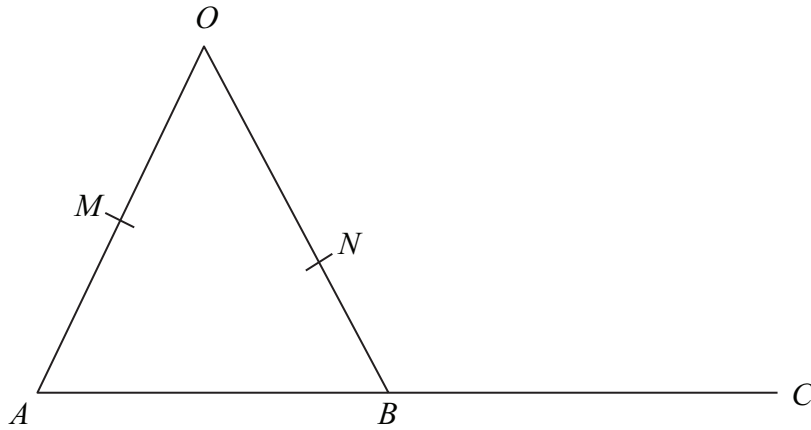
(3)

R is the midpoint of the straight line PRY .

(b) Use a vector method to show that WXY is a straight line.

(2)





OMA , ONB and ABC are straight lines.

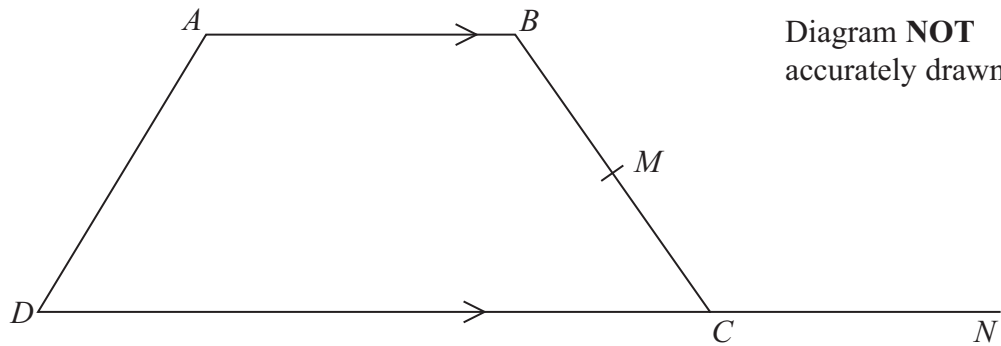
M is the midpoint of OA .

B is the midpoint of AC .

$\vec{OA} = 6\mathbf{a}$ $\vec{OB} = 6\mathbf{b}$ $\vec{ON} = k\mathbf{b}$ where k is a scalar quantity.

Given that MNC is a straight line, find the value of k .





AB is parallel to DC

$$DC = 2AB$$

M is the midpoint of BC

$$\vec{AD} = 2\mathbf{b}$$

$$\vec{AB} = 4\mathbf{a}$$

(a) Find \vec{BM} in terms of \mathbf{a} and \mathbf{b} .

Give your answer in its simplest form.

.....
(2)

N is the point such that DCN is a straight line and $DC : CN = 2 : 1$

(b) Show that AMN is a straight line.

(2)



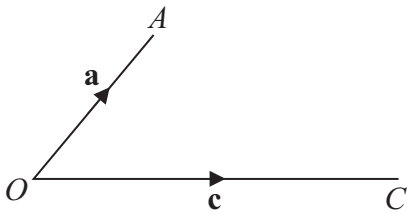


Diagram **NOT**
accurately drawn

In the diagram $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$.

(a) Find \overrightarrow{CA} in terms of \mathbf{a} and \mathbf{c} .

.....
(1)

(b) The point B is such that $\overrightarrow{AB} = \frac{1}{2} \mathbf{c}$.

Give the mathematical name for the quadrilateral $OABC$.

.....
(1)

(c) The point P is such that $\overrightarrow{OP} = \mathbf{a} + k\mathbf{c}$, where $k \geq 0$

State the two conditions relating to $\mathbf{a} + k\mathbf{c}$ that must be true for $OAPC$ to be a rhombus.

(2)



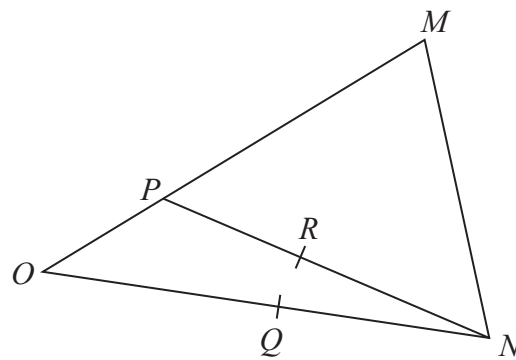


Diagram **NOT**
accurately drawn

OMN is a triangle.

P is the point on OM such that $OP = \frac{1}{4} OM$

Q is the midpoint of ON

R is the midpoint of PQ

$$\vec{OP} = \mathbf{p} \quad \vec{OQ} = \mathbf{q}$$

(a) Find, in terms of \mathbf{p} and \mathbf{q} ,

(i) \vec{MN}

(ii) \vec{PR}

.....

.....

(2)

(b) Use a vector method to prove that QR is parallel to OP

(2)

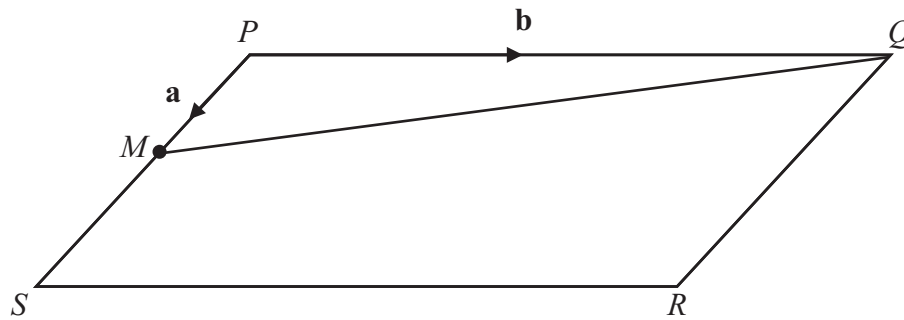


The diagram shows a parallelogram, $PQRS$.

M is the midpoint of PS .

$$\overrightarrow{PM} = \mathbf{a} \quad \overrightarrow{PQ} = \mathbf{b}$$

Diagram **NOT**
accurately drawn



(a) Find, in terms of \mathbf{a} and/or \mathbf{b} ,

(i) \overrightarrow{PS}

.....

(ii) \overrightarrow{PR}

.....

(iii) \overrightarrow{MQ}

.....

(3)

N is the point on MQ such that $MN = \frac{1}{3}MQ$

(b) Use a vector method to prove that PNR is a straight line.

(2)



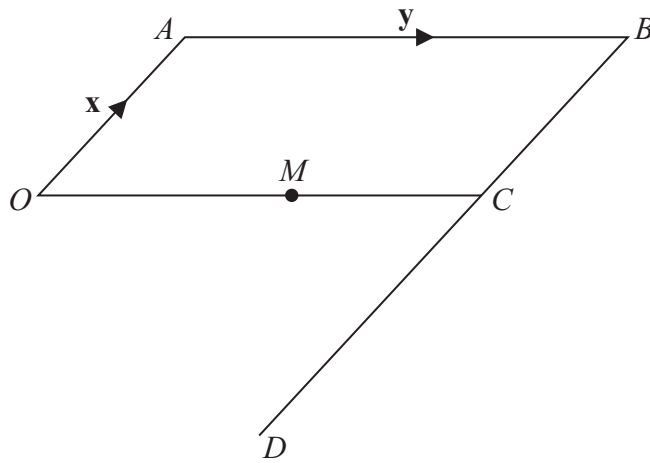


Diagram **NOT**
accurately drawn

$OABC$ is a parallelogram.

BCD is a straight line.

$BD = 3BC$.

M is the midpoint of OC .

$\vec{OA} = \mathbf{x}$ $\vec{AB} = \mathbf{y}$

(a) Find, in terms of \mathbf{x} and \mathbf{y} ,

(i) \vec{AM}

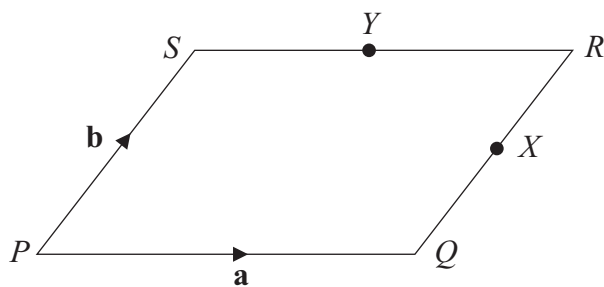
(ii) \vec{OD}

(2)

(b) Use your answers to (a)(i) and (ii) to write down two different geometric facts about the lines AM and OD .

(2)





$PQRS$ is a parallelogram.

X is the midpoint of QR and Y is the midpoint of SR .

$\vec{PQ} = \mathbf{a}$ and $\vec{PS} = \mathbf{b}$.

(a) Write down, in terms of \mathbf{a} and \mathbf{b} , expressions for

(i) \vec{PX}

.....

(ii) \vec{PY}

.....

(iii) \vec{QS}

.....

(3)

(b) Use a vector method to show that XY is parallel to QS and that $XY = \frac{1}{2}QS$.

(2)



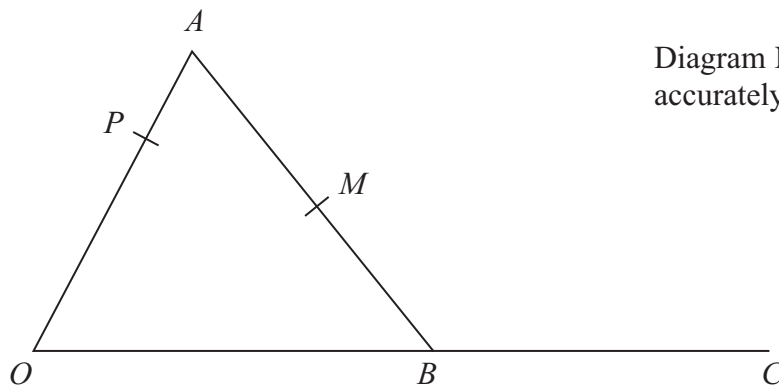


Diagram **NOT**
accurately drawn

OAB is a triangle.

P is the point on OA such that $OP:PA = 2:1$

C is the point such that B is the midpoint of OC .

M is the midpoint of AB .

$$\vec{OA} = 6\mathbf{a}$$

$$\vec{OB} = 4\mathbf{b}$$

Show that PMC is a straight line.



$OABC$ is a parallelogram.

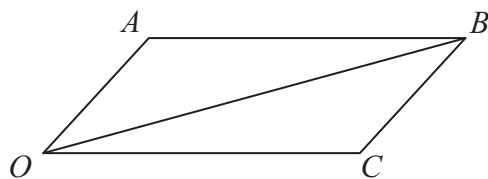


Diagram **NOT**
accurately drawn

$$\vec{OA} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \quad \vec{OC} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}.$$

- (a) Find the vector \vec{OB} as a column vector.

$$\begin{pmatrix} \\ \end{pmatrix}$$

.....

(1)

X is the point on OB such that $OX = kOB$, where $0 < k < 1$

- (b) Find, in terms of k , the vectors

(i) \vec{OX} ,

.....

(ii) \vec{AX} ,

.....

(iii) \vec{XC} .

.....

(3)

- (c) Find the value of k for which $\vec{AX} = \vec{XC}$.

.....

(2)

- (d) Use your answer to part (c) to show that the diagonals of the parallelogram $OABC$ bisect one another.

.....

.....

.....

(2)

