# Pearson Edexcel International GCSE in Mathematics (Specification A) (Modular) (4XMAH)

## Two-year Scheme of Work: Higher Tier

For first teaching from September 2024

# Introduction

This Scheme of Work is a suggested teaching order to enable the completion of the **International GCSE Mathematics (Specification A) (Modular) Higher Tier** qualification content. It provides an approximate teaching time of 160 hours, with 20 hours for revision and practice of examination questions. There is a separate Scheme of Work for the Foundation Tier.

It is not intended to be prescriptive. This document is editable to allow for any adaptations you may wish to make to best suit your teaching style and learner needs.

This Scheme of Work is based on a five-term (sixth term to be used for revision) teaching model over two years for Higher Tier students. This Scheme of Work matches the Higher Tier course planner.

For schools that start their school year in September, the teaching towards Unit 1 assumes two full terms of teaching and then revision time before the Unit 1 examination, in the summer series. Following the Unit 1 examination, the Scheme of Work then assumes that students will move on to study some of the content for Unit 2. As the Unit 1 examinations are in May/June, it is assumed that students will be starting their Unit 2 studies before they complete the end of the first full year of the course.

The Scheme of Work therefore assumes just less than 3 full terms of studying and revision before the Unit 2 examinations. If your students complete their studies early towards the Unit 1 examination, you can begin teaching the Unit 2 content before moving back to revision for the Unit 1 examination.

This document can also be used directly as a Scheme of Work for the International GCSE Mathematics (Specification A) Linear (4MA1) qualification.

Each content area contains:

- Specification references
- Recommended teaching time, although this is adaptable according to individual teaching needs
- Objectives for students
- Possible success criteria for students
- Opportunities for reasoning/problem solving
- Common misconceptions
- Notes for general mathematical teaching points.

Teachers should be aware that the guided learning hours/teaching hours are approximate and should be used as a guideline only.

## Unit 1 course overview

| Content area reference | Content area title | Guided Learning Hours (GLH) | Year and Term |
|---|---|---|---|
| 1 | Decimals | 2 | |
| 2 | Fractions and percentages | 2 | |
| 3 | Ratio and proportion 1 | 2 | |
| 4 | Surds and powers | 3 | |
| 5 | Degree of accuracy | 4 | |
| 6 | Set language, notation and Venn diagrams | 4 | **Year 1, Term 1, Total GLH 35** |
| 7 | Algebraic manipulation 1 | 5 | |
| 8 | Linear equations | 2 | |
| 9 | Linear graphs | 4 | |
| 10 | Quadratic equations, inequalities and graphs | 7 | |
| 11 | Compound measures | 4 | |
| 12 | Geometry of shapes 1 | 5 | |
| 13 | Perimeter, area and volume 1 | 5 | |
| 14 | Pythagoras' theorem and trigonometry | 6 | **Year 1, Term 2, Total GLH 35** |
| 15 | Advanced trigonometry | 7 | |
| 16 | Graphical representation of data 1 | 3 | |
| 17 | Probability | 5 | |
| | **Unit 1 total** | **70** | |
| | **Revision of Unit 1** | **10** | **Year 1, Term 3 - 1st half** |

## Unit 2 course overview

| Content area reference | Content area title | Guided Learning Hours (GLH) | Year and Term |
|---|---|---|---|
| 18 | Special numbers | 2 | **Year 1, Term 3 - 2nd half, Total GLH 20** |
| 19 | Percentages | 3 | |
| 20 | Ratio and proportion 2 | 4 | |
| 21 | Indices and standard form | 3 | |
| 22 | Proof | 4 | |
| 23 | Expressions, formulae and rearranging formulae | 4 | |
| 24 | Inequalities | 3 | **Year 2, Term 1, Total GLH 35** |
| 25 | Sequences | 4 | |
| 26 | Graphs of inequalities | 4 | |
| 27 | Harder graphs and transformation of graphs | 6 | |
| 28 | Simultaneous equations | 4 | |
| 29 | Function notation | 6 | |
| 30 | Calculus | 7 | |
| 31 | Geometry of shapes 2 | 2 | |
| 32 | Constructions and bearings | 4 | **Year 2, Term 2, Total GLH 35** |
| 33 | Perimeter, area and volume 2 | 4 | |
| 34 | Transformations | 4 | |
| 35 | Circle properties | 6 | |
| 36 | Similar shapes | 5 | |
| 37 | Vectors | 6 | |
| 38 | Graphical representation of data 2 | 2 | |
| 39 | Statistical measures | 4 | |
| | **Unit 2 total** | **90** | |
| | **Revision of Unit 2** | **10** | **Year 2, Term 3** |
| | **Unit 1 and Unit 2: Total GLH** | **180** | |

# Unit 1: Higher Tier

**It is assumed that students being prepared for the Higher Tier will have knowledge of Unit 1 Foundation Tier subject content.**

| **1. Decimals** | **Teaching time**<br>1-3 hours |
|---|---|

**OBJECTIVES**

| Foundation Ref | Higher Ref | |
|---|---|---|
| | **H1.3A** | convert recurring decimals into fractions |
| **F1.8B** | | round to a given number of significant figures or decimal places |
| **F1.8D** | | use estimation to evaluate approximations to numerical calculations |
| **F1.11A** | | use a scientific electronic calculator to determine numerical results |

**POSSIBLE SUCCESS CRITERIA**

Estimate the value of $\dfrac{34.5 \times 7.34}{0.154}$

Change $0.4\dot{5}$ into a fraction in its simplest form.

**OPPORTUNITIES FOR REASONING/PROBLEM SOLVING**

Use of decimals within a problem.

Show algebraically that $3.0\dot{1}$ can be written as $3\dfrac{1}{90}$

Links with other areas of mathematics can be made by using surds in Pythagoras' theorem and when using trigonometric ratios.

**COMMON MISCONCEPTIONS**

Significant figure and decimal place rounding are often confused.
Some students may think 35 934 = 36 to two significant figures.

**NOTES**

The expectation for Higher Tier is that much of this work will be reinforced throughout the course.
Make sure students are absolutely clear about the difference between significant figures and decimal places.

## 2. Fractions and percentages

**Teaching time**
1-3 hours

### OBJECTIVES

| Foundation Ref | Higher Ref | |
|---|---|---|
| **F1.6B** | | express a given number as a percentage of another number |
| **F1.6C** | | express a percentage as a fraction and as a decimal |
| **F1.2D** | | order fractions and calculate a given fraction of a given quantity |
| **F1.2E** | | express a given number as a fraction of another number |
| **F1.2F** | | use common denominators to add and subtract fractions and mixed numbers |
| **F1.2G** | | convert a fraction to a decimal or a percentage |
| **F1.2H** | | understand and use unit fractions as multiplicative inverses |
| **F1.2I** | | multiply and divide fractions and mixed numbers |

### POSSIBLE SUCCESS CRITERIA

Express a given number as a fraction of another, including where the fraction is, for example, greater than 1, e.g. $\frac{120}{100} = 1\frac{2}{10} = 1\frac{1}{5}$

Prove whether a fraction is terminating or recurring.

Convert a fraction to a decimal including where the fraction is greater than 1

Convince me that 0.125 is $\frac{1}{8}$

Work out 56 cm as a percentage of 2.5 m.

### OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Many of these topics provide opportunities for reasoning in real-life contexts, particularly percentages.
Calculate original values and evaluate statements in relation to this value justifying which statement is correct.

### COMMON MISCONCEPTIONS

The larger the denominator, the larger the fraction.

Incorrect links between fractions and decimals, such as thinking that $\frac{1}{5} = 0.15$, 5% = 0.5, 4% = 0.4, etc.

### NOTES

Ensure that you include fractions where only one of the denominators needs to be changed, in addition to where both need to be changed for addition and subtraction.

Include multiplying and dividing integers by fractions. Encourage use of the fraction button.

Students should be reminded of basic percentages.

| **3. Ratio and proportion 1** | **Teaching time**<br>1-3 hours |
|---|---|

## OBJECTIVES

| Foundation Ref | Higher Ref | |
|---|---|---|
| **F1.10A** | | use and apply number in everyday personal, domestic or community life |
| **F1.10B** | | carry out calculations using standard units of mass, length, area, volume and capacity |
| **F1.10C** | | understand and carry out calculations using time, and carry out calculations using money, including converting between currencies |

## POSSIBLE SUCCESS CRITERIA

The concept of perimeter and area by measuring lengths of sides will be familiar to students.
Students should know the various metric units.

## OPPORTUNITIES FOR REASONING/PROBLEM-SOLVING

Encourage students to draw a sketch where one is not provided.
Emphasise the functional elements with carpets, tiles for walls, boxes in a larger box, etc. Best value and minimum cost can be incorporated too.
Ensure that examples use different metric units of length, including decimals.
Know the impact of estimating their answers and whether it is an overestimate or underestimate in relation to a given context.

## COMMON MISCONCEPTIONS

Students often get the concepts of area and perimeter confused.

## NOTES

Emphasise the importance of reading the question carefully.
Include ratios with decimals 0.2 : 1

## 4. Surds and powers

**Teaching time**
2-4 hours

### OBJECTIVES

| Foundation Ref | Higher Ref | |
|---|---|---|
| | **H1.4A** | understand the meaning of surds |
| | **H1.4B** | manipulate surds, including rationalising a denominator |
| | **H1.4C** | use index laws to simplify and evaluate numerical expressions involving integer, fractional and negative powers |
| **F1.4C** | | use index notation and index laws for multiplication and division of positive and negative integer powers including zero |

### POSSIBLE SUCCESS CRITERIA

Prove that the square root of 45 lies between 6 and 7

Simplify $\sqrt{40}$

Rationalise the denominator of $\frac{5}{\sqrt{10}}$ ; $\frac{6}{1+\sqrt{2}}$

Evaluate $(2^3 \times 2^5) \div 2^4$, $4^0$, $8^{-\frac{2}{3}}$

Work out the value of $n$ in $40 = 5 \times 2^n$

### OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems that use indices instead of integers will provide rich opportunities to apply the knowledge in this unit in other areas of mathematics.

### COMMON MISCONCEPTIONS

The order of operations is often not applied correctly when squaring negative numbers, and many calculators will reinforce this misconception.

### NOTES

Students need to know how to enter negative numbers into their calculator.
Use the term negative number and not minus number to avoid confusion with calculations.

| 5. Degree of accuracy | Teaching time<br>3-5 hours |
|---|---|

**OBJECTIVES**

| Foundation Ref | Higher Ref | |
|---|---|---|
| **F1.8C** | | identify upper and lower bounds where values are given to a degree of accuracy |
| | **H1.8A** | solve problems using upper and lower bounds where values are given to a degree of accuracy |

**POSSIBLE SUCCESS CRITERIA**

Round 16,000 people to the nearest 1000
Round 1100 g to 1 significant figure.
Work out the upper and lower bounds of a formula where all terms are given to 1 decimal place.
Be able to justify that measurements to the nearest whole unit may be inaccurate by up to one half in either direction.

**OPPORTUNITIES FOR REASONING/PROBLEM SOLVING**

This content area provides many opportunities for students to evaluate their answers and provide counterarguments in mathematical and real-life contexts, in addition to requiring them to understand the implications of rounding their answers.

**COMMON MISCONCEPTIONS**

Students readily accept the rounding for lower bounds but take some convincing in relation to upper bounds.

**NOTES**

Students should use 'half a unit above' and 'half a unit below' to find upper and lower bounds.
Encourage use of a number line when introducing the concept.

## 6. Set language, notation and Venn diagrams

**Teaching time**
3-5 hours

### OBJECTIVES

| Foundation Ref | Higher Ref | |
|---|---|---|
| **F1.5A** | | understand the definition of a set |
| **F1.5B** | | use the set notation $\cup$, $\cap$ and $\in$ and $\notin$ |
| **F1.5C** | | understand the concept of the universal set and the empty set and the symbols for these sets |
| **F1.5D** | | understand and use the complement of a set |
| **F1.5E** | | use Venn diagrams to represent sets |
| **F6.3D** | | find probabilities from a Venn diagram |
| | **H1.5A** | understand sets defined in algebraic terms, and understand and use subsets |
| | **H1.5B** | use Venn diagrams to represent sets and the number of elements in sets |
| | **H1.5C** | use the notation n(*A*) for the number of elements in the set *A* |
| | **H1.5D** | use sets in practical situations |

### POSSIBLE SUCCESS CRITERIA

Universal set is {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
$A$ = {1, 2, 3, 4, 5, 6}, $B$ = {2, 4, 6, 8}; Write down $A \cap B$, $A \cup B$
$C$ = {1, 3, 5}; write down $C'$
Is $4 \in C$, is $4 \in A$, is $C$ a subset of $A$?
Find n($A$).
Draw a Venn diagram to show the universal set, $A$, $B$ and $C$
If a number is picked at random, find P($A \cap B$)

### OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Given Universal set is {1, 2, 3, 4, 5, 6, 7, 8, 9, 10
$A$ = {5, 7, 9} and $B$ = {1, 3, 5, 7}
Write down a possible set $C$ so that $A \cap C$ = {7} and $C$ has 4 members.

### COMMON MISCONCEPTIONS

$A$ = {5, 7, 9} and $B$ = {1, 3, 5, 7} then $A \cup B$ = {1, 3, 5, 5, 7, 7, 9}

### NOTES

When drawing a Venn diagram, it is a good idea to put members in the intersection first.

| 7. Algebraic manipulation | Teaching time<br>4-6 hours |
|---|---|

**OBJECTIVES**

| Foundation Ref | Higher Ref | |
|---|---|---|
| **F2.1B** | | understand that algebraic expressions follow the generalised rules of arithmetic |
| **F2.1C** | | use index notation for positive and negative integer powers (including zero) |
| **F2.2A** | | evaluate expressions by substituting numerical values for letters |
| **F2.2E** | | expand the product of two simple linear expressions |
| **F2.2F** | | understand the concept of a quadratic expression and be able to factorise such expressions (limited to $x^2 + bx + c$) |
| **F2.7A** | | solve quadratic equations by factorisation (limited to $x^2 + bx + c = 0$) |
| | **H2.1A** | use index notation involving fractional, negative and zero powers |
| **F2.1D** | | use index laws in simple cases |
| **F2.2B** | | collect like terms |
| **F2.2C** | | multiply a single term over a bracket |
| **F2.2D** | | take out common factors |
| | **H2.2A** | expand the product of two or more linear expressions |
| | **H2.2B** | understand the concept of a quadratic expression and be able to factorise such expressions |
| | **H2.2C** | manipulate algebraic fractions where the numerator and/or the denominator can be numeric, linear or quadratic |
| | **H2.2D** | complete the square for a given quadratic expression |

**POSSIBLE SUCCESS CRITERIA**

Simplify $4p - 2q^2 + 1 - 3p + 5q^2$.

Simplify $z^4 \times z^3$, $y^3 \div y^2$, $(a^7)^2$, $\left(8x^6y^4\right)^{\frac{1}{3}}$

Factorise $15x^2y - 35x^2y^2$; $6x^2 - 7x + 1$

Expand and simplify $3(t - 1) + 57$; $(3x + 2)(4x - 1)$; $(x + 7)(x - 1)(2x + 1)$

Use fractions when working in algebraic situations.

**OPPORTUNITIES FOR REASONING/PROBLEM SOLVING**

Evaluate statements and justify which answer is correct by providing a counterargument by way of a correct solution.

**COMMON MISCONCEPTIONS**

When expanding two linear expressions, poor number skills involving negatives and times tables will become evident.

**NOTES**

Students will be asked to show ‘algebraic working’ when solving equations. Solutions with no working will score no marks.
Students can leave their answer in fraction form where appropriate. Emphasise that fractions are more accurate in calculations than rounded percentage or decimal equivalents.

## 8. Linear equations

**Teaching time**
1-3 hours

**OBJECTIVES**

| Foundation Ref | Higher Ref | |
|---|---|---|
| **F2.4A** | | solve linear equations, with integer or fractional coefficients, in one unknown in which the unknown appears on either side or both sides of the equation |
| **F2.4B** | | set up simple linear equations from given data |

**POSSIBLE SUCCESS CRITERIA**

Solve 5(x + 3) = 2x – 7

Solve $\frac{2x-1}{3}-\frac{x+1}{2}=5$

**OPPORTUNITIES FOR REASONING/PROBLEM SOLVING**

Problems that require students to justify why certain values in a solution can be ignored.
Set up and solve problems involving linear equations.

**COMMON MISCONCEPTIONS**

When solving equations like $\frac{2x-1}{3}-\frac{x+1}{2}=5$ the common error is to forget to use the negative sign when expanding brackets.

**NOTES**

Students can leave their answers in fractional form where appropriate.
Ensure that correct language is used to avoid reinforcing misconceptions: for example, 0.15 should never be read as 'zero point fifteen.'

## 9. Linear graphs

**Teaching time**
3-5 hours

**OBJECTIVES**

| Foundation Ref | Higher Ref | |
|---|---|---|
| **F3.3A** | | interpret information presented in a range of linear and non-linear graphs |
| **F3.3B** | | understand and use conventions for rectangular Cartesian coordinates |
| **F3.3C** | | plot points ($x$, $y$) in any of the four quadrants or locate points with given coordinates |
| **F3.3D** | | determine the coordinates of points identified by geometrical information |
| **F3.3E** | | determine the coordinates of the midpoint of a line segment, given the coordinates of the two end points |
| **F3.3F** | | draw and interpret straight line conversion graphs |
| **F3.3G** | | find the gradient of a straight line |
| **F3.3H** | | recognise that equations of the form $y = mx + c$ are straight line graphs with gradient m and intercept on the $y$-axis at the point (0, $c$) |
| **F3.3I** | | recognise, generate points and plot graphs of linear and quadratic functions |
| | **H3.3F** | calculate the gradient of a straight line given the coordinates of two points |
| | **H3.3G** | find the equation of a straight line parallel to a given line; find the equation of a straight line perpendicular to a given line |

**POSSIBLE SUCCESS CRITERIA**

Interpret a description of a journey into a distance–time or speed–time graph.
Calculate various measures given a graph.
Find the equation of the line passing through two coordinates by calculating the gradient first.
Understand that the form $y = mx + c$ or $ax + by = c$ represents a straight line.
Find an equation of the line that goes through (1, 2) and is parallel to $3y + 2x = 5$
Find an equation of the line that goes through (1, 2) and is perpendicular to $3y + 2x = 5$

**OPPORTUNITIES FOR REASONING/PROBLEM SOLVING**

Speed/distance graphs can provide opportunities for interpreting non-mathematical problems as a sequence of mathematical processes, whilst also requiring students to justify their reasons why one vehicle is faster than another.
Given an equation of a line, provide a counterargument as to whether or not another equation of a line is parallel or perpendicular to the first line.
Decide if lines are parallel or perpendicular without drawing them and provide reasons.

## COMMON MISCONCEPTIONS

Reading scales incorrectly is a common cause of errors.
Students can find visualisation of a question difficult, especially when dealing with gradients resulting from negative coordinates.

## NOTES

Careful annotation should be encouraged: it is good practice to label the axes and check that students understand the scales.
Use various measures in the distance–time and velocity–time graphs, including miles, kilometres, seconds, and hours.

## 10. Quadratic equations, inequalities and graphs

**Teaching time**
6-8 hours

### OBJECTIVES

| Foundation Ref | Higher Ref | |
|---|---|---|
| | **H2.7A** | solve quadratic equations by factorisation |
| | **H2.7B** | solve quadratic equations by using the quadratic formula or completing the square |
| | **H2.7C** | form and solve quadratic equations from data given in a context |
| **F3.3I** | | recognise, generate points and plot graphs of quadratic functions |
| | **H3.3A** | recognise, plot and draw graphs with equation:<br>$y = Ax^3 + Bx^2 + Cx + D$ in which:<br>(i) the constants are integers and some could be zero<br>(ii) the letters $x$ and $y$ can be replaced with any other two letters or:<br>$$y = Ax^3 + Bx^2 + Cx + D + \frac{E}{x} + \frac{F}{x^2}$$<br>in which:<br>(i) the constants are numerical and at least three of them are zero<br>(ii) the letters $x$ and $y$ can be replaced with any other two letters or:<br>$y = \sin x, y = \cos x, y = \tan x$ for angles of any size (in degrees) |

### POSSIBLE SUCCESS CRITERIA

Solve $3x^2 + 4 = 100$
Solve $2x^2 + 3x + 1 = 0$
Draw the graph of $y = x^2 + 5x + 6$
Know that the quadratic formula can be used to solve all quadratic equations, and often provides a more efficient method than factorising or completing the square.
Have an understanding of solutions that can be written in surd form.
Select and use the correct mathematical techniques to draw graphs.
Identify a variety of functions by the shape of the graph.

### OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems that require students to set up and solve a quadratic equation.
Match equations of quadratics, cubics, reciprocal, trig functions with their graphs by recognising the shape or by sketching.

**COMMON MISCONCEPTIONS**

All working must be shown when solving quadratic equations, including substitution into the quadratic formula.

**NOTES**

Remind students to use brackets for negative numbers when using a calculator and remind them of the importance of knowing when to leave answers in surd form.
Reinforce the fact that some problems may produce one inappropriate solution, which can be ignored.
Clear presentation of working out is essential.
Link with graphical representations.

## 11. Compound measures

**Teaching time**
3-5 hours

### OBJECTIVES

| Foundation Ref | Higher Ref | |
|---|---|---|
| **F4.4G** | | use compound measure such as speed, density and pressure |
| **F4.9A** | | convert measurements within the metric system to include linear and area units |
| **F4.10F** | | convert between units of volume within the metric system |

### POSSIBLE SUCCESS CRITERIA

Find the speed given distance and time.
Find the distance (in km) given the speed (in km/h) and the time (in minutes).
Recall and use the formula for density.
Give the formula for pressure, use it to find one of the variables.
Change 4 $m^2$ into $cm^2$.
Change 45 $mm^2$ into $cm^2$.
Change 3000 $cm^3$ into $m^3$.

### OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Find the mass of an object, having first to find its volume.
Work out the average speed of a journey.

### COMMON MISCONCEPTIONS

Using inconsistent units when solving problems.
Converting time into a decimal incorrectly, e.g. writing 1 hour 15 minutes as 1.15 hours.

### NOTES

Practise converting time into decimals.
Ensure that conversions between metric units are known.
Ensure that consistent units are used when solving problems.

| 12. Geometry of shapes 1 | Teaching time<br>4-6 hours |
|---|---|

## OBJECTIVES

| Foundation Ref | Higher Ref | |
|---|---|---|
| F4.1B | | use angle properties of intersecting lines, parallel lines and angles on a straight line |
| F4.1C | | understand the exterior angle of a triangle property and the angle sum of a triangle property |
| F4.1D | | understand the terms 'isosceles', 'equilateral' and 'right-angled triangles' and the angle properties of these triangles |
| F4.2B | | understand and use the term 'quadrilateral' and the angle sum property of quadrilaterals |
| F4.2C | | understand and use the properties of the parallelogram, rectangle, square, rhombus, trapezium and kite |
| | H4.7A | provide reasons, using standard geometrical statements, to support numerical values for angles obtained in any geometrical context involving lines, polygons and circles |

## POSSIBLE SUCCESS CRITERIA

Name all quadrilaterals that have a specific property.
What is the same and what is different between families of polygons?

## OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Multi-step "angle chasing"-style problems that involve justifying how students have found a specific angle will provide opportunities to develop a chain of reasoning.
Geometrical problems involving algebra, whereby equations can be formed and solved, allow students the opportunity to make and use connections with different parts of mathematics.

## COMMON MISCONCEPTIONS

Some students will think that all trapezia are isosceles, or a square is only square if 'horizontal', or a 'non-horizontal' square is called a diamond.
Incorrectly identifying the 'base angles' (i.e. the equal angles) of an isosceles triangle when not drawn horizontally.

## NOTES

Students must be encouraged to use geometrical language appropriately, 'quote' the appropriate reasons for angle calculations and show step-by-step deduction when solving multi-step problems.
Emphasise that diagrams in examinations are seldom drawn accurately.

## 13. Perimeter, area and volume 1

**Teaching time**
4-6 hours

### OBJECTIVES

| Foundation Ref | Higher Ref | |
|---|---|---|
| **F4.9B** | | find the perimeter of shapes made from triangles and rectangles |
| **F4.9C** | | find the area of simple shapes using the formulae for the areas of triangles and rectangles |
| **F4.9D** | | find the area of parallelograms and trapezia |
| **F4.9E** | | find circumferences and areas of circles using relevant formulae; find perimeters and areas of semicircles |
| | **H4.9A** | find perimeters and areas of sectors of circles |
| **F4.10B** | | understand the terms 'face', 'edge' and 'vertex' in the context of 3D solids |

### POSSIBLE SUCCESS CRITERIA

Calculate the area and/or perimeter of shapes with different units of measurement.

Understand that answers in terms of $\pi$ are more accurate.

Calculate the perimeters and/or areas of circles and sectors of circles given the radius or diameter and vice versa.

### OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Using compound shapes or combinations of polygons that require students to subsequently interpret their result in a real-life context.

Multi-step problems, including the requirement to form and solve equations, provide links with other areas of mathematics.

### COMMON MISCONCEPTIONS

Students often get the concepts of area and perimeter confused.

Students often get the concepts of surface area and volume confused.

### NOTES

Encourage students to draw a sketch where one is not provided.

Ensure that examples use different metric units of length, including decimals.

Emphasise the need to learn the circle formulae.

Ensure that students know it is more accurate to leave answers in terms of $\pi$, but only when asked to do so.

## 14. Pythagoras' theorem and trigonometry

**Teaching time**
5-7 hours

### OBJECTIVES

| Foundation Ref | Higher Ref | |
|---|---|---|
| **F4.8A** | | know, understand and use Pythagoras' theorem in two dimensions |
| **F4.8B** | | know, understand and use sine, cosine and tangent of acute angles to determine lengths and angles of a right-angled triangle |
| **F4.8C** | | apply trigonometrical methods to solve problems in two dimensions |
| | **H4.8A** | understand and use sine, cosine and tangent of obtuse angles |
| | **H4.8B** | understand and use angles of elevation and depression |

### POSSIBLE SUCCESS CRITERIA

Does 2, 3, 6 give a right-angled triangle?
Justify when to use Pythagoras' theorem and when to use trigonometry.

### OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Combined triangle problems that involve consecutive application of Pythagoras' theorem or a combination of Pythagoras' theorem and the trigonometric ratios.
Link to 'real-life' situations, e.g. link with bearings and scale drawings.

### COMMON MISCONCEPTIONS

Answers may be displayed on a calculator in surd form.
Students forget to square root their final answer, or round their answer prematurely.

### NOTES

Students may need reminding about surds.
Scale drawings are not acceptable.
Calculators need to be in degree mode.
Use a suitable mnemonic to remember SOHCAHTOA.
Use Pythagoras' theorem and trigonometry together.

## 15. Advanced trigonometry

**Teaching time**
6-8 hours

### OBJECTIVES

| Foundation Ref | Higher Ref | |
|---|---|---|
| | **H4.8C** | understand and use the sine and cosine rules for any triangle |
| | **H4.8D** | use Pythagoras' theorem in three dimensions |
| | **H4.8E** | understand and use the formula $\frac{1}{2}ab\sin C$ for the area of a triangle |
| | **H4.8F** | apply trigonometrical methods to solve problems in three dimensions, including finding the angle between a line and a plane |

### POSSIBLE SUCCESS CRITERIA

Find the area of a segment of a circle given the radius and length of the chord.
Justify when to use the cosine rule, sine rule, Pythagoras' theorem or normal trigonometric ratios to solve problems.

### OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Triangles formed in a semicircle can provide links with other areas of mathematics.
Multi-step problems requiring the use of both the sine rule and cosine rule.

### COMMON MISCONCEPTIONS

Not using the correct rule or attempting to use 'normal trig' in non-right-angled triangles.
When finding angles, students will often be unable to rearrange the cosine rule or fail to find the inverse of cos $\theta$.

### NOTES

The cosine rule is used when we have SAS and used to find the side opposite the 'included' angle or when we have SSS to find an angle.
Ensure that finding angles with 'normal trig' is refreshed prior to this topic.
Students may find it useful to be reminded of simple geometrical facts, i.e. the shortest side is always opposite the smallest angle in a triangle.
In multi-step questions emphasise the importance of not rounding prematurely and using exact values where appropriate.

| 16. Graphical representation of data 1 | Teaching time 2-4 hours |
|---|---|

## OBJECTIVES

| Foundation Ref | Higher Ref | |
|---|---|---|
| | **H6.1A** | construct and interpret histograms |
| **F6.1A(i)** | | use different methods of presenting data |

## POSSIBLE SUCCESS CRITERIA

Construct histograms from frequency tables.
Know the appropriate uses of histograms.
Use and understand frequency density from histograms.
Design and use two-way tables for discrete and grouped data.
Use information provided to complete a two-way table.

## OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Students should be able to provide a correct solution as a counterargument to statements involving the "averages", e.g. Susan states that the median is 15, she is wrong. Explain why.
Find the median from a histogram.

## COMMON MISCONCEPTIONS

Labelling axes incorrectly in terms of the scales, and also using 'Frequency' instead of 'Frequency Density.'
Students often confuse the methods involved with estimating the mean and histograms when dealing with data tables.
Histograms are often not well understood with the height used for frequency rather than the area.

## NOTES

Ensure that axes are clearly labelled.

## 17. Probability

**Teaching time**
4-6 hours

### OBJECTIVES

| Foundation Ref | Higher Ref | |
|---|---|---|
| **F6.3A** | | understand the language of probability |
| **F6.3B** | | understand and use the probability scale |
| **F6.3C** | | understand and use estimates or measures of probability from theoretical models |
| **F6.3D** | | find probabilities from a Venn diagram |
| **F6.3E** | | understand the concepts of a sample space and an event, and how the probability of an event happening can be determined from the sample space |
| **F6.3F** | | list all the outcomes for single events and for two successive events in a systematic way |
| **F6.3G** | | estimate probabilities from previously collected data |
| **F6.3H** | | calculate the probability of the complement of an event happening |
| **F6.3I** | | use the addition rule of probability for mutually exclusive events |
| **F6.3J** | | understand and use the term 'expected frequency' |
| | **H6.3A** | draw and use tree diagrams |

### POSSIBLE SUCCESS CRITERIA

If the probability of outcomes are $X$, $2X$, $4X$, $3X$, calculate $X$.
Draw a Venn diagram of students studying French, German or both, and then calculate the probability that a student studies French, given that they also study German.
Use a tree diagram to find the probability of a combined event.

### OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Students should be given the opportunity to justify the probability of events happening or not happening in real-life and abstract contexts.

### COMMON MISCONCEPTIONS

Probability without replacement is best illustrated visually and by initially working out probability 'with' replacement.
Not using fractions or decimals when working with probability trees.

### NOTES

Encourage students to work 'across' the branches, working out the probability of each successive event.
The probability of the combinations of outcomes should = 1
If a question says, for example, that 'two counters are taken from a bag' then, by implication, this is a non-replacement probability question.

# Unit 2: Higher Tier

**It is assumed that students being prepared for Unit 2 Higher Tier will have knowledge of Unit 1 and Unit 2 Foundation Tier and Unit 1 Higher Tier subject content.**

| **18. Special numbers** | **Teaching time** 1-3 hours |
|---|---|

### OBJECTIVES

| Foundation Ref | Higher Ref | |
|---|---|---|
| **F1.4D** | | express integers as a product of powers of prime factors |
| **F1.4E** | | find highest common factors (HCF) and lowest common multiples (LCM) |

### POSSIBLE SUCCESS CRITERIA

What is the value of $2^5$?
Find the HCF and LCM of 12 and 20
Understand that every number can be written as a unique product of its prime factors.
Recall prime numbers up to 100
Understand the meaning of prime factor.
Write a number as a product of its prime factors.

### OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems that use indices instead of integers will provide rich opportunities to apply the knowledge in this unit in other areas of mathematics.
Calculations involving how many items to buy.

### COMMON MISCONCEPTIONS

1 is a prime number.
Particular emphasis should be made on the definition of “product” as multiplication, as many students get confused and think it relates to addition.

### NOTES

Students need to be encouraged to learn squares from 2 × 2 to 15 × 15 and cubes of 2, 3, 4, 5 and 10, and corresponding square and cube roots.

## 19. Percentages

**Teaching time**
2-4 hours

### OBJECTIVES

| Foundation Ref | Higher Ref | |
|---|---|---|
| **F1.6D** | | understand the multiplicative nature of percentages as operators |
| **F1.6E** | | solve simple percentage problems, including percentage increase and decrease |
| **F1.6F** | | use reverse percentages |
| **F1.6G** | | use compound interest and depreciation |
| | **H1.6A** | use repeated percentage change |
| | **H1.6B** | solve compound interest problems |

### POSSIBLE SUCCESS CRITERIA

Be able to work out the price of a deposit, given the price of a sofa is £480 and the deposit is 15% of the price, without a calculator.
Find fractional percentages of amounts, with and without using a calculator.
Work out the interest earned when £5600 is invested for 3 years at 2.5% compound interest.
Find the original price when the sale price of an item is £68 following a reduction of 15%.

### OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Many of these topics provide opportunities for reasoning in real-life contexts, particularly percentages.
Calculate original values and evaluate statements in relation to this value, justifying which statement is correct.

### COMMON MISCONCEPTIONS

It is not possible to have a percentage greater than 100%.

### NOTES

Amounts of money should always be rounded to the nearest penny, except where successive calculations are done (i.e. compound interest, which is covered in a later unit).
Emphasise the use of percentages in real-life situations.

| 20. Ratio and proportion 2 | Teaching time 3-5 hours |
|---|---|

## OBJECTIVES

| Foundation Ref | Higher Ref | |
|---|---|---|
| **F1.7A** | | use ratio notation, including reduction to its simplest form and its various links to fraction notation |
| **F1.7B** | | divide a quantity in a given ratio or ratios |
| **F1.7C** | | use the process of proportionality to evaluate unknown quantities |
| **F1.7D** | | calculate an unknown quantity from quantities that vary in direct proportion |
| **F1.7E** | | solve word problems about ratio and proportion |

## POSSIBLE SUCCESS CRITERIA

Write/interpret a ratio to describe a situation such as 1 blue for every 2 red …, 3 adults for every 10 children …
Recognise that two paints mixed red to yellow 5 : 4 and 20 : 16 are the same colour.
When a quantity is split in the ratio 3 : 5, what fraction does each person get?
Find amounts for three people when amount for one given.

## OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems involving sharing in a ratio that include percentages rather than specific numbers can provide links with other areas of mathematics.
In a youth club, the ratio of the number of boys to the number of girls is 3 : 2. 30% of the boys are under the age of 14 and 60% of the girls are under the age of 14. What percentage of the youth club is under the age of 14?

## COMMON MISCONCEPTIONS

Students often identify a ratio-style problem and then divide by the number given in the question, without fully understanding the question.

## NOTES

Three-part ratios are usually difficult for students to understand.
Also include using decimals to find quantities.
Use a variety of measures in ratio and proportion problems.

## 21. Indices and standard form

**Teaching time**
2-4 hours

### OBJECTIVES

| Foundation Ref | Higher Ref | |
|---|---|---|
| **F1.9A** | | calculate with and interpret numbers in the form $a \times 10^n$ where $n$ is an integer and $1 \leqslant a < 10$ |
| **F4.4B** | | calculate time intervals in terms of the 24-hour and 12-hour clock |
| **F4.4C** | | make sensible estimates of a range of measures |
| | **H1.9A** | solve problems involving standard form |

### POSSIBLE SUCCESS CRITERIA

Write 51080 in standard form.
Write $3.74 \times 10^{-6}$ as an ordinary number.

### OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Evaluate statements and justify which answer is correct by providing a counterargument by way of a correct solution.

### COMMON MISCONCEPTIONS

Some students may think that any number multiplied by a power of 10 qualifies as a number written in standard form.

### NOTES

Standard form is used in science and there are lots of cross-curricular opportunities.
Students need to be given plenty of practice in using standard form with calculators.

## 22. Proof

**Teaching time**
3-5 hours

**OBJECTIVES**

| Foundation Ref | Higher Ref | |
|---|---|---|
| | **H2.2E** | use algebra to support and construct proofs |

**POSSIBLE SUCCESS CRITERIA**

Solve 'Show that' and proof questions using consecutive integers ($n$, $n$ + 1), squares $a^2$, $b^2$, even numbers 2$n$, odd numbers 2$n$ +1.

**OPPORTUNITIES FOR REASONING/PROBLEM SOLVING**

Evaluate statements and justify which answer is correct by providing a counterargument by way of a correct solution.
Formal proof is an ideal opportunity for students to provide a clear logical chain of reasoning providing links with other areas of mathematics.

**COMMON MISCONCEPTIONS**

Some students do not recognise that "Show that" is a command phrase in exam questions to indicate that they need to write down the stages of their working. Answers, for example from a calculator, without working will not score any marks.

**NOTES**

Students will be asked to show 'algebraic working' when solving equations. Solutions with no working will score no marks.
Students can leave their answer in fraction form where appropriate. Emphasise that fractions are more accurate in calculations than rounded percentage or decimal equivalents.

## 23. Expressions, formulae and rearranging formulae

**Teaching time**
3-5 hours

### OBJECTIVES

| Foundation Ref | Higher Ref | |
|---|---|---|
| **F2.3A** | | understand that a letter may represent an unknown number or a variable |
| **F2.3B** | | use correct notational conventions for algebraic expressions and formulae |
| **F2.3C** | | substitute positive and negative integers, decimals and fractions for words and letters in expressions and formulae |
| **F2.3D** | | use formulae from mathematics and other real-life contexts expressed initially in words or diagrammatic form and convert to letters and symbols |
| **F2.3E** | | derive a formula or expression |
| **F2.3F** | | change the subject of a formula where the subject appears once |
| | **H2.3A** | understand the process of manipulating formulae or equations to change the subject, to include cases where the subject may appear twice or a power of the subject occurs |
| | **H2.5A** | set up problems involving direct or inverse proportion and relate algebraic solutions to graphical representation of the equations |

### POSSIBLE SUCCESS CRITERIA

Find the value of $3x^2 - 2x$ for different values of $x$.
Find the value $a$ in $v^2 = u^2 + 2as$ given values of the other variables.
Make $a$ the subject of $v^2 = u^2 + 2as$
Make $y$ the subject of $t = \sqrt{\frac{2-3y}{4}}$
Make $t$ the subject of $a = \frac{2t+b}{3-t}$
Given that $y$ is inversely proportional to $x^2$, and that when $x = 2$, $y = 3$, find a formula for $y$ in terms of $x$.

### OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Justify and infer relationships in real-life scenarios to direct and inverse proportion such as, ice cream sales and sunshine.

### COMMON MISCONCEPTIONS

Confusing direct and inverse proportion.

### NOTES

Students should be reminded to show all stages in their working.
Consider using science contexts for problems involving inverse proportionality, e.g. volume of gas inversely proportional to the pressure or frequency is inversely proportional to wavelength.

| **24. Inequalities** | **Teaching time**<br>2-4 hours |
|---|---|

**OBJECTIVES**

| Foundation Ref | Higher Ref | |
|---|---|---|
| **F2.8A** | | understand and use the symbols<br>$>, <, \geqslant$ and $\leqslant$ |
| **F2.8B** | | understand and use the convention for open and closed intervals on a number line |
| **F2.8C** | | solve simple linear inequalities in one variable and represent the solution set on a number line |
| | **H2.8A** | solve quadratic inequalities in one unknown and represent the solution set on a number line |

**POSSIBLE SUCCESS CRITERIA**

Use inequality symbols to compare numbers.
Given a list of numbers, represent them on a number line using the correct notation.
Solve equations involving inequalities.
Solve $4x + 5 > x + 1$
Know that the quadratic formula can be used to solve all quadratic equations, and often provides a more efficient method than factorising or completing the square.
Have an understanding of solutions that can be written in surd form.
Solve $x^2 < 9$; $2x^2 + 3x + 1 < 0$

**OPPORTUNITIES FOR REASONING/PROBLEM SOLVING**

Problems that require students to justify why certain values in a solution can be ignored.
Set up and solve problems involving linear equations.
Problems that require students to set up and solve a quadratic equation or inequality.

**COMMON MISCONCEPTIONS**

When solving inequalities, students often state their final answer as a number quantity, and exclude the inequality or change it to =
Some students believe that –6 is greater than –3
When solving equations like $\frac{2x-1}{3} - \frac{x+1}{2} = 5$ the common error is to forget to use the negative sign when expanding brackets.

**NOTES**

Emphasise the importance of leaving their answer as an inequality (and not changing it to =).
Students can leave their answers in fractional form where appropriate.
Ensure that correct language is used to avoid reinforcing misconceptions: for example, 0.15 should never be read as ‘zero point fifteen’, and 5 > 3 should be read as ‘five is greater than 3’, not ‘5 is bigger than 3’.

## 25. Sequences

**Teaching time**
3-5 hours

**OBJECTIVES**

| Foundation Ref | Higher Ref | |
|---|---|---|
| **F3.1A** | | generate terms of a sequence using term-to-term and position-to-term definitions of the sequence |
| **F3.1B** | | find subsequent terms of an integer sequence and the rule for generating it |
| **F3.1C** | | use linear expressions to describe the *n*th term of arithmetic sequences |
| | **H3.1A** | understand and use common difference ($d$) and first term ($a$) in an arithmetic sequence |
| | **H3.1B** | know and use *n*th term = $a + (n - 1)d$ |
| | **H3.1C** | find the sum of the first $n$ terms of an arithmetic series ($Sn$) |

**POSSIBLE SUCCESS CRITERIA**

Given a sequence, 'which is the 1st term greater than 50?'
Given the sequence 12, 7, 2, -3... find an expression in terms of $n$ for the *n*th term.
Be able to solve problems involving sequences from real-life situations, such as:
What is the amount of money after $x$ months saving the same amount, or the height of a tree that grows 6 m per year?
Given the sequence 5, 8, 11, 14... find the $50^{th}$ term, the sum of the first 50 terms.

**OPPORTUNITIES FOR REASONING/PROBLEM SOLVING**

Evaluate statements about whether or not specific numbers or patterns are in a sequence and justify the reasons.

**COMMON MISCONCEPTIONS**

Students struggle to relate the position of the term to "$n$".
Writing $n + 3$ instead of $3n - 1$ for the *n*th term of 2, 5, 8, 11...

**NOTES**

Emphasise use of $3n$ meaning $3 \times n$.
Students need to be clear on the description of the pattern in words, the difference between the terms and the algebraic description of the *n*th term.

## 26. Graphs of inequalities

**Teaching time**
3-5 hours

### OBJECTIVES

| Foundation Ref | Higher Ref | |
|---|---|---|
| **F2.8D** | | represent simple linear inequalities on rectangular Cartesian graphs |
| **F2.8E** | | identify regions on rectangular Cartesian graphs defined by simple linear inequalities |
| | **H2.8B** | identify harder examples of regions defined by linear inequalities |

### POSSIBLE SUCCESS CRITERIA

Show the region defined by $x < 3$, $y > 1$, $y < 3x + 2$

Be able to state the solution set of $X^2 - 3X - 10 < 0$ as $\{X: X < -3\} \cup \{X: X > 5\}$.

### OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Match equations to their graphs and to real-life scenarios.

### COMMON MISCONCEPTIONS

Students can find visualisation of a question difficult, especially when dealing with gradients resulting from negative coordinates.

When solving inequalities students often state their final answer as a number quantity and exclude the inequality or change it to =

### NOTES

Encourage students to sketch what information they are given in a question – emphasise that it is a sketch.

Careful annotation should be encouraged – it is good practice to label the axes and check that students understand the scales.

## 27. Harder graphs and transformation of graphs

**Teaching time**
5-7 hours

### OBJECTIVES

| Foundation Ref | Higher Ref | |
|---|---|---|
| | **H3.3B** | apply to the graph of $y = f(x)$ the transformations<br>$y = f(x) + a$, $y = f(ax)$, $y = f(x + a)$,<br>$y = af(x)$ for linear, quadratic, sine and cosine functions |
| | **H3.3C** | interpret and analyse transformations of functions and write the functions algebraically |
| | **H3.3D** | find the gradients of non-linear graphs |
| | **H3.3E** | find the intersection points of two graphs, one linear ($y_1$) and one non-linear ($y_2$) and recognise that the solutions correspond to the solutions of $(y_2 - y_1) = 0$ |

### POSSIBLE SUCCESS CRITERIA

Find the gradient, at a point, of a non-linear graph.
Give the graph of $y = f(x)$, sketch the graph of $y = 2f(x)$; $y = f(x + 2)$; $y = -f(x)$
Match the characteristic shape of the graphs to their functions and transformations.

### OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Match a given list of events/processes with their graph.

### COMMON MISCONCEPTIONS

Students struggle with the concept of solutions and what they represent in concrete terms.

### NOTES

Use lots of practical examples to help model the quadratic function, e.g. draw a graph to model the trajectory of a projectile and predict when/where it will land.
Ensure axes are labelled and pencils used for drawing and transformations.
Graphical calculations or appropriate ICT will allow students to see the impact of changing variables within a function.

## 28. Simultaneous equations

**Teaching time**
3-5 hours

### OBJECTIVES

| Foundation Ref | Higher Ref | |
|---|---|---|
| | **H2.6A** | calculate the exact solution of two simultaneous equations in two unknowns |
| | **H2.6B** | interpret the equations as lines and the common solution as the point of intersection |
| | **H2.7D** | solve simultaneous equations in two unknowns, one equation being linear and the other being quadratic |

### POSSIBLE SUCCESS CRITERIA

Solve the simultaneous equations $2x + 5y = -14$; $3x - 4y = 25$
Solve the simultaneous equations $x^2 + y^2 = 18$; $2x + 1 = y$

### OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems that require students to set up and solve a pair of simultaneous equations in a real-life context, such as 2 adult tickets and 1 child ticket cost £28, and 1 adult ticket and 3 child tickets cost £34. How much does 1 adult ticket cost?
Link the solution of simultaneous equations to their graphical representation.

### COMMON MISCONCEPTIONS

Some students always discard solutions with negative values.

### NOTES

Reinforce the fact that some problems may produce one inappropriate solution, which can be ignored.
Clear presentation of working out is essential.
Link with graphical representations.

| **29. Function notation** | **Teaching time**<br>4-6 hours |
|---|---|

**OBJECTIVES**

| Foundation Ref | Higher Ref | |
|---|---|---|
| | **H3.2A** | understand the concept that a function is a mapping between elements of two sets |
| | **H3.2B** | use function notations of the form f($x$) = … and f : x $\mapsto$ … |
| | **H3.2C** | understand the terms 'domain' and 'range' and which values may need to be excluded from a domain |
| | **H3.2D** | understand and find the composite function fg and the inverse function $f^{-1}$ |

**POSSIBLE SUCCESS CRITERIA**

Given f($x$) = 3 – 5$x$; find f(2), $f^{-1}(3)$

Given g($x$) = $\frac{2}{3-x}$, write down the value of $x$ that must be omitted from any domain of $g$.

Find fg(4); gf(4)

**OPPORTUNITIES FOR REASONING/PROBLEM SOLVING**

Forming and solving equations using functions, e.g. solve f($x$) = g($x$)
Give the graph of f($x$) and use that to find f(3) and f($x$) = 2

**COMMON MISCONCEPTIONS**

Confusing gf($x$) with fg($x$)

**NOTES**

Link with algebraic manipulation and equation solving.

## 30. Calculus

**Teaching time**
6-8 hours

### OBJECTIVES

| Foundation Ref | Higher Ref | |
|---|---|---|
| | | |
| | **H3.4A** | understand the concept of a variable rate of change |
| | **H3.4B** | differentiate integer powers of $x$ |
| | **H3.4C** | determine gradients, rates of change, stationary points, turning points (maxima and minima) by differentiation and relate these to graphs |
| | **H3.4D** | distinguish between maxima and minima by considering the general shape of the graph only |
| | **H3.4E** | apply calculus to linear kinematics and to other simple practical problems |

### POSSIBLE SUCCESS CRITERIA

Differentiate $8x^3 + 3x + 2$; $\dfrac{2}{x^2} + 3x$

Find the turning point of $y = x^2 + 8x - 20$

### OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Find the values of $x$ for which the graph of $y = x^2 - x + 3$ has a gradient of 7

Given that $s = t^3 + 2t^2$ find the value of $t$ for which the particle is instantaneously at rest.

### COMMON MISCONCEPTIONS

3 differentiates to 3 (rather than 0)

### NOTES

Link with solving linear and quadratic equations.

| 31. Geometry of shapes 2 | Teaching time<br>1-3 hours |
|---|---|

**OBJECTIVES**

| Foundation Ref | Higher Ref | |
|---|---|---|
| **F4.2D** | | understand the term ‘regular polygon’ and calculate interior and exterior angles of regular polygons |
| **F4.2E** | | understand and use the angle sum of polygons |
| **F4.3A** | | identify any lines of symmetry and the order of rotational symmetry of a given two-dimensional figure |

**POSSIBLE SUCCESS CRITERIA**

Name all quadrilaterals that have a specific property.
Given the size of its exterior angle, how many sides does the polygon have?
What is the same and what is different between families of polygons?
Given a geometric diagram, find the value of a given angle and give a reason for each stage of working.

**OPPORTUNITIES FOR REASONING/PROBLEM SOLVING**

Multi-step “angle chasing”-style problems that involve justifying how students have found a specific angle will provide opportunities to develop a chain of reasoning.
Geometrical problems involving algebra, whereby equations can be formed and solved, allow students the opportunity to make and use connections with different parts of mathematics.

**COMMON MISCONCEPTIONS**

Some students will think that all trapezia are isosceles, or a square is only square if ‘horizontal’, or a ‘non-horizontal’ square is called a diamond.
Incorrectly identifying the exterior angles of polygons.

**NOTES**

Students must be encouraged to use geometrical language appropriately, ‘quote’ the appropriate reasons for angle calculations and show step-by-step deduction when solving multi-step problems.
Emphasise that diagrams in examinations are seldom drawn accurately.
Use triangles to find angle sums of polygons; this could be explored algebraically as an investigation.

## 32. Constructions and bearings

**Teaching time**
2-4 hours

### OBJECTIVES

| Foundation Ref | Higher Ref | |
|---|---|---|
| **F4.4D** | | understand angle measure including three-figure bearings |
| **F4.5B** | | construct triangles and other two-dimensional shapes using a combination of a ruler, a protractor and a pair of compasses |
| **F4.5C** | | solve problems using scale drawings |
| **F4.5D** | | use straight edge and a pair of compasses to:<br>(i) construct the perpendicular bisector of a line segment<br>(ii) construct the bisector of an angle |
| **F4.11B** | | use and interpret maps and scale drawings |

### POSSIBLE SUCCESS CRITERIA

Able to read and construct scale drawings.

When given the bearing of a point $A$ from point $B$, can work out the bearing of $B$ from $A$.

Know that scale diagrams, including bearings and maps, are 'similar' to the real-life examples.

Construct the perpendicular bisector of a given angle.

### OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems involving combinations of bearings and scale drawings can provide a rich opportunity to link with other areas of mathematics and allow students to justify their findings.

### COMMON MISCONCEPTIONS

Correct use of a protractor may be an issue.

### NOTES

Drawings should be done in pencil.

Construction lines should not be erased.

| 33. Perimeter, area and volume 2 | Teaching time<br>4-6 hours |
|---|---|

## OBJECTIVES

| Foundation Ref | Higher Ref | |
|---|---|---|
| F4.10C | | find the surface area of simple shapes using the area formulae for triangles and rectangles |
| F4.10D | | find the surface area of a cylinder |
| F4.10E | | find the volume of prisms, including cuboids and cylinders, using an appropriate formula |
| | H4.10A | find the surface area and volume of a sphere and a right circular cone using relevant formulae |

## POSSIBLE SUCCESS CRITERIA

Calculate the area and/or perimeter of shapes with different units of measurement.

Understand that answers in terms of $\pi$ are more accurate.

Calculate the perimeters and/or areas of circles and sectors of circles, given the radius or diameter and vice versa.

Work out the length, given the area of the cross-section and volume of a cuboid.

Given two solids with the same volume and the dimensions of one, write and solve an equation in terms of $\pi$ to find the dimensions of the other.

## OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Using compound shapes or combinations of polygons that require students to subsequently interpret their result in a real-life context.

Multi-step problems, including the requirement to form and solve equations, provide links with other areas of mathematics.

Combinations of 3D forms such as a cone and a sphere where the radius has to be calculated given the total height.

## COMMON MISCONCEPTIONS

Students often get the concepts of surface area and volume confused.

## NOTES

Encourage students to draw a sketch where one is not provided.

Ensure that examples use different metric units of length, including decimals.

Ensure that students know it is more accurate to leave answers in terms of $\pi$, but only when asked to do so.

## 34. Transformations

**Teaching time**
3-5 hours

### OBJECTIVES

| Foundation Ref | Higher Ref | |
|---|---|---|
| **F5.2A** | | understand that rotations are specified by a centre and an angle |
| **F5.2B** | | rotate a shape about a point through a given angle |
| **F5.2C** | | recognise that an anti-clockwise rotation is a *positive* angle of rotation and a clockwise rotation is a *negative* angle of rotation |
| **F5.2D** | | understand that reflections are specified by a mirror line |
| **F5.2E** | | construct a mirror line given an object and reflect a shape given a mirror line |
| **F5.2F** | | understand that translations are specified by a distance and direction |
| **F5.2G** | | translate a shape |
| **F5.2H** | | understand and use column vectors in translations |
| **F5.2I** | | understand that rotations, reflections and translations preserve length and angle so that a transformed shape under any of these transformations remains congruent to the original shape |
| **F5.2J** | | understand that enlargements are specified by a centre and a scale factor |
| **F5.2K** | | understand that enlargements preserve angles and not lengths |
| **F5.2L** | | enlarge a shape given the scale factor |
| **F5.2M** | | identify and give complete descriptions of transformations |

### POSSIBLE SUCCESS CRITERIA

Understand that translations are specified by a distance and direction (using a vector).
Understand that distances and angles are preserved under rotations, reflections and translations, so that any shape is congruent to its image.
Understand that similar shapes are enlargements of each other, and angles are preserved.

### OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Students should be given the opportunity to explore the effect of reflecting in two parallel mirror lines and combining transformations.

### COMMON MISCONCEPTIONS

Students often use the term ‘transformation’ when describing transformations instead of the required information.
Lines parallel to the coordinate axes often get confused.

**NOTES**

Emphasise the need to describe the transformations fully, and if asked to describe a ‘single’ transformation students should not include two types.

Find the centre of rotation, by trial and error and by using tracing paper. Include centres on or inside shapes.

## 35. Circle properties

**Teaching time**
5-7 hours

### OBJECTIVES

| Foundation Ref | Higher Ref | |
|---|---|---|
| **F4.6B** | | understand chord and tangent properties of circles |
| | **H4.6A** | understand and use the internal and external intersecting chord properties |
| | **H4.6B** | recognise the term 'cyclic quadrilateral' |
| | **H4.6C** | understand and use angle properties of the circle including:<br>(i) angle subtended by an arc at the centre of a circle is twice the angle subtended at any point on the remaining part of the circumference<br>(ii) angle subtended at the circumference by a diameter is a right angle<br>(iii) angles in the same segment are equal<br>(iv) the sum of the opposite angles of a cyclic quadrilateral is 180°<br>(v) the alternate segment theorem |

### POSSIBLE SUCCESS CRITERIA

Justify clearly missing angles on diagrams using the various circle theorems, giving a reason for each stage in working.

### OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems that involve a clear chain of reasoning and provide counterarguments to statements.
Can be linked to other areas of mathematics by incorporating trigonometry and Pythagoras' theorem.

### COMMON MISCONCEPTIONS

Much of the confusion arises from mixing up the diameter and the radius.
There is often confusion when identifying cyclic quadrilaterals.

### NOTES

Reasoning needs to be carefully constructed and correct notation should be used throughout.
Students should label any diagrams clearly, as this will assist them; particular emphasis should be made on labelling any radii in the first instance.

| 36. Similar shapes | Teaching time<br>4-6 hours |
|---|---|

## OBJECTIVES

| Foundation Ref | Higher Ref | |
|---|---|---|
| **F4.2F** | | understand congruence as meaning the same shape and size |
| **F4.2G** | | understand that two or more polygons with the same shape and size are said to be congruent to each other |
| **F4.11A** | | understand and use the geometrical properties that similar figures have corresponding lengths in the same ratio but corresponding angles remain unchanged |
| | **H4.11A** | understand that areas of similar figures are in the ratio of the square of corresponding sides |
| | **H4.11B** | understand that volumes of similar figures are in the ratio of the cube of corresponding sides |
| | **H4.11C** | use areas and volumes of similar figures in solving problems |

## POSSIBLE SUCCESS CRITERIA

Recognise that all corresponding angles in similar shapes are equal in size when the corresponding lengths of sides are not.
Understand that enlargement does not have the same effect on area and volume.
Given the volumes of two similar shapes and the surface area of one, find the surface area of the other shape.

## OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Multi-step questions that require calculating missing lengths of similar shapes prior to calculating the area of the shape or using this information in trigonometry or Pythagoras' problems.

## COMMON MISCONCEPTIONS

Students commonly use the same scale factor for length, area and volume.

## NOTES

Encourage students to consider what happens to the area when a 1 cm square is enlarged by a scale factor of 3.
Ensure that examples involving given volumes are used, requiring the cube root to be calculated to find the length scale factor.

## 37. Vectors

**Teaching time**
5-7 hours

### OBJECTIVES

| Foundation Ref | Higher Ref | |
|---|---|---|
| | **H5.1A** | understand that a vector has both magnitude and direction |
| | **H5.1B** | understand and use vector notation including column vectors |
| | **H5.1C** | multiply vectors by scalar quantities |
| | **H5.1D** | add and subtract vectors |
| | **H5.1E** | calculate the modulus (magnitude) of a vector |
| | **H5.1F** | find the resultant of two or more vectors |
| | **H5.1G** | apply vector methods for simple geometrical proofs |

### POSSIBLE SUCCESS CRITERIA

Add and subtract vectors algebraically and use column vectors.
Solve geometric problems and produce proofs.
Find the magnitude of a vector.

### OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

"Show that"-type questions are an ideal opportunity for students to provide a clear logical chain of reasoning, providing links with other areas of mathematics, in particular algebra.
Find the area of a parallelogram defined by given vectors.

### COMMON MISCONCEPTIONS

Students find it difficult to understand that parallel vectors are equal as they are in different locations in the plane.

### NOTES

Students find manipulation of column vectors relatively easy compared to pictorial and algebraic manipulation methods – encourage them to draw any vectors they calculate on the picture.
Geometry of a hexagon provides a good source of parallel, reverse and multiples of vectors.
Remind students to underline vectors or use an arrow above them, or they will be regarded as just lengths.
Extend geometric proofs by showing that the medians of a triangle intersect at a single point.

## 38. Graphical representation of data 2

**Teaching time**
1-3 hours

### OBJECTIVES

| Foundation Ref | Higher Ref | |
|---|---|---|
| | **H6.1B** | construct cumulative frequency diagrams from tabulated data |
| | **H6.1C** | use cumulative frequency diagrams |

### POSSIBLE SUCCESS CRITERIA

Construct cumulative frequency graphs from frequency tables.
Compare two data sets and justify their comparisons based on measures extracted from their diagrams, where appropriate, in terms of the context of the data.

### OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Interpret two or more data sets from cumulative frequency graphs and relate the key measures in the context of the data.

### COMMON MISCONCEPTIONS

Labelling axes incorrectly in terms of the scales, and also using 'Frequency' instead of 'Frequency Density' or 'Cumulative Frequency.'
Students often confuse the methods involved with cumulative frequency, estimating the mean and histograms when dealing with data tables.

### NOTES

Ensure that axes are clearly labelled.

## 39. Statistical measures

**Teaching time**
3-5 hours

### OBJECTIVES

| Foundation Ref | Higher Ref | |
|---|---|---|
| **F6.2A** | | understand the concept of average |
| **F6.2B** | | calculate the mean, median, mode and range for a discrete data set |
| **F6.2C** | | calculate an estimate for the mean for grouped data |
| **F6.2D** | | identify the modal class for grouped data |
| | **H6.2A** | estimate the median from a cumulative frequency diagram |
| | **H6.2B** | understand the concept of a measure of spread |
| | **H6.2C** | find the interquartile range from a discrete data set |
| | **H6.2D** | estimate the interquartile range from a cumulative frequency diagram |
| | | |
| **F6.1Aii** | | use different methods of presenting data |
| **F6.1B** | | use appropriate methods of tabulation to enable the construction of statistical diagrams |
| **F6.1C** | | interpret statistical diagrams |

### POSSIBLE SUCCESS CRITERIA

Be able to state the median, mode, mean and range from a small data set.
Be able to find the interquartile range from a discrete data set.
Estimate the mean from a grouped frequency table.
Estimate the median and interquartile range from a cumulative frequency graph. Compare two data sets and justify their comparisons based on measures extracted from their diagrams, where appropriate, in terms of the context of the data.

### OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Students should be able to provide reasons for choosing to use a specific average to support a point of view.
Given the mean, median and mode of five positive whole numbers, can you find the numbers?
Students should be able to provide a correct solution as a counterargument to statements involving the "averages", e.g. Susan states that the median is 15, she is wrong. Explain why.
Comparing diagrams to decide the most appropriate.

### COMMON MISCONCEPTIONS

Students often forget the difference between continuous and discrete data.
Often the $\sum(m \times f)$ is divided by the number of classes rather than $\sum f$ when estimating the mean.

**NOTES**

Encourage students to cross out the midpoints ($m$) of each group once they have used these numbers to work out $m \times f$. This helps students to avoid summing $m$ instead of $f$.
Remind students how to find the midpoint of two numbers.
Emphasise that continuous data is measured, i.e. length, weight, and discrete data can be counted, i.e. number of shoes.