American High School Mathematics Examination
Years 1950 — 1999
Updated on: April 30, 2021

Comments and suggestions to DrYuFromShanghai@QQ.com

	1999	1998	1997	1996	1995	1994	1993	1992	1991	1990	1989	1988	1987	1986	1985	1984	1983	1982	1981	1980	1979	1978	1977	1976	1975
Q1	E	E	C	D	B	C	D	B	E	E	C	D	D	B	A	C	E	E	E	C	D	B	D	B	B
Q2	A	E	D	A	D	B	D	B	E	E	D	D	E	A	E	B	B	A	C	D	D	C	D	B	D
Q3	E	D	D	E	B	B	E	C	A	C	D	A	C	D	D	C	A	C	D	E	C	D	E	E	A
Q4	A	E	A	D	B	A	E	A	C	B	D	B	B	B	C	B	D	E	C	C	E	B	C	C	A
Q5	C	C	C	E	C	E	A	B	E	B	E	E	B	A	E	D	D	C	C	B	D	C	A	C	B
Q6	D	C	B	E	C	A	B	D	E	D	A	A	A	C	B	B	C	D	A	A	A	E	D	C	E
Q7	B	D	D	B	C	E	E	B	B	A	C	D	C	E	E	C	B	B	B	B	E	E	E	C	E
Q8	D	D	D	D	C	C	D	E	C	E	D	B	C	E	E	D	A	B	A	A	C	D	B	A	D
Q9	D	D	C	B	D	D	D	E	D	B	B	C	B	C	B	D	D	B	A	E	E	B	B	D	C
Q10	C	A	D	D	E	B	C	D	B	D	C	D	D	E	E	B	D	A	E	D	D	B	E	D	A
Q11	A	D	D	D	C	D	A	B	B	C	A	C	E	B	B	A	B	C	C	D	B	C	B	B	E
Q12	C	A	E	B	D	D	E	C	D	D	C	A	D	B	D	B	E	A	E	C	B	E	D	C	B
Q13	C	E	E	D	C	B	D	C	D	E	B	E	A	E	E	A	E	D	E	B	A	B	E	A	D
Q14	A	A	B	C	E	B	B	E	C	A	B	A	B	D	C	A	E	C	A	A	C	C	B	A	E
Q15	E	C	D	B	D	B	D	B	B	C	E	A	D	C	E	A	C	D	B	B	E	A	D	B	A
Q16	C	B	D	D	E	B	D	E	D	C	B	C	D	C	B	E	D	B	E	B	E	E	D	E	C
Q17	C	E	A	E	E	D	A	B	D	C	D	C	E	B	B	C	C	C	B	D	C	D	B	A	D
Q18	E	A	E	D	D	C	E	D	D	C	B	B	D	E	D	E	B	D	C	D	C	C	B	E	D
Q19	C	C	D	D	C	C	D	D	B	B	E	B	B	A	A	D	A	B	B	D	A	C	A	B	D
Q20	E	C	A	C	B	B	B	D	E	C	B	E	A	E	D	C	C	D	B	C	C	A	E	E	B
Q21	B	C	C	D	E	C	B	A	A	E	C	E	B	B	C	C	D	E	D	A	B	A	B	B	D
Q22	C	C	E	B	E	C	C	B	B	D	A	A	C	C	D	B	B	E	D	E	A	D	C	A	E
Q23	E	D	D	B	D	E	B	E	C	B	D	B	D	E	C	D	A	A	C	C	C	C	B	A	C
Q24	B	C	B	B	A	C	E	C	D	D	B	C	B	D	C	E	E	A	D	D	E	A	D	C	E
Q25	B	B	B	B	B	A	E	E	D	B	B	E	C	B	B	D	B	D	A	C	B	D	E	D	B
Q26	D	B	A	B	C	A	C	B	C	A	C	D	B	C	E	B	D	B	D	E	B	B	B	C	C
Q27	A	E	D	D	E	D	B	D	C	C	D	D	B	C	C	C	E	C	C	E	E	C	C	A	E
Q28	E	B	E	C	E	C	D	B	B	B	D	D	D	C	B	C	C	B	D	C	D	E	A	B	A
Q29	C	D	B	C	C	A	B	D	B	D	B	A	A	B	C	A	C	A	E	A	E	D	B	B	C
Q30	D	E	C	E	D	C	D	B	B	E	A	E	E	B	E	B	C	D	D	B	B	E	A	E	B
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1974	1973	1972	1971	1970	1969	1968	1967	1966	1965	1964	1963	1962	1961	1960	1959	1958	1957	1956	1955	1954	1953	1952	1951	1950	
D	D	D	D	E	B	D	C	B	C	E	D	D	C	E	B	C	B	A	D	E	B	B	B	C	Q1
B	C	B	B	A	A	B	D	E	D	C	A	A	E	C	D	D	E	D	B	C	D	B	D	D	Q2
A	B	B	B	C	E	A	B	D	B	D	E	B	E	B	E	B	E	D	B	C	D	A	B	E	Q3
D	D	D	D	B	E	C	C	B	C	A	D	B	B	C	C	E	C	E	E	E	D	C	D	A	Q4
B	D	A	A	D	B	A	D	A	A	A	E	C	C	C	A	A	B	C	D	A	C	A	B	A	Q5
D	C	C	C	B	C	E	D	C	B	B	B	D	E	B	D	B	A	B	B	D	A	E	D	C	Q6
D	A	E	E	E	A	E	E	A	E	C	A	C	A	D	D	A	E	C	B	B	D	B	B	B	Q7
A	E	D	D	B	D	B	A	B	A	C	D	D	B	D	A	D	A	B	D	C	C	A	C	C	Q8
B	E	A	A	C	C	E	E	A	E	E	C	B	C	E	C	C	B	D	D	C	D	D	D	A	Q9
B	A	D	D	D	C	C	A	E	A	A	B	A	D	E	D	E	C	B	A	C	C	B	C	D	Q10
A	B	A	A	E	B	B	B	C	B	D	B	B	C	D	D	C	E	A	C	A	C	C	C	A	Q11
B	D	B	B	C	A	C	B	E	D	E	E	C	A	D	A	A	C	E	D	C	B	E	C	C	Q12
D	D	C	C	D	B	B	E	E	E	A	E	B	E	B	D	B	C	C	C	B	B	E	C	D	Q13
A	C	B	B	A	A	E	C	C	A	C	A	B	E	E	D	C	D	B	A	E	E	B	E	D	Q14
B	D	C	C	E	D	D	D	D	B	B	C	D	B	B	C	D	B	A	D	D	B	C	E	E	Q15
A	B	B	B	C	E	E	B	B	C	E	C	C	E	C	D	D	D	C	E	D	D	E	C	B	Q16
C	E	E	E	E	D	C	A	C	E	D	A	B	D	A	E	E	A	D	C	A	B	C	D	C	Q17
D	C	A	A	A	B	D	B	A	B	C	A	A	A	E	E	A	B	E	A	D	C	D	D	E	Q18
A	D	D	D	C	B	E	E	B	C	A	B	C	B	C	B	B	A	D	B	D	C	D	E	C	Q19
D	C	E	E	A	C	A	A	C	C	B	D	A	A	D	B	C	A	C	A	B	D	E	C	D	Q20
B	B	C	C	B	E	D	B	E	D	D	E	E	D	E	C	E	E	E	B	C	D	B	C	B	Q21
E	A	E	E	A	C	E	A	A	A	C	D	D	C	A	C	C	A	B	D	B	B	B	B	D	Q22
B	D	D	D	D	A	B	B	A	D	D	B	E	B	C	A	A	C	C	C	D	B	A	B	B	Q23
A	D	B	B	B	E	C	A	B	E	A	B	A	A	A	C	B	B	D	E	A	A	B	D	E	Q24
C	E	C	C	E	D	C	A	D	D	B	A	C	A	D	D	E	B	C	E	D	C	D	A	D	Q25
C	E	E	E	B	B	E	C	C	E	C	E	E	C	B	A	B	E	A	E	B	A	C	E	E	Q26
A	A	D	D	A	E	B	C	A	A	E	C	E	D	E	A	A	A	C	B	D	D	E	E	B	Q27
D	C	E	E	A	E	D	E	B	B	D	D	D	D	B	E	B	B	D	E	B	D	C	C	B	Q28
B	A	C	C	D	C	A	C	B	A	E	C	A	B	A	D	E	D	D	E	A	D	A	D	B	Q29
A	B	A	A	E	D	A	D	D	B	D	C	D	D	C	B	C	D	B	B	B	D	A	C	A	Q30
	C	C	C	B	D	D	C	D	C	B	D	C	D	D	B	B	B	E	D	A	A	D	D	C	Q31
	A	B	B	C	C	C	E	B	C	C	A	E	C	A	C	E	E	B	B	E	D	D	D	D	Q32
	C	C	C	A	A	A	D	D	D	B	A	A	B	A	B	B	E	E	A	D	C	B	C	D	Q33
	C	A	A	C	B	B	A	B	B	C	B	E	A	C	A	A	C	A	C	D	C	E	A	D	Q34
	E	D	D	D	B	D	B	C	D	B	B	B	D	C	E	D	D	B	B	A	E	A	A	B	Q35
							C	E	E	E	C	C	B	B	D	D	D	E	E	C	C	E	B	A	Q36
							A	C	C	D	D	D	C	A	B	A	C	E	B	B	E	B	D	E	Q37
							E	D	E	D	E	B	A	D	B	D	D	D	B	B	C	D	A	B	Q38
							B	E	B	A	D	C	B	E	D	C	E	C	B	E	A	A	A	E	Q39
							D	A	D	A	C	B	A	A	C	B	D	A	A	C	C	E	C	D	Q40
															D	C	D	C	A	B	E	C	B	D	Q41
															E	E	C	A	C	D	E	D	C	D	Q42
															B	D	B	C	E	B	C	A	E	E	Q43
															C	E	E	B	A	E	A	C	E	C	Q44
															B	B	A	A	A	D	E	E	A	A	Q45
															B	E	E	A	B	E	D	C	A	E	Q46
															A	D	A	D	C	B	D	D	D	C	Q47
															B	E	A	B	B	C	D	A	C	C	Q48
															B	D	C	E	C	C	E	C	D	D	Q49
															D	C	B	B	C	A	B	D	D	E	Q50

Comments and suggestions to DrYuFromShanghai@QQ.com

Q1. $\sqrt{8}+\sqrt{18}=$

A) $\sqrt{20}$ **B)** $2(\sqrt{2}+\sqrt{3})$ **C)** 7 **D)** $5\sqrt{2}$ **E)** $2\sqrt{13}$

Q2. Triangles ABC and XYZ are similar, with A corresponding to X and B to Y. If $AB=3, BC=4$, and $XY=5$, then YZ is:

A) $3\frac{3}{4}$ **B)** 6 **C)** $6\frac{1}{4}$ **D)** $6\frac{2}{3}$ **E)** 8

Q3. Four rectangular paper strips of length 10 and width 1 are put flat on a table and overlap perpendicularly as shown.

How much area of the table is covered?

A) 36 **B)** 40 **C)** 44 **D)** 98 **E)** 100

Q4. The slope of the line $\frac{x}{3}+\frac{y}{2}=1$ is

A) $-\frac{3}{2}$ **B)** $-\frac{2}{3}$ **C)** $\frac{1}{3}$ **D)** $\frac{2}{3}$ **E)** $\frac{3}{2}$

Q5. If b and c are constants and $(x+2)(x+b)=x^2+cx+6$, then c is

A) -5 **B)** -3 **C)** -1 **D)** 3 **E)** 5

Q6. A figure is an equiangular parallelogram if and only if it is a

A) rectangle **B)** regular polygon **C)** rhombus **D)** square **E)** trapezoid

Q7. Estimate the time it takes to send 60 blocks of data over a communications channel if each block consists of 512 "chunks" and the channel can transmit 120 chunks per second.

A) 0.04 seconds **B)** 0.4 seconds **C)** 4 seconds **D)** 4 minutes **E)** 4 hours

Q8. If $\frac{b}{a}=2$ and $\frac{c}{b}=3$, what is the ratio of $a+b$ to $b+c$?

A) $\frac{1}{3}$ **B)** $\frac{3}{8}$ **C)** $\frac{3}{5}$ **D)** $\frac{2}{3}$ **E)** $\frac{3}{4}$

Q9. An $8' \times 10'$ table sits in the corner of a square room, as in Figure 1 below.

The owners desire to move the table to the position shown in Figure 2. The side of the room is S feet. What is the smallest integer value of S for which the table can be moved as desired without tilting it or taking it apart?

A) 11 **B)** 12 **C)** 13 **D)** 14 **E)** 15

Q10. In an experiment, a scientific constant C is determined to be 2.43865 with an error of at most ± 0.00312. The experimenter wishes to announce a value for C in which every digit is significant. That is, whatever C is, the announced value must be the correct result when C is rounded to that number of digits. The most accurate value the experimenter can announce for C is

A) 2 **B)** 2.4 **C)** 2.43 **D)** 2.44 **E)** 2.439

Q11. On each horizontal line in the figure below, the five large dots indicate the populations of cities A, B, C, D and E in the year indicated.

Which city had the greatest percentage increase in population from 1970 to 1980?

A) A **B)** B **C)** C **D)** D **E)** E

Q12. Each integer 1 through 9 is written on a separate slip of paper and all nine slips are put into a hat. Jack picks one of these slips at random and puts it back. Then Jill picks a slip at random. Which digit is most likely to be the units digit of the sum of Jack's integer and Jill's integer?

A) 0 **B)** 1 **C)** 8 **D)** 9

E) each digit is equally likely

Q13. If $\sin(x) = 3\cos(x)$ then what is $\sin(x) \cdot \cos(x)$?

A) $\frac{1}{6}$ **B)** $\frac{1}{5}$ **C)** $\frac{2}{9}$ **D)** $\frac{1}{4}$ **E)** $\frac{3}{10}$

Q14. For any real number a and positive integer k, define

$$\binom{a}{k} = \frac{a(a-1)(a-2)\cdots(a-(k-1))}{k(k-1)(k-2)\cdots(2)(1)}.$$

What is

$$\binom{-\frac{1}{2}}{100} \div \binom{\frac{1}{2}}{100}?$$

A) -199 **B)** -197 **C)** -1 **D)** 197 **E)** 199

Q15. If a and b are integers such that $x^2 - x - 1$ is a factor of $ax^3 + bx^2 + 1$, then b is

A) -2 **B)** -1 **C)** 0 **D)** 1 **E)** 2

Q16. ABC and $A'B'C'$ are equilateral triangles with parallel sides and the same center, as in the figure.

The distance between side BC and side $B'C'$ is $\frac{1}{6}$ the altitude of $\triangle ABC$. The ratio of the area of $\triangle A'B'C'$ to the area of $\triangle ABC$ is

A) $\frac{1}{36}$ **B)** $\frac{1}{6}$ **C)** $\frac{1}{4}$ **D)** $\frac{\sqrt{3}}{4}$ **E)** $\frac{9+8\sqrt{3}}{36}$

Q17. If $|x| + x + y = 10$ and $x + |y| - y = 12$, find $x + y$

A) -2 **B)** 2 **C)** $\frac{18}{5}$ **D)** $\frac{22}{3}$ **E)** 22

Q18. At the end of a professional bowling tournament, the top 5 bowlers have a playoff. First #5 bowls #4. The loser receives 5^{th} prize and the winner bowls #3 in another game. The loser of this game receives 4^{th} prize and the winner bowls #2. The loser of this game receives 3^{rd} prize and the winner bowls #1. The winner of this game gets 1st prize and the loser gets 2nd prize. In how many orders can bowlers #1 through #5 receive the prizes?

A) 10 **B)** 16 **C)** 24 **D)** 120 **E)** none of these

Q19. Simplify

$$\frac{bx(a^2x^2 + 2a^2y^2 + b^2y^2) + ay(a^2x^2 + 2b^2x^2 + b^2y^2)}{bx + ay}$$

A) $a^2x^2 + b^2y^2$ **B)** $(ax + by)^2$ **C)** $(ax + by)(bx + ay)$

D) $2(a^2x^2 + b^2y^2)$ **E)** $(bx + ay)^2$

Q20. In one of the adjoining figures a square of side 2 is dissected into four pieces so that E and F are the midpoints of opposite sides and AG is perpendicular to BF. These four pieces can then be reassembled into a rectangle as shown in the second figure. The ratio of height to base, XY/YZ, in this rectangle is

A) 4 **B)** $1 + 2\sqrt{3}$ **C)** $2\sqrt{5}$ **D)** $\frac{8+4\sqrt{3}}{3}$ **E)** 5

Q21. The complex number z satisfies $z + |z| = 2 + 8i$. What is $|z|^2$? Note: if $z = a + bi$, then $|z| = \sqrt{a^2 + b^2}$.

A) 68 **B)** 100 **C)** 169 **D)** 208 **E)** 289

Q22. For how many integers x does a triangle with side lengths $10, 24$ and x have all its angles acute?

A) 4 **B)** 5 **C)** 6 **D)** 7 **E)** more than 7

Q23. The six edges of a tetrahedron $ABCD$ measure $7, 13, 18, 27, 36$ and 41 units. If the length of edge AB is 41, then the length of edge CD is

A) 7 **B)** 13 **C)** 18 **D)** 27 **E)** 36

Q24. An isosceles trapezoid is circumscribed around a circle. The longer base of the trapezoid is 16, and one of the base angles is arcsin(.8). Find the area of the trapezoid.

A) 72 **B)** 75 **C)** 80 **D)** 90

E) not uniquely determined

Q25. X, Y and Z are pairwise disjoint sets of people. The average ages of people in the sets $X, Y, Z, X \cup Y, X \cup Z$ and $Y \cup Z$ are $37, 23, 41, 29, 39.5$ and 33 respectively. Find the average age of the people in set $X \cup Y \cup Z$.

A) 33 **B)** 33.5 **C)** $33.6\overline{6}$ **D)** $33.83\overline{3}$ **E)** 34

Q26. Suppose that p and q are positive numbers for which

$$\log_9(p) = \log_{12}(q) = \log_{16}(p+q).$$

What is the value of $\frac{q}{p}$?

A) $\frac{4}{3}$ **B)** $\frac{1+\sqrt{3}}{2}$ **C)** $\frac{8}{5}$ **D)** $\frac{1+\sqrt{5}}{2}$ **E)** $\frac{16}{9}$

Q27. In the figure, $AB \perp BC, BC \perp CD$, and BC is tangent to the circle with center O and diameter AD. In which one of the following cases is the area of $ABCD$ an integer?

A) $AB = 3, CD = 1$ **B)** $AB = 5, CD = 2$ **C)** $AB = 7, CD = 3$ **D)** $AB = 9, CD = 4$ **E)** $AB = 11, CD = 5$

Q28. An unfair coin has probability p of coming up heads on a single toss. Let w be the probability that, in 5 independent toss of this coin, heads come up exactly 3 times. If $w = 144/625$, then

A) p must be $\frac{2}{5}$

B) p must be $\frac{3}{5}$

C) p must be greater than $\frac{3}{5}$

D) p is not uniquely determined

E) there is no value of p for which $w = \dfrac{144}{625}$

Q29. You plot weight (y) against height (x) for three of your friends and obtain the points $(x_1, y_1), (x_2, y_2), (x_3, y_3)$. If

$$x_1 < x_2 < x_3 \quad \text{and} \quad x_3 - x_2 = x_2 - x_1$$

which of the following is necessarily the slope of the line which best fits the data? "Best fits" means that the sum of the squares of the vertical distances from the data points to the line is smaller than for any other line.

A) $\dfrac{y_3 - y_1}{x_3 - x_1}$ **B)** $\dfrac{(y_2 - y_1) - (y_3 - y_2)}{x_3 - x_1}$ **C)** $\dfrac{2y_3 - y_1 - y_2}{2x_3 - x_1 - x_2}$

D) $\dfrac{y_2 - y_1}{x_2 - x_1} + \dfrac{y_3 - y_2}{x_3 - x_2}$ **E)** none of these

Q30. Let $f(x) = 4x - x^2$. Give x_0, consider the sequence defined by $x_n = f(x_{n-1})$ for all $n \geq 1$. For how many real numbers x_0 will the sequence $x_0, x_1, x_2, \ldots$ take on only a finite number of different values?

A) 0 **B)** 1 or 2 **C)** 3, 4, 5 or 6

D) more than 6 but finitely many **E)** ∞

Q1. $(-1)^{5^2} + 1^{2^5} =$

A) -7 **B)** -2 **C)** 0 **D)** 1 **E)** 57

Q2. $\sqrt{\frac{1}{9} + \frac{1}{16}} =$

A) $\frac{1}{5}$ **B)** $\frac{1}{4}$ **C)** $\frac{2}{7}$ **D)** $\frac{5}{12}$ **E)** $\frac{7}{12}$

Q3. A square is cut into three rectangles along two lines parallel to a side, as shown. If the perimeter of each of the three rectangles is 24, then the area of the original square is

A) 24 **B)** 36 **C)** 64 **D)** 81 **E)** 96

Q4. In the figure, $ABCD$ is an isosceles trapezoid with side lengths $AD = BC = 5$, $AB = 4$, and $DC = 10$. The point C is on $\overline{DF}$ and B is the midpoint of hypotenuse $\overline{DE}$ in right triangle DEF. Then $CF =$

A) 3.25 **B)** 3.5 **C)** 3.75 **D)** 4.0 **E)** 4.25

Q5. Toothpicks of equal length are used to build a rectangular grid as shown. If the grid is 20 toothpicks high and 10 toothpicks wide, then the number of toothpicks used is

A) 30 **B)** 200 **C)** 410 **D)** 420 **E)** 430

Q6. If $a, b > 0$ and the triangle in the first quadrant bounded by the co-ordinate axes and the graph of $ax + by = 6$ has area 6, then $ab =$

A) 3 **B)** 6 **C)** 12 **D)** 108 **E)** 432

Q7. In $\triangle ABC$, $\angle A = 100^\circ$, $\angle B = 50^\circ$, $\angle C = 30^\circ$, $\overline{AH}$ is an altitude, and $\overline{BM}$ is a median. Then $\angle MHC =$

A) 15° **B)** 22.5° **C)** 30° **D)** 40° **E)** 45°

Q8. For how many integers n between 1 and 100 does $x^2 + x - n$ factor into the product of two linear factors with integer coefficients?

A) 0 **B)** 1 **C)** 2 **D)** 9 **E)** 10

Q9. Mr. and Mrs. Zeta want to name their baby Zeta so that its monogram (first, middle, and last initials) will be in alphabetical order with no letter repeated. How many such monograms are possible?

A) 276 **B)** 300 **C)** 552 **D)** 600 **E)** 15600

Q10. Consider the sequence defined recursively by $u_1 = a$ (any positive number), and $u_{n+1} = -1/(u_n + 1)$, $n = 1, 2, 3, \ldots$ For which of the following values of n must $u_n = a$?

A) 14 **B)** 15 **C)** 16 **D)** 17 **E)** 18

Q11. Let a, b, c, and d be positive integers with $a < 2b$, $b < 3c$, and $c < 4d$. If $d < 100$, the largest possible value for a is

A) 2367 **B)** 2375 **C)** 2391 **D)** 2399 **E)** 2400

Q12. The traffic on a certain east-west highway moves at a constant speed of 60 miles per hour in both directions. An eastbound driver passes 20 west-bound vehicles in a five-minute interval. Assume vehicles in the westbound lane are equally spaced. Which of the following is closest to the number of westbound vehicles present in a 100-mile section of highway?

A) 100 **B)** 120 **C)** 200 **D)** 240 **E)** 400

Q13. Two strips of width 1 overlap at an angle of α as shown. The area of the overlap (shown shaded) is

A) $\sin\alpha$ **B)** $\dfrac{1}{\sin\alpha}$ **C)** $\dfrac{1}{1-\cos\alpha}$ **D)** $\dfrac{1}{\sin^2\alpha}$ **E)** $\dfrac{1}{(1-\cos\alpha)^2}$

Q14. $\cot 10 + \tan 5 =$

A) $\csc 5$ **B)** $\csc 10$ **C)** $\sec 5$ **D)** $\sec 10$ **E)** $\sin 15$

Q15. In $\triangle ABC$, $AB = 5$, $BC = 7$, $AC = 9$, and D is on $\overline{AC}$ with $BD = 5$. Find the ratio of $AD : DC$.

A) $4:3$ **B)** $7:5$ **C)** $11:6$ **D)** $13:5$ **E)** $19:8$

Q16. A lattice point is a point in the plane with integer coordinates. How many lattice points are on the line segment whose endpoints are $(3, 17)$ and $(48, 281)$? (Include both endpoints of the segment in your count.)

A) 2 **B)** 4 **C)** 6 **D)** 16 **E)** 46

Q17. The perimeter of an equilateral triangle exceeds the perimeter of a square by 1989 cm. The length of each side of the triangle exceeds the length of each side of the square by d cm. The square has perimeter greater than 0. How many positive integers are NOT possible value for d?

A) 0 **B)** 9 **C)** 221 **D)** 663 **E)** ∞

Q18. The set of all real numbers for which

$$x + \sqrt{x^2+1} - \frac{1}{x+\sqrt{x^2+1}}$$

is a rational number is the set of all

A) integers x **B)** rational x **C)** real x

D) x for which $\sqrt{x^2+1}$ is rational

E) x for which $x + \sqrt{x^2+1}$ is rational

Q19. A triangle is inscribed in a circle. The vertices of the triangle divide the circle into three arcs of lengths 3, 4, and 5. What is the area of the triangle?

A) 6 **B)** $\frac{18}{\pi^2}$ **C)** $\frac{9}{\pi^2}(\sqrt{3}-1)$ **D)** $\frac{9}{\pi^2}(\sqrt{3}-1)$ **E)** $\frac{9}{\pi^2}(\sqrt{3}+3)$

Q20. Let x be a real number selected uniformly at random between 100 and 200. If $\lfloor\sqrt{x}\rfloor = 12$, find the probability that $\lfloor\sqrt{100x}\rfloor = 120$. ($\lfloor v\rfloor$ means the greatest integer less than or equal to v.)

A) $\frac{2}{25}$ **B)** $\frac{241}{2500}$ **C)** $\frac{1}{10}$ **D)** $\frac{96}{625}$ **E)** 1

Q21. A square flag has a red cross of uniform width with a blue square in the center on a white background as shown. (The cross is symmetric with respect to each of the diagonals of the square.) If the entire cross (both the red arms and the blue center) takes up 36% of the area of the flag, what percent of the area of the flag is blue?

A) 0.5 **B)** 1 **C)** 2 **D)** 3 **E)** 6

Q22. A child has a set of 96 distinct blocks. Each block is one of 2 materials (plastic, wood), 3 sizes (small, medium, large), 4 colors (blue, green, red, yellow), and 4 shapes (circle, hexagon, square, triangle). How many blocks in the set differ from the ’plastic medium red circle’ in exactly 2 ways? (The ’wood medium red square’ is such a block)

A) 29 **B)** 39 **C)** 48 **D)** 56 **E)** 62

Q23. A particle moves through the first quadrant as follows. During the first minute it moves from the origin to $(1, 0)$. Thereafter, it continues to follow the directions indicated in the figure, going back and forth between the positive x and y axes, moving one unit of distance parallel to an axis in each minute. At which point will the particle be after exactly 1989 minutes?

A) $(35, 44)$ **B)** $(36, 45)$ **C)** $(37, 45)$ **D)** $(44, 35)$ **E)** $(45, 36)$

Q24. Five people are sitting at a round table. Let $f \geq 0$ be the number of people sitting next to at least 1 female and $m \geq 0$ be the number of people sitting next to at least one male. The number of possible ordered pairs (f, m) is

A) 7 **B)** 8 **C)** 9 **D)** 10 **E)** 11

Q25. In a certain cross country meet between 2 teams of 5 runners each, a runner who finishes in the nth position contributes n to his teams score. The team with the lower score wins. If there are no ties among the runners, how many different winning scores are possible?

A) 10 **B)** 13 **C)** 27 **D)** 120 **E)** 126

Q26. A regular octahedron is formed by joining the centers of adjoining faces of a cube. The ratio of the volume of the octahedron to the volume of the cube is

A) $\frac{\sqrt{3}}{12}$ **B)** $\frac{\sqrt{6}}{16}$ **C)** $\frac{1}{6}$ **D)** $\frac{\sqrt{2}}{8}$ **E)** $\frac{1}{4}$

Q27. Let n be a positive integer. If the equation $2x + 2y + z = n$ has 28 solutions in positive integers x, y, and z, then n must be either

A) 14 or 15 **B)** 15 or 16 **C)** 16 or 17 **D)** 17 or 18 **E)** 18 or 19

Q28. Find the sum of the roots of $\tan^2 x - 9\tan x + 1 = 0$ that are between $x = 0$ and $x = 2\pi$ radians.

A) $\frac{\pi}{2}$ **B)** π **C)** $\frac{3\pi}{2}$ **D)** 3π **E)** 4π

Q29. What is the value of the sum

$$S = \sum_{k=0}^{49} (-1)^k \binom{99}{2k} = \binom{99}{0} - \binom{99}{2} + \binom{99}{4} - \cdots - \binom{99}{98}?$$

A) -2^{50} **B)** -2^{49} **C)** 0 **D)** 2^{49} **E)** 2^{50}

Q30. Suppose that 7 boys and 13 girls line up in a row. Let S be the number of places in the row where a boy and a girl are standing next to each other. For example, for the row GBBGGGBGBGGGBGBGGBGG we have that $S = 12$. The average value of S (if all possible orders of these 20 people are considered) is closest to

A) 9 **B)** 10 **C)** 11 **D)** 12 **E)** 13

Q1. If $\dfrac{\frac{x}{4}}{2} = \dfrac{4}{\frac{x}{2}}$, then $x =$

A) $\pm\frac{1}{2}$ **B)** ± 1 **C)** ± 2 **D)** ± 4 **E)** ± 8

Q2. $\left(\frac{1}{4}\right)^{-\frac{1}{4}} =$

A) -16 **B)** $-\sqrt{2}$ **C)** $-\frac{1}{16}$ **D)** $\frac{1}{256}$ **E)** $\sqrt{2}$

Q3. The consecutive angles of a trapezoid form an arithmetic sequence. If the smallest angle is 75°, then the largest angle is

A) 95° **B)** 100° **C)** 105° **D)** 110° **E)** 115°

Q4. Let $ABCD$ be a parallelogram with $\angle ABC = 120^\circ$, $AB = 16$ and $BC = 10$.

Extend $\overline{CD}$ through D to E so that $DE = 4$. If $\overline{BE}$ intersects $\overline{AD}$ at F, then FD is closest to

A) 1 **B)** 2 **C)** 3 **D)** 4 **E)** 5

Q5. Which of these numbers is largest?

A) $\sqrt{\sqrt[3]{5\cdot 6}}$ **B)** $\sqrt{6\sqrt[3]{5}}$ **C)** $\sqrt{5\sqrt[3]{6}}$ **D)** $\sqrt[3]{5\sqrt{6}}$ **E)** $\sqrt[3]{6\sqrt{5}}$

Q6. Points A and B are 5 units apart. How many lines in a given plane containing A and B are 2 units from A and 3 units from B?

A) 0 **B)** 1 **C)** 2 **D)** 3 **E)** more than 3

Q7. A triangle with integral sides has perimeter 8. The area of the triangle is

A) $2\sqrt{2}$ **B)** $\frac{16}{9}\sqrt{3}$ **C)** $2\sqrt{3}$ **D)** 4 **E)** $4\sqrt{2}$

Q8. The number of real solutions of the equation

$$|x-2| + |x-3| = 1$$

is

A) 0 **B)** 1 **C)** 2 **D)** 3

E) more than 3

Q9. Each edge of a cube is colored either red or black. Every face of the cube has at least one black edge. The smallest number possible of black edges is

A) 2 **B)** 3 **C)** 4 **D)** 5 **E)** 6

Q10. An $10 \times 10 \times 10$ wooden cube is formed by gluing together 10^3 unit cubes. What is the greatest number of unit cubes that can be seen from a single point?

A) 328 **B)** 329 **C)** 330 **D)** 331 **E)** 332

Q11. How many positive integers less than 50 have an odd number of positive integer divisors?

A) 3 **B)** 5 **C)** 7 **D)** 9 **E)** 11

Q12. Let f be the function defined by $\mathrm{f}(x) = ax^2 - \sqrt{2}$ for some positive a. If $\mathrm{f}(\mathrm{f}(\sqrt{2})) = -\sqrt{2}$ then $a =$

A) $\frac{2-\sqrt{2}}{2}$ **B)** $\frac{1}{2}$ **C)** $2-\sqrt{2}$ **D)** $\frac{\sqrt{2}}{2}$ **E)** $\frac{2+\sqrt{2}}{2}$

Q13. If the following instructions are carried out by a computer, which value of X will be printed because of instruction 5?

1) START X AT 3 AND S AT 0.
2) INCREASE THE VALUE OF X BY 2.
3) INCREASE THE VALUE OF S BY THE VALUE OF X.
4) IF S IS AT LEAST 10000, THEN GO TO INSTRUCTION 5; OTHERWISE, GO TO INSTRUCTION 2. AND PROCEED FROM THERE.
5) PRINT THE VALUE OF X.
6) STOP.

A) 19 **B)** 21 **C)** 23 **D)** 199 **E)** 201

Q14. An acute isosceles triangle, ABC, is inscribed in a circle.

Through B and C, tangents to the circle are drawn, meeting at point D. If $\angle ABC = \angle ACB = 2\angle D$ and x is the radian measure of $\angle A$, then $x =$

A) $\frac{3\pi}{7}$ **B)** $\frac{4\pi}{9}$ **C)** $\frac{5\pi}{11}$ **D)** $\frac{6\pi}{13}$ **E)** $\frac{7\pi}{15}$

Q15. Four whole numbers, when added three at a time, give the sums $180, 197, 208$ and 222. What is the largest of the four numbers?

A) 77 **B)** 83 **C)** 89 **D)** 95

E) cannot be determined from the given information

Q16. At one of George Washington's parties, each man shook hands with everyone except his spouse, and no handshakes took place between women. If 13 married couples attended, how many handshakes were there among these 26 people?

A) 78 **B)** 185 **C)** 234 **D)** 312 **E)** 325

Q17. How many of the numbers, $100, 101, \cdots, 999$ have three different digits in increasing order or in decreasing order?

A) 120 **B)** 168 **C)** 204 **D)** 216 **E)** 240

Q18. First a is chosen at random from the set $\{1, 2, 3, \cdots, 99, 100\}$, and then b is chosen at random from the same set. The probability that the integer $3^a + 7^b$ has units digit 8 is

A) $\frac{1}{16}$ **B)** $\frac{1}{8}$ **C)** $\frac{3}{16}$ **D)** $\frac{1}{5}$ **E)** $\frac{1}{4}$

Q19. For how many integers N between 1 and 1990 is the improper fraction $\frac{N^2+7}{N+4}$ <u>*not*</u> in lowest terms?

A) 0 **B)** 86 **C)** 90 **D)** 104 **E)** 105

Q20. In the figure $ABCD$ is a quadrilateral with right angles at A and C.

Points E and F are on $\overline{AC}$, and $\overline{DE}$ and $\overline{BF}$ are perpendicular to $\overline{AC}$. If $AE = 3, DE = 5$, and $CE = 7$, then $BF =$

A) 3.6 **B)** 4 **C)** 4.2 **D)** 4.5 **E)** 5

Q21. Consider a pyramid $P - ABCD$ whose base $ABCD$ is square and whose vertex P is equidistant from A, B, C and D. If $AB = 1$ and $\angle APB = 2\theta$, then the volume of the pyramid is

A) $\frac{\sin(\theta)}{6}$ **B)** $\frac{\cot(\theta)}{6}$ **C)** $\frac{1}{6\sin(\theta)}$ **D)** $\frac{1-\sin(2\theta)}{6}$ **E)** $\frac{\sqrt{\cos(2\theta)}}{6\sin(\theta)}$

Q22. If the six solutions of $x^6 = -64$ are written in the form $a + bi$, where a and b are real, then the product of those solutions with $a > 0$ is

A) -2 **B)** 0 **C)** 2i **D)** 4 **E)** 16

Q23. If $x, y > 0, \log_y(x) + \log_x(y) = \frac{10}{3}$ and $xy = 144$, then $\frac{x+y}{2} =$

A) $12\sqrt{2}$ **B)** $13\sqrt{3}$ **C)** 24 **D)** 30 **E)** 36

Q24. All students at Adams High School and at Baker High School take a certain exam. The average scores for boys, for girls, and for boys and girls combined, at Adams HS and Baker HS are shown in the table, as is the average for boys at the two schools combined. What is the average score for the girls at the two schools combined?

Average Scores

Category	Adams	Baker	Adams&Baker
Boys	71	81	79
Girls	76	90	?
Boys&Girls	74	84	

A) 81 **B)** 82 **C)** 83 **D)** 84 **E)** 85

Q25. Nine congruent spheres are packed inside a unit cube in such a way that one of them has its center at the center of the cube and each of the others is tangent to the center sphere and to three faces of the cube. What is the radius of each sphere?

A) $1 - \frac{\sqrt{3}}{2}$ **B)** $\frac{2\sqrt{3}-3}{2}$ **C)** $\frac{\sqrt{2}}{6}$ **D)** $\frac{1}{4}$ **E)** $\frac{\sqrt{3}(2-\sqrt{2})}{4}$

Q26. Ten people form a circle. Each picks a number and tells it to the two neighbors adjacent to him in the circle. Then each person computes and announces the average of the numbers of his two neighbors. The average announced by each person was (in order around the circle) 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 (NOT the original number the person picked). The number picked by the person who announced the average 6 was

A) 1 **B)** 5 **C)** 6 **D)** 10

E) not uniquely determined from the given information

Q27. Which of these triples could *not* be the lengths of the three altitudes of a triangle?

A) $1, \sqrt{3}, 2$ **B)** $3, 4, 5$ **C)** $5, 12, 13$ **D)** $7, 8, \sqrt{113}$ **E)** $8, 15, 17$

Q28. A quadrilateral that has consecutive sides of lengths $70, 90, 130$ and 110 is inscribed in a circle and also has a circle inscribed in it. The point of tangency of the inscribed circle to the side of length 130 divides that side into segments of length x and y. Find $|x - y|$.

A) 12 **B)** 13 **C)** 14 **D)** 15 **E)** 16

Q29. A subset of the integers $1, 2, \cdots, 100$ has the property that none of its members is 3 times another. What is the largest number of members such a subset can have?

A) 50 **B)** 66 **C)** 67 **D)** 76 **E)** 78

Q30. If $R_n = \frac{1}{2}(a^n + b^n)$ where $a = 3 + 2\sqrt{2}$ and $b = 3 - 2\sqrt{2}$, and $n = 0, 1, 2, \cdots$, then R_{12345} is an integer. Its units digit is

A) 1 **B)** 3 **C)** 5 **D)** 7 **E)** 9

Q1. If for any three distinct numbers a, b, and c we define $\mathrm{f}(a,b,c)=\dfrac{c+a}{c-b}$, then $\mathrm{f}(1,-2,-3)$ is

A) -2 **B)** $-\frac{2}{5}$ **C)** $-\frac{1}{4}$ **D)** $\frac{2}{5}$ **E)** 2

Q2. $|3-\pi|=$

A) $\frac{1}{7}$ **B)** 0.14 **C)** $3-\pi$ **D)** $3+\pi$ **E)** $\pi-3$

Q3. $(4^{-1}-3^{-1})^{-1}=$

A) -12 **B)** -1 **C)** $\frac{1}{12}$ **D)** 1 **E)** 12

Q4. Which of the following triangles cannot exist?

A) An acute isosceles triangle

B) An isosceles right triangle

C) An obtuse right triangle

D) A scalene right triangle

E) A scalene obtuse triangle

Q5. In the arrow-shaped polygon [see figure], the angles at vertices A, C, D, E and F are right angles, $BC = FG = 5, CD = FE = 20, DE = 10$, and $AB = AG$.

The area of the polygon is closest to

A) 288 **B)** 291 **C)** 294 **D)** 297 **E)** 300

Q6. If $x \geq 0$, then $\sqrt{x\sqrt{x\sqrt{x}}}=$

A) $x\sqrt{x}$ **B)** $x\sqrt[4]{x}$ **C)** $\sqrt[8]{x}$ **D)** $\sqrt[8]{x^3}$ **E)** $\sqrt[8]{x^7}$

Q7. If $x=\frac{a}{b}$, $a \neq b$ and $b \neq 0$, then $\dfrac{a+b}{a-b}=$

A) $\dfrac{x}{x+1}$ **B)** $\dfrac{x+1}{x-1}$ **C)** 1 **D)** $x-\dfrac{1}{x}$ **E)** $x+\dfrac{1}{x}$

Q8. Liquid X does not mix with water. Unless obstructed, it spreads out on the surface of water to form a circular film 0.1cm thick. A rectangular box measuring 6cm by 3cm by 12cm is filled with liquid X. Its contents are poured onto a large body of water. What will be the radius, in centimeters, of the resulting circular film?

A) $\frac{\sqrt{216}}{\pi}$ **B)** $\sqrt{\frac{216}{\pi}}$ **C)** $\sqrt{\frac{2160}{\pi}}$ **D)** $\frac{216}{\pi}$ **E)** $\frac{2160}{\pi}$

Q9. From time $t=0$ to time $t=1$ a population increased by $i\%$, and from time $t=1$ to time $t=2$ the population increased by $j\%$. Therefore, from time $t=0$ to time $t=2$ the population increased by

A) $(i+j)\%$ **B)** $ij\%$ **C)** $(i+ij)\%$
D) $\left(i+j+\frac{ij}{100}\right)\%$ **E)** $\left(i+j+\frac{i+j}{100}\right)\%$

Q10. Point P is 9 units from the center of a circle of radius 15. How many different chords of the circle contain P and have integer lengths?

A) 11 **B)** 12 **C)** 13 **D)** 14 **E)** 29

Q11. Jack and Jill run 10 km. They start at the same point, run 5 km up a hill, and return to the starting point by the same route. Jack has a 10 minute head start and runs at the rate of 15 km/hr uphill and 20 km/hr downhill. Jill runs 16 km/hr uphill and 22 km/hr downhill. How far from the top of the hill are they when they pass each other going in opposite directions (in km)?

A) $\frac{5}{4}$ **B)** $\frac{35}{27}$ **C)** $\frac{27}{20}$ **D)** $\frac{7}{3}$ **E)** $\frac{28}{49}$

Q12. The measures (in degrees) of the interior angles of a convex hexagon form an arithmetic sequence of integers. Let m be the measure of the largest interior angle of the hexagon. The largest possible value of m, in degrees, is

A) 165 **B)** 167 **C)** 170 **D)** 175 **E)** 179

Q13. Horses X, Y and Z are entered in a three-horse race in which ties are not possible. The odds against X winning are $3:1$ and the odds against Y winning are $2:3$, what are the odds against Z winning? (By "odds against H winning are $p:q$" we mean the probability of H winning the race is $\frac{q}{p+q}$.)

A) 3:20 **B)** 5:6 **C)** 8:5 **D)** 17:3 **E)** 20:3

Q14. If x is the cube of a positive integer and d is the number of positive integers that are divisors of x, then d could be

A) 200 **B)** 201 **C)** 202 **D)** 203 **E)** 204

Q15. A circular table has 60 chairs around it. There are N people seated at this table in such a way that the next person seated must sit next to someone. What is the smallest possible value for N?

A) 15 **B)** 20 **C)** 30 **D)** 40 **E)** 58

Q16. One hundred students at Century High School participated in the AHSME last year, and their mean score was 100. The number of non-seniors taking the AHSME was 50% more than the number of seniors, and the mean score of the seniors was 50% higher than that of the non-seniors. What was the mean score of the seniors?

A) 100 **B)** 112.5 **C)** 120 **D)** 125 **E)** 150

Q17. A positive integer N is a "palindrome" if the integer obtained by reversing the sequence of digits of N is equal to N. The year 1991 is the only year in the current century with the following 2 properties:

(a) It is a palindrome

(b) It factors as a product of a 2-digit prime palindrome and a 3-digit prime palindrome.

How many years in the millennium between 1000 and 2000 have properties (a) and (b)?

A) 1 **B)** 2 **C)** 3 **D)** 4 **E)** 5

Q18. If S is the set of points z in the complex plane such that $(3+4i)z$ is a real number, then S is a

A) right triangle **B)** circle **C)** hyperbola **D)** line **E)** parabola

Q19. Triangle ABC has a right angle at $C, AC = 3$ and $BC = 4$. Triangle ABD has a right angle at A and $AD = 12$. Points C and D are on opposite sides of $\overline{AB}$.

The line through D parallel to $\overline{AC}$ meets $\overline{CB}$ extended at E. If

$$\frac{DE}{DB} = \frac{m}{n},$$

where m and n are relatively prime positive integers, then $m+n =$

A) 25 **B)** 128 **C)** 153 **D)** 243 **E)** 256

Q20. The sum of all real x such that $(2^x - 4)^3 + (4^x - 2)^3 = (4^x + 2^x - 6)^3$ is

A) $\frac{3}{2}$ **B)** 2 **C)** $\frac{5}{2}$ **D)** 3 **E)** $\frac{7}{2}$

Q21. For all real numbers x except $x = 0$ and $x = 1$, the function $\mathrm{f}(x)$ is defined by $\mathrm{f}(x/(x-1)) = 1/x$. Suppose $0 \le t \le \pi/2$. What is the value of $\mathrm{f}(\sec^2 t)$?

A) $\sin^2\theta$ **B)** $\cos^2\theta$ **C)** $\tan^2\theta$ **D)** $\cot^2\theta$ **E)** $\csc^2\theta$

Q22. Two circles are externally tangent.

Lines $\overline{PAB}$ and $\overline{PA'B'}$ are common tangents with A and A' on the smaller circle B and B' on the larger circle. If $PA = AB = 4$, then the area of the smaller circle is

A) 1.44π **B)** 2π **C)** 2.56π **D)** $\sqrt{8}\pi$ **E)** 4π

Q23. If $ABCD$ is a 2×2 square, E is the midpoint of $\overline{AB}$, F is the midpoint of $\overline{BC}$, $\overline{AF}$ and $\overline{DE}$ intersect at I, and $\overline{BD}$ and $\overline{AF}$ intersect at H,

then the area of quadrilateral $BEIH$ is

A) $\frac{1}{3}$ **B)** $\frac{2}{5}$ **C)** $\frac{7}{15}$ **D)** $\frac{8}{15}$ **E)** $\frac{3}{5}$

Q24. The graph, G of $y = \log_{10} x$ is rotated $90°$ counter-clockwise about the origin to obtain a new graph G'. Which of the following is an equation for G'?

A) $y = \log_{10}\left(\frac{x+90}{9}\right)$ **B)** $y = \log_x 10$ **C)** $y = \frac{1}{x+1}$

D) $y = 10^{-x}$ **E)** $y = 10^x$

Q25. If $T_n = 1 + 2 + 3 + \cdots + n$ and

$$P_n = \frac{T_2}{T_2 - 1} \cdot \frac{T_3}{T_3 - 1} \cdot \frac{T_4}{T_4 - 1} \cdot \cdots \cdot \frac{T_n}{T_n - 1}$$

for $n = 2, 3, 4, \cdots$, then P_{1991} is closest to which of the following numbers?

A) 2.0 **B)** 2.3 **C)** 2.6 **D)** 2.9 **E)** 3.2

Q26. An n-digit positive integer is cute if its n digits are an arrangement of the set $\{1, 2, \ldots, n\}$ and its first k digits form an integer that is divisible by k, for $k = 1, 2, \ldots, n$. For example, 321 is a cute 3-digit integer because 1 divides 3, 2 divides 32, and 3 divides 321. How many cute 6-digit integers are there?

A) 0 **B)** 1 **C)** 2 **D)** 3 **E)** 4

Q27. If $x + \sqrt{x^2 - 1} + \frac{1}{x - \sqrt{x^2 - 1}} = 20$ then $x^2 + \sqrt{x^4 - 1} + \frac{1}{x^2 + \sqrt{x^4 - 1}} =$

A) 5.05 **B)** 20 **C)** 51.005 **D)** 61.25 **E)** 400

Q28. Initially an urn contains 100 white and 100 black marbles. Repeatedly 3 marbles are removed (at random) from the urn and replaced with some marbles from a pile outside the urn as follows: 3 blacks are replaced with 1 black, or 2 blacks and 1 white are replaced with a white and a black, or 1 black and 2 whites are replaced with 2 whites, or 3 whites are replaced with a black and a white. Which of the following could be the contents of the urn after repeated applications of this procedure?

A) 2 black **B)** 2 white **C)** 1 black

D) 1 black and 1 white **E)** 1 white

Q29. Equilateral triangle ABC has P on AB and Q on AC. The triangle is folded along PQ so that vertex A now rests at A' on side BC. If $BA' = 1$ and $A'C = 2$ then the length of the crease PQ is

A) $\frac{8}{5}$ **B)** $\frac{7}{20}\sqrt{21}$ **C)** $\frac{1+\sqrt{5}}{2}$ **D)** $\frac{13}{8}$ **E)** $\sqrt{3}$

Q30. For any set S, let $|S|$ denote the number of elements in S, and let $n(S)$ be the number of subsets of S, including the empty set and the set S itself. If A, B, and C are sets for which $n(A) + n(B) + n(C) = n(A \cup B \cup C)$ and $|A| = |B| = 100$, then what is the minimum possible value of $|A \cap B \cap C|$?

A) 96 **B)** 97 **C)** 98 **D)** 99 **E)** 100

Q1. If $3(4x+5\pi)=P$ then $6(8x+10\pi)=$

A) $2P$ **B)** $4P$ **C)** $6P$ **D)** $8P$ **E)** $18P$

Q2. An urn is filled with coins and beads, all of which are either silver or gold. Twenty percent of the objects in the urn are beads. Forty percent of the coins in the urn are silver. What percent of objects in the urn are gold coins?

A) 40% **B)** 48% **C)** 52% **D)** 60% **E)** 80%

Q3. If $m>0$ and the points $(m,3)$ and $(1,m)$ lie on a line with slope m, then $m=$

A) 1 **B)** $\sqrt{2}$ **C)** $\sqrt{3}$ **D)** 2 **E)** $\sqrt{5}$

Q4. If a,b and c are positive integers and a and b are odd, then $3^a+(b-1)^2c$ is

A) odd for all choices of c

B) even for all choices of c

C) odd if c is even;even if c is odd

D) odd if c is odd;even if c is even

E) odd if c is not a multiple of 3; even if c is a multiple of 3

Q5. $6^6+6^6+6^6+6^6+6^6+6^6=$

A) 6^6 **B)** 6^7 **C)** 36^6 **D)** 6^{36} **E)** 36^{36}

Q6. If $x>y>0$, then $\dfrac{x^y y^x}{y^y x^x}=$

A) $(x-y)^{y/x}$ **B)** $\left(\dfrac{x}{y}\right)^{x-y}$ **C)** 1 **D)** $\left(\dfrac{x}{y}\right)^{y-x}$ **E)** $(x-y)^{x/y}$

Q7. The ratio of w to x is $4:3$, of y to z is $3:2$ and of z to x is $1:6$. What is the ratio of w to y?

A) $1:3$ **B)** $16:3$ **C)** $20:3$ **D)** $27:4$ **E)** $12:1$

Q8. A square floor is tiled with congruent square tiles. The tiles on the two diagonals of the floor are black. The rest of the tiles are white. If there are 101 black tiles, then the total number of tiles is

A) 121 **B)** 625 **C)** 676 **D)** 2500 **E)** 2601

Q9. Five equilateral triangles, each with side $2\sqrt{3}$, are arranged so they are all on the same side of a line containing one side of each vertex. Along this line, the midpoint of the base of one triangle is a vertex of the next. The area of the region of the plane that is covered by the union of the five triangular regions is

A) 10 **B)** 12 **C)** 15 **D)** $10\sqrt{3}$ **E)** $12\sqrt{3}$

Q10. The number of positive integers k for which the equation

$$kx - 12 = 3k$$

has an integer solution for x is

A) 3 **B)** 4 **C)** 5 **D)** 6 **E)** 7

Q11.

The ratio of the radii of two concentric circles is $1:3$. If $\overline{AC}$ is a diameter of the larger circle, $\overline{BC}$ is a chord of the larger circle that is tangent to the smaller circle, and $AB = 12$, then the radius of the larger circle is

A) 13 **B)** 18 **C)** 21 **D)** 24 **E)** 26

Q12. Let $y = mx + b$ be the image when the line $x - 3y + 11 = 0$ is reflected across the x-axis. The value of $m + b$ is

A) -6 **B)** -5 **C)** -4 **D)** -3 **E)** -2

Q13. How many pairs of positive integers (a,b) with $a + b \leq 100$ satisfy the equation

$$\frac{a + b^{-1}}{a^{-1} + b} = 13?$$

A) 1 **B)** 5 **C)** 7 **D)** 9 **E)** 13

Q14. Which of the following equations have the same graph?

I) $y = x - 2$

II) $y = \dfrac{x^2 - 4}{x + 2}$

III) $(x + 2)y = x^2 - 4$

A) I and II only **B)** I and III only **C)** II and III only **D)** I,II,and III

E) None. All of the equations have different graphs.

Q15. Let $i = \sqrt{-1}$. Define a sequence of complex numbers by

$$z_1 = 0, z_{n+1} = z_n^2 + i \text{ for } n \geq 1.$$

In the complex plane, how far from the origin is z_{111}?

A) 1 **B)** $\sqrt{2}$ **C)** $\sqrt{3}$ **D)** $\sqrt{110}$ **E)** $\sqrt{2^{55}}$

Q16. If

$$\frac{y}{x-z} = \frac{x+y}{z} = \frac{x}{y}$$

for three positive numbers x, y and z, all different, then $\frac{x}{y} =$

A) $\frac{1}{2}$ **B)** $\frac{3}{5}$ **C)** $\frac{2}{3}$ **D)** $\frac{5}{3}$ **E)** 2

Q17. The 2-digit integers from 19 to 92 are written consecutively to form the integer $N = 192021 \cdots 9192$. Suppose that 3^k is the highest power of 3 that is a factor of N. What is k?

A) 0 **B)** 1 **C)** 2 **D)** 3 **E)** more than

Q18. The increasing sequence of positive integers $a_1, a_2, a_3, \cdots$ has the property that

$$a_{n+2} = a_n + a_{n+1} \text{ for all } n \geq 1.$$

If $a_7 = 120$, then a_8 is

A) 128 **B)** 168 **C)** 193 **D)** 194 **E)** 210

Q19. For each vertex of a solid cube, consider the tetrahedron determined by the vertex and the midpoints of the three edges that meet at that vertex. The portion of the cube that remains when these eight tetrahedra are cut away is called a cuboctahedron. The ratio of the volume of the cuboctahedron to the volume of the original cube is closest to which of these?

A) 75% **B)** 78% **C)** 81% **D)** 84% **E)** 87%

Q20. Part of an "n-pointed regular star" is shown.

It is a simple closed polygon in which all $2n$ edges are congruent, angles $A_1, A_2, \cdots, A_n$ are congruent, and angles $B_1, B_2, \cdots, B_n$ are congruent. If the acute angle at A_1 is $10°$ less than the acute angle at B_1, then $n =$

A) 12 **B)** 18 **C)** 24 **D)** 36 **E)** 60

Q21. For a finite sequence $A = (a_1, a_2, \ldots, a_n)$ of numbers, the "Cesáro sum" of A is defined to be $\frac{S_1 + \cdots + S_n}{n}$, where $S_k = a_1 + \cdots + a_k$ and $1 \leq k \leq n$. If the Cesáro sum of the 99-term sequence $(a_1, \ldots, a_{99})$ is 1000, what is the Cesáro sum of the 100-term sequence $(1, a_1, \ldots, a_{99})$?

A) 991 **B)** 999 **C)** 1000 **D)** 1001 **E)** 1009

Q22. Ten points are selected on the positive x-axis,X^+, and five points are selected on the positive y-axis,Y^+. The fifty segments connecting the ten points on X^+ to the five points on Y^+ are drawn. What is the maximum possible number of points of intersection of these fifty segments that could lie in the interior of the first quadrant?

A) 250 **B)** 450 **C)** 500 **D)** 1250 **E)** 2500

Q23. Let S be a subset of $\{1, 2, 3, \ldots, 50\}$ such that no pair of distinct elements in S has a sum divisible by 7. What is the maximum number of elements in S?

A) 6 **B)** 7 **C)** 14 **D)** 22 **E)** 23

Q24. Let $ABCD$ be a parallelogram of area 10 with $AB = 3$ and $BC = 5$. Locate E, F and G on segments $\overline{AB}, \overline{BC}$ and $\overline{AD}$, respectively, with $AE = BF = AG = 2$. Let the line through G parallel to $\overline{EF}$ intersect $\overline{CD}$ at H. The area of quadrilateral $EFHG$ is

A) 4 **B)** 4.5 **C)** 5 **D)** 5.5 **E)** 6

Q25. In $\triangle ABC$, $\angle ABC = 120^\circ, AB = 3$ and $BC = 4$. If perpendiculars constructed to $\overline{AB}$ at A and to $\overline{BC}$ at C meet at D, then $CD =$

A) 3 **B)** $\frac{8}{\sqrt{3}}$ **C)** 5 **D)** $\frac{11}{2}$ **E)** $\frac{10}{\sqrt{3}}$

Q26. Semicircle $\widehat{AB}$ has center C and radius 1. Point D is on $\widehat{AB}$ and $\overline{CD} \perp \overline{AB}$.

Extend $\overline{BD}$ and $\overline{AD}$ to E and F, respectively, so that circular arcs $\widehat{AE}$ and $\widehat{BF}$ have B and A as their respective centers. Circular arc $\widehat{EF}$ has center D. The area of the shaded "smile" $AEFBDA$, is

A) $(2 - \sqrt{2})\pi$ **B)** $2\pi - \pi\sqrt{2} - 1$ **C)** $(1 - \frac{\sqrt{2}}{2})\pi$ **D)** $\frac{5\pi}{2} - \pi\sqrt{2} - 1$ **E)** $(3 - 2\sqrt{2})\pi$

Q27. A circle of radius r has chords $\overline{AB}$ of length 10 and $\overline{CD}$ of length 7. When $\overline{AB}$ and $\overline{CD}$ are extended through B and C, respectively, they intersect at P, which is outside of the circle. If $\angle APD = 60^\circ$ and $BP = 8$, then $r^2 =$

A) 70 **B)** 71 **C)** 72 **D)** 73 **E)** 74

Q28. Let $i = \sqrt{-1}$. The product of the real parts of the roots of $z^2 - z = 5 - 5i$ is

A) -25 **B)** -6 **C)** -5 **D)** $\frac{1}{4}$ **E)** 25

Q29. An unfair coin has a $2/3$ probability of turning up heads. If this coin is tossed 50 times, what is the probability that the total number of heads is even?

A) $25\left(\frac{2}{3}\right)^{50}$ **B)** $\frac{1}{2}\left(1 - \frac{1}{3^{50}}\right)$ **C)** $\frac{1}{2}$ **D)** $\frac{1}{2}\left(1 + \frac{1}{3^{50}}\right)$ **E)** $\frac{2}{3}$

Q30. Let $ABCD$ be an isosceles trapezoid with bases $AB = 92$ and $CD = 19$. Suppose $AD = BC = x$ and a circle with center on $\overline{AB}$ is tangent to segments $\overline{AD}$ and $\overline{BC}$. If m is the smallest possible value of x, then m^2=

A) 1369 **B)** 1679 **C)** 1748 **D)** 2109 **E)** 8825

Q1. For integers a, b, and c define $\boxed{\text{a,b,c}}$ to mean $a^b - b^c + c^a$. Then $\boxed{\text{1,-1,2}}$ equals:

A) -4 **B)** -2 **C)** 0 **D)** 2 **E)** 4

Q2. In $\triangle ABC$, $\angle A = 55°$, $\angle C = 75°$, D is on side $\overline{AB}$ and E is on side $\overline{BC}$.

If $DB = BE$, then $\angle BED =$

A) $50°$ **B)** $55°$ **C)** $60°$ **D)** $65°$ **E)** $70°$

Q3. $\dfrac{15^{30}}{45^{15}} =$

A) $\left(\dfrac{1}{3}\right)^{15}$ **B)** $\left(\dfrac{1}{3}\right)^{2}$ **C)** 1 **D)** 3^{15} **E)** 5^{15}

Q4. Define the operation "$\circ$" by $x \circ y = 4x - 3y + xy$, for all real numbers x and y. For how many real numbers y does $3 \circ y = 12$?

A) 0 **B)** 1 **C)** 3 **D)** 4 **E)** more than 4

Q5. Last year a bicycle cost \$160 and a cycling helmet \$40. This year the cost of the bicycle increased by 5%, and the cost of the helmet increased by 10%. The percent increase in the combined cost of the bicycle and the helmet is:

A) 6% **B)** 7% **C)** 7.5% **D)** 8% **E)** 15%

Q6. $\sqrt{\dfrac{8^{10} + 4^{10}}{8^4 + 4^{11}}} =$

A) $\sqrt{2}$ **B)** 16 **C)** 32 **D)** $(12)^{\frac{2}{3}}$ **E)** 512.5

Q7. The symbol R_k stands for an integer whose base-ten representation is a sequence of k ones. For example, $R_3 = 111$, $R_5 = 11111$, etc. When R_{24} is divided by R_4, the quotient $Q = R_{24}/R_4$ is an integer whose base-ten representation is a sequence containing only ones and zeroes. The number of zeros in Q is:

A) 10 **B)** 11 **C)** 12 **D)** 13 **E)** 15

Q8. Let C_1 and C_2 be circles of radius 1 that are in the same plane and tangent to each other. How many circles of radius 3 are in this plane and tangent to both C_1 and C_2?

A) 2 **B)** 4 **C)** 5 **D)** 6 **E)** 8

Q9. Country A has $c\%$ of the world's population and $d\%$ of the worlds wealth. Country B has $e\%$ of the world's population and $f\%$ of its wealth. Assume that the citizens of A share the wealth of A equally,and assume that those of B share the wealth of B equally. Find the ratio of the wealth of a citizen of A to the wealth of a citizen of B.

A) $\frac{cd}{ef}$ **B)** $\frac{ce}{ef}$ **C)** $\frac{cf}{de}$ **D)** $\frac{de}{cf}$ **E)** $\frac{df}{ce}$

Q10. Let r be the number that results when both the base and the exponent of a^b are tripled, where $a, b > 0$. If r equals the product of a^b and x^b where $x > 0$, then $x =$

A) 3 **B)** $3a^2$ **C)** $27a^2$ **D)** $2a^{3b}$ **E)** $3a^{2b}$

Q11. If $log_2(log_2(log_2(x))) = 2$, then how many digits are in the base-ten representation for x?

A) 5 **B)** 7 **C)** 9 **D)** 11 **E)** 13

Q12. If $\text{f}(2x) = \frac{2}{2+x}$ for all $x > 0$, then $2\text{f}(x) =$

A) $\frac{2}{1+x}$ **B)** $\frac{2}{2+x}$ **C)** $\frac{4}{1+x}$ **D)** $\frac{4}{2+x}$ **E)** $\frac{8}{4+x}$

Q13. A square of perimeter 20 is inscribed in a square of perimeter 28. What is the greatest distance between a vertex of the inner square and a vertex of the outer square?

A) $\sqrt{58}$ **B)** $\frac{7\sqrt{5}}{2}$ **C)** 8 **D)** $\sqrt{65}$ **E)** $5\sqrt{3}$

Q14. The convex pentagon $ABCDE$ has $\angle A = \angle B = 120°, EA = AB = BC = 2$ and $CD = DE = 4$. What is the area of ABCDE?

A) 10 **B)** $7\sqrt{3}$ **C)** 15 **D)** $9\sqrt{3}$ **E)** $12\sqrt{5}$

Q15. For how many values of n will an n-sided regular polygon have interior angles with integral measures?

A) 16 **B)** 18 **C)** 20 **D)** 22 **E)** 24

Q16. Consider the non-decreasing sequence of positive integers

$$1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, \cdots$$

in which the n^{th} positive integer appears n times. The remainder when the 1993^{rd} term is divided by 5 is

A) 0 **B)** 1 **C)** 2 **D)** 3 **E)** 4

Q17. Amy painted a dartboard over a square clock face using the "hour positions" as boundaries.[See figure.] If t is the area of one of the eight triangular regions such as that between 12 o'clock and 1 o'clock, and q is the area of one of the four corner quadrilaterals such as that between 1 o'clock and 2 o'clock, then $\frac{q}{t} =$

A) $2\sqrt{3}-2$ **B)** $\frac{3}{2}$ **C)** $\frac{\sqrt{5}+1}{2}$ **D)** $\sqrt{3}$ **E)** 2

Q18. Al and Barb start their new jobs on the same day. Al's schedule is 3 work-days followed by 1 rest-day. Barb's schedule is 7 work-days followed by 3 rest-days. On how many of their first 1000 days do both have rest-days on the same day?

A) 48 **B)** 50 **C)** 72 **D)** 75 **E)** 100

Q19. How many ordered pairs (m, n) of positive integers are solutions to

$$\frac{4}{m} + \frac{2}{n} = 1?$$

A) 1 **B)** 2 **C)** 3 **D)** 4 **E)** more than 6

Q20. Consider the equation $10z^2 - 3iz - k = 0$, where z is a complex variable and $i^2 = -1$. Which of the following statements is true?

A) For all positive real numbers k, both roots are pure imaginary.

B) For all negative real numbers k, both roots are pure imaginary.

C) For all pure imaginary numbers k, both roots are real and rational.

D) For all pure imaginary numbers k, both roots are real and irrational.

E) For all complex numbers k, neither root is real.

Q21. Let $a_1, a_2, \cdots, a_k$ be a finite arithmetic sequence with $a_4 + a_7 + a_{10} = 17$ and $a_4 + a_5 + \cdots + a_{13} + a_{14} = 77$. If $a_k = 13$, then $k =$

A) 16 **B)** 18 **C)** 20 **D)** 22 **E)** 24

Q22. Twenty cubical blocks are arranged as shown. First, 10 are arranged in a triangular pattern; then a layer of 6, arranged in a triangular pattern, is centered on the 10; then a layer of 3, arranged in a triangular pattern, is centered on the 6; and finally one block is centered on top of the third layer. The blocks in the bottom layer are numbered 1 through 10 in some order. Each block in layers 2,3 and 4 is assigned the number which is the sum of numbers assigned to the three blocks on which it rests. Find the smallest possible number which could be assigned to the top block.

A) 55 **B)** 83 **C)** 114 **D)** 137 **E)** 144

Q23. Points A, B, C and D are on a circle of diameter 1, and X is on diameter $\overline{AD}$. If $BX = CX$ and $3\angle BAC = \angle BXC = 36°$, then $AX =$

A) $\cos(6°)\cos(12°)\sec(18°)$

B) $\cos(6°)\sin(12°)\csc(18°)$

C) $\cos(6°)\sin(12°)\sec(18°)$

D) $\sin(6°)\sin(12°)\csc(18°)$

E) $\sin(6°)\sin(12°)\sec(18°)$

Q24. A box contains 3 shiny pennies and 4 dull pennies. One by one, pennies are drawn at random from the box and not replaced. If the probability is a/b that it will take more than four draws until the third shiny penny appears and a/b is in lowest terms, then $a + b =$

A) 11 **B)** 20 **C)** 35 **D)** 58 **E)** 66

Q25. Let S be the set of points on the rays forming the sides of a $120°$ angle, and let P be a fixed point inside the angle on the angle bisector.

Consider all distinct equilateral triangles PQR with Q and R in S. (Points Q and R may be on the same ray, and switching the names of Q and R does not create a distinct triangle.) There are

A) exactly 2 such triangles

B) exactly 3 such triangles

C) exactly 7 such triangles

D) exactly 15 such triangles

E) more than 15 such triangles

Q26. Find the largest positive value attained by the function $f(x) = \sqrt{8x - x^2} - \sqrt{14x - x^2 - 48}$, x a real number.

A) $\sqrt{7} - 1$ **B)** 3 **C)** $2\sqrt{3}$ **D)** 4 **E)** $\sqrt{55} - \sqrt{5}$

Q27. The sides of $\triangle ABC$ have lengths $6, 8,$ and 10.

A circle with center P and radius 1 rolls around the inside of $\triangle ABC$, always remaining tangent to at least one side of the triangle. When P first returns to its original position, through what distance has P traveled?

A) 10 **B)** 12 **C)** 14 **D)** 15 **E)** 17

Q28. How many triangles with positive area are there whose vertices are points in the xy-plane whose coordinates are integers (x, y) satisfying $1 \le x \le 4$ and $1 \le y \le 4$?

A) 496 **B)** 500 **C)** 512 **D)** 516 **E)** 560

Q29. Which of the following could NOT be the lengths of the external diagonals of a right regular prism [a "box"]? (An *external diagonal* is a diagonal of one of the rectangular faces of the box.)

A) $\{4,5,6\}$ **B)** $\{4,5,7\}$ **C)** $\{4,6,7\}$ **D)** $\{5,6,7\}$ **E)** $\{5,7,8\}$

Q30. Given $0 \le x_0 < 1$, let

$$x_n = \begin{cases} 2x_{n-1} & \text{if } 2x_{n-1} < 1 \\ 2x_{n-1} - 1 & \text{if } 2x_{n-1} \ge 1 \end{cases}$$

for all integers $n > 0$. For how many x_0 is it true that $x_0 = x_5$?

A) 0 **B)** 1 **C)** 5 **D)** 31 **E)** ∞

Q1. $4^4 \cdot 9^4 \cdot 4^9 \cdot 9^9 =$

A) 13^{13} **B)** 13^{36} **C)** 36^{13} **D)** 36^{36} **E)** 1296^{26}

Q2. A large rectangle is partitioned into four rectangles by two segments parallel to its sides. The areas of three of the resulting rectangles are shown. What is the area of the fourth rectangle?

A) 10 **B)** 15 **C)** 20 **D)** 21 **E)** 25

Q3. How many of the following are equal to $x^x + x^x$ for all $x > 0$?

I) $2x^x$

II) x^{2x}

III) $(2x)^x$

IV) $(2x)^{2x}$

A) 0 **B)** 1 **C)** 2 **D)** 3 **E)** 4

Q4. In the xy-plane, the segment with endpoints $(-5, 0)$ and $(25, 0)$ is the diameter of a circle. If the point $(x, 15)$ is on the circle, then $x =$

A) 10 **B)** 12.5 **C)** 15 **D)** 17.5 **E)** 20

Q5. Pat intended to multiply a number by 6 but instead divided by 6. Pat then meant to add 14 but instead subtracted 14. After these mistakes, the result was 16. If the correct operations had been used, the value produced would have been

A) less than 400

B) between 400 and 600

C) between 600 and 800

D) between 800 and 1000

E) greater than 1000

Q6. In the sequence

$$\ldots, a, b, c, d, 0, 1, 1, 2, 3, 5, 8, \ldots$$

each term is the sum of the two terms to its left. Find a.

A) -3 **B)** -1 **C)** 0 **D)** 1 **E)** 3

Q7. Squares $ABCD$ and $EFGH$ are congruent, $AB = 10$, and G is the center of square $ABCD$. The area of the region in the plane covered by these squares is

A) 75 **B)** 100 **C)** 125 **D)** 150 **E)** 175

Q8. In the polygon shown, each side is perpendicular to its adjacent sides, and all 28 of the sides are congruent. The perimeter of the polygon is 56. The area of the region bounded by the polygon is

A) 84 **B)** 96 **C)** 100 **D)** 112 **E)** 196

Q9. If $\angle A$ is four times $\angle B$, and the complement of $\angle B$ is four times the complement of $\angle A$, then $\angle B =$

A) $10°$ **B)** $12°$ **C)** $15°$ **D)** $18°$ **E)** $22.5°$

Q10. For distinct real numbers x and y, let $M(x, y)$ be the larger of x and y and let $m(x, y)$ be the smaller of x and y. If $a < b < c < d < e$, then

$$M(M(a, m(b, c)), m(d, m(a, e))) =$$

A) a **B)** b **C)** c **D)** d **E)** e

Q11. Three cubes of volume $1, 8$ and 27 are glued together at their faces. The smallest possible surface area of the resulting configuration is

A) 36 **B)** 56 **C)** 70 **D)** 72 **E)** 74

Q12. If $\mathrm{i}^2 = -1$, then $(\mathrm{i} - \mathrm{i}^{-1})^{-1} =$

A) 0 **B)** $-2\mathrm{i}$ **C)** $2\mathrm{i}$ **D)** $-\frac{\mathrm{i}}{2}$ **E)** $\frac{\mathrm{i}}{2}$

Q13. In triangle ABC, $AB = AC$. If there is a point P strictly between A and B such that $AP = PC = CB$, then $\angle A =$

A) $30°$ **B)** $36°$ **C)** $48°$ **D)** $60°$ **E)** $72°$

Q14. Find the sum of the arithmetic series

$$20 + 20\frac{1}{5} + 20\frac{2}{5} + \cdots + 40$$

A) 3000 **B)** 3030 **C)** 3150 **D)** 4100 **E)** 6000

Q15. For how many n in $\{1, 2, 3, \ldots, 100\}$ is the tens digit of n^2 odd?

A) 10 **B)** 20 **C)** 30 **D)** 40 **E)** 50

Q16. Some marbles in a bag are red and the rest are blue. If one red marble is removed, then one-seventh of the remaining marbles are red. If two blue marbles are removed instead of one red, then one-fifth of the remaining marbles are red. How many marbles were in the bag originally?

A) 8 **B)** 22 **C)** 36 **D)** 57 **E)** 71

Q17. An 8 by $2\sqrt{2}$ rectangle has the same center as a circle of radius 2. The area of the region common to both the rectangle and the circle is

A) 2π **B)** $2\pi + 2$ **C)** $4\pi - 4$ **D)** $2\pi + 4$ **E)** $4\pi - 2$

Q18. Triangle ABC is inscribed in a circle, and $\angle B = \angle C = 4\angle A$. If B and C are adjacent vertices of a regular polygon of n sides inscribed in this circle, then $n =$

A) 5 **B)** 7 **C)** 9 **D)** 15 **E)** 18

Q19. Label one disk "1", two disks "2", three disks "3", . . . , fifty disks "50". Put these $1 + 2 + 3 + \cdots + 50 = 1275$ labeled disks in a box. Disks are then drawn from the box at random without replacement. The minimum number of disks that must be drawn to guarantee drawing at least ten disks with the same label is

A) 10 **B)** 51 **C)** 415 **D)** 451 **E)** 501

Q20. Suppose x, y, z is a geometric sequence with common ratio r and $x \neq y$. If $x, 2y, 3z$ is an arithmetic sequence, then r is

A) $\frac{1}{4}$ **B)** $\frac{1}{3}$ **C)** $\frac{1}{2}$ **D)** 2 **E)** 4

Q21. Find the number of counter examples to the statement:

" If N is an odd positive integer the sum of whose digits is 4 and none of whose digits is 0, then N is prime. "

A) 0 **B)** 1 **C)** 2 **D)** 3 **E)** 4

Q22. Nine chairs in a row are to be occupied by six students and Professors Alpha, Beta and Gamma. These three professors arrive before the six students and decide to choose their chairs so that each professor will be between two students. In how many ways can Professors Alpha, Beta and Gamma choose their chairs?

A) 12 **B)** 36 **C)** 60 **D)** 84 **E)** 630

Q23. In the xy-plane, consider the L-shaped region bounded by horizontal and vertical segments with vertices at $(0,0),(0,3),(3,3),(3,1),(5,1)$ and $(5,0)$. The slope of the line through the origin that divides the area of this region exactly in half is

A) $\frac{2}{7}$ **B)** $\frac{1}{3}$ **C)** $\frac{2}{3}$ **D)** $\frac{3}{4}$ **E)** $\frac{7}{9}$

Q24. A sample consisting of five observations has an arithmetic mean of 10 and a median of 12. The smallest value that the range (largest observation minus smallest) can assume for such a sample is

A) 2 **B)** 3 **C)** 5 **D)** 7 **E)** 10

Q25. If x and y are non-zero real numbers such that

$$|x| + y = 3 \quad \text{and} \quad |x|y + x^3 = 0,$$

then the integer nearest to $x - y$ is

A) -3 **B)** -1 **C)** 2 **D)** 3 **E)** 5

Q26. A regular polygon of m sides is exactly enclosed (no overlaps, no gaps) by m regular polygons of n sides each. (Shown here for $m = 4, n = 8$.) If $m = 10$, what is the value of n?

A) 5 **B)** 6 **C)** 14 **D)** 20 **E)** 26

Q27. A bag of popping corn contains $\frac{2}{3}$ white kernels and $\frac{1}{3}$ yellow kernels. Only $\frac{1}{2}$ of the white kernels will pop, whereas $\frac{2}{3}$ of the yellow ones will pop. A kernel is selected at random from the bag, and pops when placed in the popper. What is the probability that the kernel selected was white?

A) $\frac{1}{2}$ **B)** $\frac{5}{9}$ **C)** $\frac{4}{7}$ **D)** $\frac{3}{5}$ **E)** $\frac{2}{3}$

Q28. In the xy-plane, how many lines whose x-intercept is a positive prime number and whose y-intercept is a positive integer pass through the point $(4, 3)$?

A) 0 **B)** 1 **C)** 2 **D)** 3 **E)** 4

Q29. Points A, B and C on a circle of radius r are situated so that $AB = AC, AB > r$, and the length of minor arc BC is r. If angles are measured in radians, then $AB/BC =$

A) $\frac{1}{2}\csc\frac{1}{4}$ **B)** $2\cos\frac{1}{2}$ **C)** $4\sin\frac{1}{2}$ **D)** $\csc\frac{1}{2}$ **E)** $2\sec\frac{1}{2}$

Q30. When n standard 6-sided dice are rolled, the probability of obtaining a sum of 1994 is greater than zero and is the same as the probability of obtaining a sum of S. The smallest possible value of S is

A) 333 **B)** 335 **C)** 337 **D)** 339 **E)** 341

Q1. Kim earned scores of 87,83, and 88 on her first three mathematics examinations. If Kim receives a score of 90 on the fourth exam, then her average will

A) remain the same **B)** increase by 1 **C)** increase by 2 **D)** increase by 3 **E)** increase by 4

Q2. If $\sqrt{2+\sqrt{x}} = 3$, then $x =$

A) 1 **B)** $\sqrt{7}$ **C)** 7 **D)** 49 **E)** 121

Q3. The total in-store price for an appliance is $99.99. A television commercial advertises the same product for three easy payments of $29.98 and a one-time shipping and handling charge of $9.98. How many cents are saved by buying the appliance from the television advertiser?

A) 6 **B)** 7 **C)** 8 **D)** 9 **E)** 10

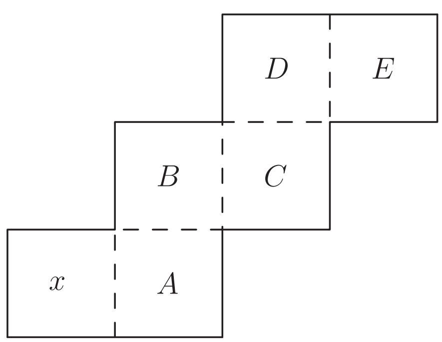
Q4. If M is 30% of Q, Q is 20% of P, and N is 50% of P, then $\frac{M}{N} =$

A) $\frac{3}{250}$ **B)** $\frac{3}{25}$ **C)** 1 **D)** $\frac{6}{5}$ **E)** $\frac{4}{3}$

Q5. A rectangular field is 300 feet wide and 400 feet long. Random sampling indicates that there are, on the average, three ants per square inch through out the field. [12 inches = 1 foot.] Of the following, the number that most closely approximates the number of ants in the field is

A) 500 thousand **B)** 5 million **C)** 50 million **D)** 500 million **E)** 5 billion

Q6. The figure shown can be folded into the shape of a cube. In the resulting cube, which of the lettered faces is opposite the face marked x?



A) A **B)** B **C)** C **D)** D **E)** E

Q7. The radius of Earth at the equator is approximately 4000 miles. Suppose a jet flies once around Earth at a speed of 500 miles per hour relative to Earth. If the flight path is a negligible height above the equator, then, among the following choices, the best estimate of the number of hours of flight is:

A) 8 **B)** 25 **C)** 50 **D)** 75 **E)** 100

Q8. In $\triangle ABC$, $\angle C = 90°$, $AC = 6$ and $BC = 8$. Points D and E are on $\overline{AB}$ and $\overline{BC}$, respectively, and $\angle BED = 90°$. If $DE = 4$, then $BD =$

A) 5 **B)** $\frac{16}{3}$ **C)** $\frac{20}{3}$ **D)** $\frac{15}{2}$ **E)** 8

Q9. Consider the figure consisting of a square, its diagonals, and the segments joining the midpoints of opposite sides. The total number of triangles of any size in the figure is

A) 10 **B)** 12 **C)** 14 **D)** 16 **E)** 18

Q10. The area of the triangle bounded by the lines $y = x, y = -x$ and $y = 6$ is

A) 12 **B)** $12\sqrt{2}$ **C)** 24 **D)** $24\sqrt{2}$ **E)** 36

Q11. How many base 10 four-digit numbers, $N = \underline{abcd}$, satisfy all three of the following conditions?

(i) $4,000 \le N < 6,000$;

(ii) N is a multiple of 5;

(iii) $3 \le b < c \le 6$.

A) 10 **B)** 18 **C)** 24 **D)** 36 **E)** 48

Q12. Let f be a linear function with the properties that $\mathrm{f}(1) \le \mathrm{f}(2), \mathrm{f}(3) \ge \mathrm{f}(4)$, and $\mathrm{f}(5) = 5$. Which of the following is true?

A) $\mathrm{f}(0) < 0$ **B)** $\mathrm{f}(0) = 0$ **C)** $\mathrm{f}(1) < \mathrm{f}(0) < \mathrm{f}(-1)$ **D)** $\mathrm{f}(0) = 5$ **E)** $\mathrm{f}(0) > 5$

Q13. The addition below is incorrect. The display can be made correct by changing one digit d, wherever it occurs, to another digit e. Find the sum of d and e.

$$\begin{array}{ccccccc} & 7 & 4 & 2 & 5 & 8 & 6 \\ + & 8 & 2 & 9 & 4 & 3 & 0 \\ \hline 1 & 2 & 1 & 2 & 0 & 1 & 6 \end{array}$$

A) 4 **B)** 6 **C)** 8 **D)** 10 **E)** more than 10

Q14. If $f(x) = ax^4 - bx^2 + x + 5$ and $f(-3) = 2$, then $f(3) =$

A) -5 **B)** -2 **C)** 1 **D)** 3 **E)** 8

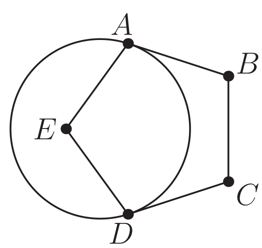
Q15. Five points on a circle are numbered 1,2,3,4, and 5 in clockwise order. A bug jumps in a clockwise direction from one point to another around the circle; if it is on an odd-numbered point, it moves one point, and if it is on an even-numbered point, it moves two points. If the bug begins on point 5, after 1995 jumps it will be on point

A) 1 **B)** 2 **C)** 3 **D)** 4 **E)** 5

Q16. Anita attends a baseball game in Atlanta and estimates that there are 50,000 fans in attendance. Bob attends a baseball game in Boston and estimates that there are 60,000 fans in attendance. A league official who knows the actual numbers attending the two games note that: i. The actual attendance in Atlanta is within 10% of Anita's estimate. ii. Bob's estimate is within 10% of the actual attendance in Boston. To the nearest 1,000, the largest possible difference between the numbers attending the two games is

A) 10000 **B)** 11000 **C)** 20000 **D)** 21000 **E)** 22000

Q17. Given regular pentagon $ABCDE$, a circle can be drawn that is tangent to $\overline{DC}$ at D and to $\overline{AB}$ at A. The number of degrees in minor arc AD is



A) 72 **B)** 108 **C)** 120 **D)** 135 **E)** 144

Q18. Two rays with common endpoint O forms a 30° angle. Point A lies on one ray, point B on the other ray, and $AB = 1$. The maximum possible length of OB is

A) 1 **B)** $\frac{1+\sqrt{3}}{\sqrt{2}}$ **C)** $\sqrt{3}$ **D)** 2 **E)** $\frac{4}{\sqrt{3}}$

Q19. Equilateral triangle DEF is inscribed in equilateral triangle ABC such that $\overline{DE} \perp \overline{BC}$. The ratio of the area of $\triangle DEF$ to the area of $\triangle ABC$ is

A) $\frac{1}{6}$ **B)** $\frac{1}{4}$ **C)** $\frac{1}{3}$ **D)** $\frac{2}{5}$ **E)** $\frac{1}{2}$

Q20. If a, b and c are three (not necessarily different) numbers chosen randomly and with replacement from the set $\{1, 2, 3, 4, 5\}$, the probability that $ab + c$ is even is

A) $\frac{2}{5}$ **B)** $\frac{59}{125}$ **C)** $\frac{1}{2}$ **D)** $\frac{64}{125}$ **E)** $\frac{3}{5}$

Q21. Two nonadjacent vertices of a rectangle are $(4, 3)$ and $(-4, -3)$, and the coordinates of the other two vertices are integers. The number of such rectangles is

A) 1 **B)** 2 **C)** 3 **D)** 4 **E)** 5

Q22. A pentagon is formed by cutting a triangular corner from a rectangular piece of paper. The five sides of the pentagon have lengths $13, 19, 20, 25$ and 31, although this is not necessarily their order around the pentagon. The area of the pentagon is

A) 459 **B)** 600 **C)** 680 **D)** 720 **E)** 745

Q23. The sides of a triangle have lengths $11, 15$, and k, where k is an integer. For how many values of k is the triangle obtuse?

A) 5 **B)** 7 **C)** 12 **D)** 13 **E)** 14

Q24. There exist positive integers A, B and C, with no common factor greater than 1, such that

$$A \log_{200} 5 + B \log_{200} 2 = C.$$

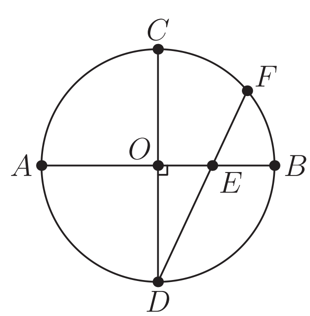
What is $A + B + C$?

A) 6 **B)** 7 **C)** 8 **D)** 9 **E)** 10

Q25. A list of five positive integers has mean 12 and range 18. The mode and median are both 8. How many different values are possible for the second largest element of the list?

A) 4 **B)** 6 **C)** 8 **D)** 10 **E)** 12

Q26. In the figure, $\overline{AB}$ and $\overline{CD}$ are diameters of the circle with center O, $\overline{AB} \perp \overline{CD}$, and chord $\overline{DF}$ intersects $\overline{AB}$ at E. If $DE = 6$ and $EF = 2$, then the area of the circle is



A) 23π **B)** $\frac{47}{2}\pi$ **C)** 24π **D)** $\frac{49}{2}\pi$ **E)** 25π

Q27. Consider the triangular array of numbers with 0,1,2,3,... along the sides and interior numbers obtained by adding the two adjacent numbers in the previous row. Rows 1 through 6 are shown.

0
1 1
2 2 2
3 4 4 3
4 7 8 7 4
5 11 15 15 11 5

Let f(n) denote the sum of the numbers in row n. What is the remainder when f(100) is divided by 100?

A) 12 **B)** 30 **C)** 50 **D)** 62 **E)** 74

Q28. Two parallel chords in a circle have lengths 10 and 14, and the distance between them is 6. The chord parallel to these chords and midway between them is of length $\sqrt{a}$ where a is

A) 144 **B)** 156 **C)** 168 **D)** 176 **E)** 184

Q29. For how many three-element sets of distinct positive integers $\{a, b, c\}$ is it true that $a \times b \times c = 2310$?

A) 32 **B)** 36 **C)** 40 **D)** 43 **E)** 45

Q30. A large cube is formed by stacking 27 unit cubes. A plane is perpendicular to one of the internal diagonals of the large cube and bisects that diagonal. The number of unit cubes that the plane intersects is

A) 16 **B)** 17 **C)** 18 **D)** 19 **E)** 20

Q1. The addition below is incorrect. What is the largest digit that can be changed to make the addition correct?

```
  6 4 1
  8 5 2
 +9 7 3
-------
2 4 5 6
```

A) 4 **B)** 5 **C)** 6 **D)** 7 **E)** 8

Q2. Each day Walter gets 3 dollars for doing his chores or 5 dollars for doing them exceptionally well. After 10 days of doing his chores daily, Walter has received a total of 36 dollars. On how many days did Walter do them exceptionally well?

A) 3 **B)** 4 **C)** 5 **D)** 6 **E)** 7

Q3. $\frac{(3!)!}{3!} =$

A) 1 **B)** 2 **C)** 6 **D)** 40 **E)** 120

Q4. Six numbers from a list of nine integers are $7, 8, 3, 5, 9$ and 5. The largest possible value of the median of all nine numbers in this list is

A) 5 **B)** 6 **C)** 7 **D)** 8 **E)** 9

Q5. Given that $0 < a < b < c < d$, which of the following is the largest?

A) $\frac{a+b}{c+d}$ **B)** $\frac{a+d}{b+c}$ **C)** $\frac{b+c}{a+d}$ **D)** $\frac{b+d}{a+c}$ **E)** $\frac{c+d}{a+b}$

Q6. If $f(x) = x^{(x+1)}(x+2)^{(x+3)}$, then $f(0) + f(-1) + f(-2) + f(-3) =$

A) $-\frac{8}{9}$ **B)** 0 **C)** $\frac{8}{9}$ **D)** 1 **E)** $\frac{10}{9}$

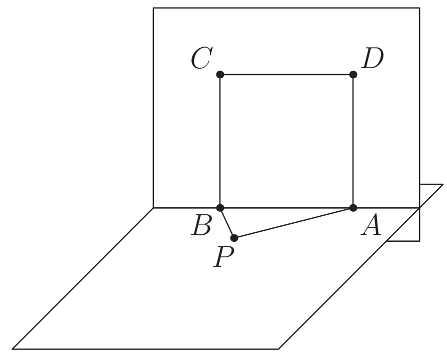
Q7. A father takes his twins and a younger child out to dinner on the twins' birthday. The restaurant charges 4.95 for the father and 0.45 for each year of a child's age, where age is defined as the age at the most recent birthday. If the bill is 9.45, which of the following could be the age of the youngest child?

A) 1 **B)** 2 **C)** 3 **D)** 4 **E)** 5

Q8. If $3 = k \cdot 2^r$ and $15 = k \cdot 4^r$, then $r =$

A) $-\log_2 5$ **B)** $\log_5 2$ **C)** $\log_{10} 5$ **D)** $\log_2 5$ **E)** $\frac{5}{2}$

Q9. Triangle PAB and square $ABCD$ are in perpendicular planes. Given that $PA = 3, PB = 4$ and $AB = 5$, what is PD?



A) 5 **B)** $\sqrt{34}$ **C)** $\sqrt{41}$ **D)** $2\sqrt{13}$ **E)** 8

Q10. How many line segments have both their endpoints located at the vertices of a given cube?

A) 12 **B)** 15 **C)** 24 **D)** 28 **E)** 56

Q11. Given a circle of radius 2, there are many line segments of length 2 that are tangent to the circle at their midpoints. Find the area of the region consisting of all such line segments.

A) $\frac{\pi}{4}$ **B)** $4 - \pi$ **C)** $\frac{\pi}{2}$ **D)** π **E)** 2π

Q12. A function f from the integers to the integers is defined as follows:

$$\text{f}(n) = \begin{cases} n+3 & \text{if n is odd} \\ \frac{n}{2} & \text{if n is even} \end{cases}$$

Suppose k is odd and $\text{f}(\text{f}(\text{f}(k))) = 27$. What is the sum of the digits of k?

A) 3 **B)** 6 **C)** 9 **D)** 12 **E)** 15

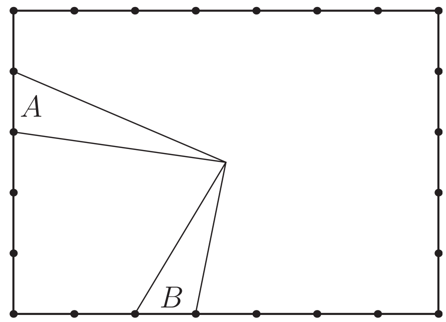
Q13. Sunny runs at a steady rate, and Moonbeam runs m times as fast, where m is a number greater than 1. If Moonbeam gives Sunny a head start of h meters, how many meters must Moonbeam run to overtake Sunny?

A) hm **B)** $\frac{h}{h+m}$ **C)** $\frac{h}{m-1}$ **D)** $\frac{hm}{m-1}$ **E)** $\frac{h+m}{m-1}$

Q14. Let $E(n)$ denote the sum of the even digits of n. For example, $E(5681) = 6 + 8 = 14$. Find $E(1) + E(2) + E(3) + \cdots + E(100)$

A) 200 **B)** 360 **C)** 400 **D)** 900 **E)** 2250

Q15. Two opposite sides of a rectangle are each divided into n congruent segments, and the endpoints of one segment are joined to the center to form triangle A. The other sides are each divided into m congruent segments, and the endpoints of one of these segments are joined to the center to form triangle B. [See figure for $n = 5, m = 7$.] What is the ratio of the area of triangle A to the area of triangle B?

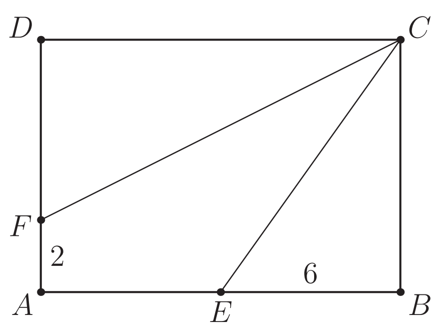


A) 1 **B)** m/n **C)** n/m **D)** $2m/n$ **E)** $2n/m$

Q16. A fair standard six-sided dice is tossed three times. Given that the sum of the first two tosses equal the third, what is the probability that at least one "2" is tossed?

A) $\frac{1}{6}$ **B)** $\frac{91}{216}$ **C)** $\frac{1}{2}$ **D)** $\frac{8}{15}$ **E)** $\frac{7}{12}$

Q17. In rectangle $ABCD$, angle C is trisected by $\overline{CF}$ and $\overline{CE}$, where E is on $\overline{AB}$, F is on $\overline{AD}$, $BE = 6$ and $AF = 2$. Which of the following is closest to the area of the rectangle $ABCD$?

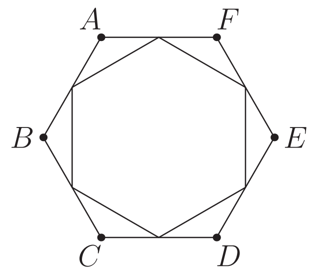


A) 110 **B)** 120 **C)** 130 **D)** 140 **E)** 150

Q18. A circle of radius 2 has center at $(2, 0)$. A circle of radius 1 has center at $(5, 0)$. A line is tangent to the two circles at points in the first quadrant. Which of the following is closest to the y-intercept of the line?

A) $\sqrt{2}/4$ **B)** $\frac{8}{3}$ **C)** $1 + \sqrt{3}$ **D)** $2\sqrt{2}$ **E)** 3

Q19. The midpoints of the sides of a regular hexagon $ABCDEF$ are joined to form a smaller hexagon. What fraction of the area of $ABCDEF$ is enclosed by the smaller hexagon?

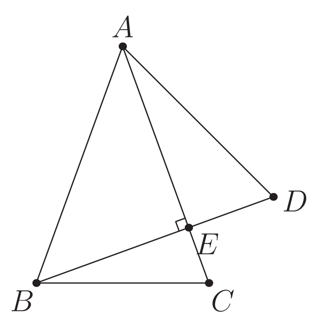


A) $\frac{1}{2}$ **B)** $\frac{\sqrt{3}}{3}$ **C)** $\frac{2}{3}$ **D)** $\frac{3}{4}$ **E)** $\frac{\sqrt{3}}{2}$

Q20. In the xy-plane, what is the length of the shortest path from $(0,0)$ to $(12,16)$ that does not go inside the circle $(x-6)^2+(y-8)^2=25$?

A) $10\sqrt{3}$ **B)** $10\sqrt{5}$ **C)** $10\sqrt{3}+\frac{5\pi}{3}$ **D)** $40\frac{\sqrt{3}}{3}$ **E)** $10+5\pi$

Q21. Triangles ABC and ABD are isosceles with $AB=AC=BD$, and BD intersects AC at E. If BD is perpendicular to AC, then $\angle C+\angle D$ is



A) $115°$ **B)** $120°$ **C)** $130°$ **D)** $135°$ **E)** not uniquely determi

Q22. Four distinct points, A, B, C, and D, are to be selected from 1996 points evenly spaced around a circle. All quadruples are equally likely to be chosen. What is the probability that the chord AB intersects the chord CD?

A) $\frac{1}{4}$ **B)** $\frac{1}{3}$ **C)** $\frac{1}{2}$ **D)** $\frac{2}{3}$ **E)** $\frac{3}{4}$

Q23. The sum of the lengths of the twelve edges of a rectangular box is 140, and the distance from one corner of the box to the farthest corner is 21. The total surface area of the box is

A) 776 **B)** 784 **C)** 798 **D)** 800 **E)** 812

Q24. The sequence $1,2,1,2,2,1,2,2,2,1,2,2,2,2,1,2,2,2,2,2,1,2,\ldots$ consists of 1’s separated by blocks of 2’s with n 2’s in the n^{th} block. The sum of the first 1234 terms of this sequence is

A) 1996 **B)** 2419 **C)** 2429 **D)** 2439 **E)** 2449

Q25. Given that $x^2 + y^2 = 14x + 6y + 6$, what is the largest possible value that $3x + 4y$ can have?

A) 72 **B)** 73 **C)** 74 **D)** 75 **E)** 76

Q26. An urn contains marbles of four colors: red, white, blue, and green. When four marbles are drawn without replacement, the following events are equally likely:

a) the selection of four red marbles;

b) the selection of one white and three red marbles;

c) the selection of one white, one blue, and two red marbles; and

d) the selection of one marble of each color.

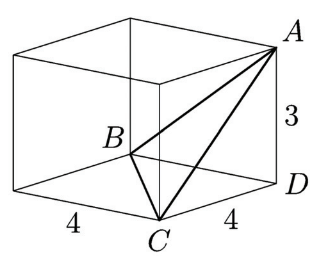
What is the smallest number of marbles satisfying the given condition?

A) 19 **B)** 21 **C)** 46 **D)** 69 **E)** more than 69

Q27. Consider two solid spherical balls, one centered at $\left(0, 0, \frac{21}{2}\right)$ with radius 6, and the other centered at $(0, 0, 1)$ with radius $\frac{9}{2}$. How many points with only integer coordinates (lattice points) are there in the intersection of the balls?

A) 7 **B)** 9 **C)** 11 **D)** 13 **E)** 15

Q28. On a $4 \times 4 \times 3$ rectangular parallelepiped, vertices A, B, and C are adjacent to vertex D. The perpendicular distance from D to the plane containing A, B, and C is closest to



A) 1.6 **B)** 1.9 **C)** 2.1 **D)** 2.7 **E)** 2.9

Q29. If n is a positive integer such that $2n$ has 28 positive divisors and $3n$ has 30 positive divisors, then how many positive divisors does $6n$ have?

A) 32 **B)** 34 **C)** 35 **D)** 36 **E)** 38

Q30. A hexagon inscribed in a circle has three consecutive sides each of length 3 and three consecutive sides each of length 5. The chord of the circle that divides the hexagon into two trapezoids, one with three sides each of length 3 and the other with three sides each of length 5, has length equal to m/n, where m and n are relatively prime positive integers. Find $m + n$.

A) 309 **B)** 349 **C)** 369 **D)** 389 **E)** 409

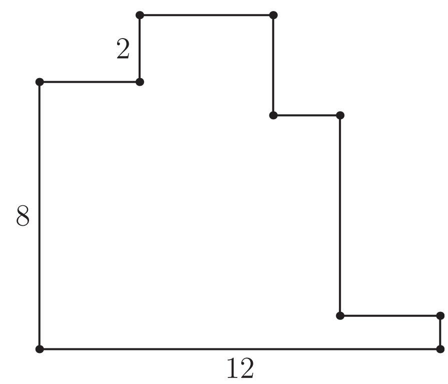
Q1. If a and b are digits for which

$$\begin{array}{cccc} & & 2 & a \\ & \times & b & 3 \\ \hline & & 6 & 9 \\ & 9 & 2 & \\ \hline & 9 & 8 & 9 \end{array}$$

then a+b =

A) 3 **B)** 4 **C)** 7 **D)** 9 **E)** 12

Q2. The adjacent sides of the decagon shown meet at right angles. What is its perimeter?



A) 22 **B)** 32 **C)** 34 **D)** 44 **E)** 50

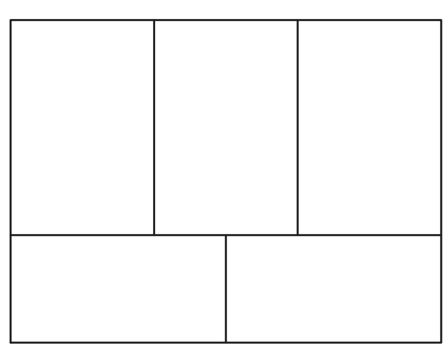
Q3. If x, y, and z are real numbers such that $(x-3)^2+(y-4)^2+(z-5)^2=0$, then $x+y+z=$

A) -12 **B)** 0 **C)** 8 **D)** 12 **E)** 50

Q4. If a is 50% larger than c, and b is 25% larger than c, then a is what percent larger than b?

A) 20% **B)** 25% **C)** 50% **D)** 100% **E)** 200%

Q5. A rectangle with perimeter 176 is divided into five congruent rectangles as shown in the diagram. What is the perimeter of one of the five congruent rectangles?



A) 35.2 **B)** 76 **C)** 80 **D)** 84 **E)** 86

Q6. Consider the sequence $1, -2, 3, -4, 5, -6, \ldots,$ whose nth term is $(-1)^{n+1} \cdot n$. What is the average of the first 200 terms of the sequence?

A) -1 **B)** -0.5 **C)** 0 **D)** 0.5 **E)** 1

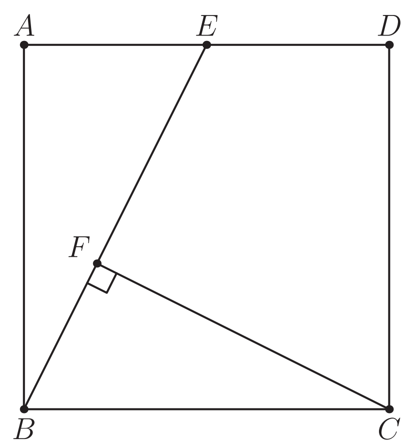
Q7. The sum of seven integers is -1. What is the maximum number of the seven integers that can be larger than 13?

A) 1 **B)** 4 **C)** 5 **D)** 6 **E)** 7

Q8. Mientka Publishing Company prices its bestseller Where's Walter? as follows: $C(n) = \begin{cases} 12n, & \text{if } 1 \le n \le 24 \\ 11n, & \text{if } 25 \le n \le 48 \\ 10n, & \text{if } 49 \le n \end{cases}$ where n is the number of books ordered, and $C(n)$ is the cost in dollars of n books. Notice that 25 books cost less than 24 books. For how many values of n is it cheaper to buy more than n books than to buy exactly n books?

A) 3 **B)** 4 **C)** 5 **D)** 6 **E)** 8

Q9. In the figure, $ABCD$ is a 2×2 square, E is the midpoint of $\overline{AD}$, and F is on $\overline{BE}$. If $\overline{CF}$ is perpendicular to $\overline{BE}$, then the area of quadrilateral $CDEF$ is



A) 2 **B)** $3 - \frac{\sqrt{3}}{2}$ **C)** $\frac{11}{5}$ **D)** $\sqrt{5}$ **E)** $\frac{9}{4}$

Q10. Two six-sided dice are fair in the sense that each face is equally likely to turn up. However, one of the dice has the 4 replaced by 3 and the other die has the 3 replaced by 4 . When these dice are rolled, what is the probability that the sum is an odd number?

A) $\frac{1}{3}$ **B)** $\frac{4}{9}$ **C)** $\frac{1}{2}$ **D)** $\frac{5}{9}$ **E)** $\frac{11}{18}$

Q11. In the sixth, seventh, eighth, and ninth basketball games of the season, a player scored 23,14, 11, and 20 points, respectively. Her points-per-game average was higher after nine games than it was after the first five games. If her average after ten games was greater than 18, what is the least number of points she could have scored in the tenth game?

A) 26 **B)** 27 **C)** 28 **D)** 29 **E)** 30

Q12. If m and b are real numbers and $mb > 0$, then the line whose equation is $y = mx + b$ "cannot" contain the point

A) $(0, 1997)$ **B)** $(0, -1997)$ **C)** $(19, 97)$ **D)** $(19, -97)$ **E)** $(1997, 0)$

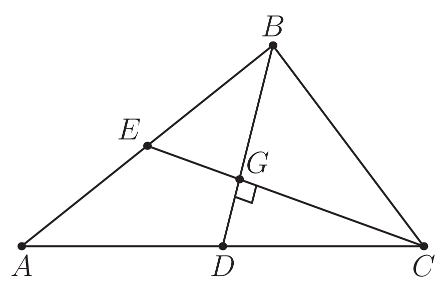
Q13. How many two-digit positive integers N have the property that the sum of N and the number obtained by reversing the order of the digits of is a perfect square?

A) 4 **B)** 5 **C)** 6 **D)** 7 **E)** 8

Q14. The number of geese in a flock increases so that the difference between the populations in year $n+2$ and year n is directly proportional to the population in year $n+1$. If the populations in the years 1994, 1995, and 1997 were 39, 60, and 123, respectively, then the population in 1996 was

A) 81 **B)** 84 **C)** 87 **D)** 90 **E)** 102

Q15. Medians BD and CE of triangle ABC are perpendicular, $BD = 8$, and $CE = 12$. The area of triangle ABC is



A) 24 **B)** 32 **C)** 48 **D)** 64 **E)** 96

Q16. The three row sums and the three column sums of the array

$$\begin{bmatrix} 4 & 9 & 2 \\ 8 & 1 & 6 \\ 3 & 5 & 7 \end{bmatrix}$$

are the same. What is the least number of entries that must be altered to make all six sums different from one another?

A) 1 **B)** 2 **C)** 3 **D)** 4 **E)** 5

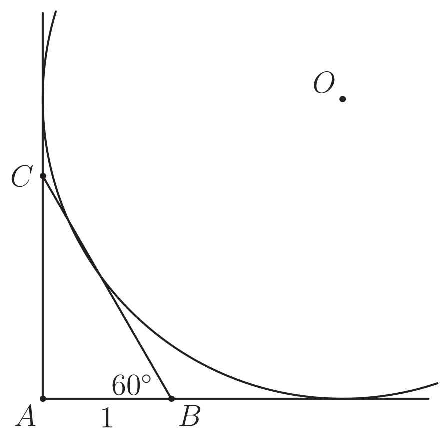
Q17. A line $x = k$ intersects the graph of $y = \log_5 x$ and the graph of $y = \log_5(x+4)$. The distance between the points of intersection is 0.5. Given that $k = a + \sqrt{b}$, where a and b are integers, what is $a + b$?

A) 6 **B)** 7 **C)** 8 **D)** 9 **E)** 10

Q18. A list of integers has mode 32 and mean 22. The smallest number in the list is 10. The median m of the list is a member of the list. If the list member m were replaced by $m + 10$, the mean and median of the new list would be 24 and $m + 10$, respectively. If were m instead replaced by $m - 8$, the median of the new list would be $m - 4$. What is m?

A) 16 **B)** 17 **C)** 18 **D)** 19 **E)** 20

Q19. A circle with center O is tangent to the coordinate axes and to the hypotenuse of the 30°-60°-90° triangle ABC as shown, where $AB = 1$. To the nearest hundredth, what is the radius of the circle?



A) 2.18 **B)** 2.24 **C)** 2.31 **D)** 2.37 **E)** 2.41

Q20. Which one of the following integers can be expressed as the sum of 100 consecutive positive integers?

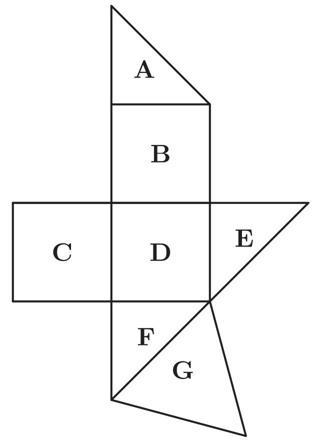
A) 1,627,384,950 **B)** 2,345,678,910 **C)** 3,579,111,300 **D)** 4,692,581,470 **E)** 5,815,937,260

Q21. For any positive integer n, let $f(n) = \begin{cases} \log_8 n, & \text{if } \log_8 n \text{ is rational,} \\ 0, & \text{otherwise.} \end{cases}$ What is $\sum_{n=1}^{1997} f(n)$?

A) $\log_8 2047$ **B)** 6 **C)** $\frac{55}{3}$ **D)** $\frac{58}{3}$ **E)** 585

Q22. Ashley, Betty, Carlos, Dick, and Elgin went shopping. Each had a whole number of dollars to spend, and together they had 56 dollars. The absolute difference between the amounts Ashley and Betty had to spend was 19 dollars. The absolute difference between the amounts Betty and Carlos had was 7 dollars, between Carlos and Dick was 5 dollars, between Dick and Elgin was 4 dollars, and between Elgin and Ashley was 11 dollars. How many dollars did Elgin have?

A) 6 **B)** 7 **C)** 8 **D)** 9 **E)** 10



Q23.

In the figure, polygons A, E, and F are isosceles right triangles; B, C, and D are squares with sides of length 1; and G is an equilateral triangle. The figure can be folded along its edges to form a polyhedron having the polygons as faces. The volume of this polyhedron is

A) $\frac{1}{2}$ **B)** $\frac{2}{3}$ **C)** $\frac{3}{4}$ **D)** $\frac{5}{6}$ **E)** $\frac{4}{3}$

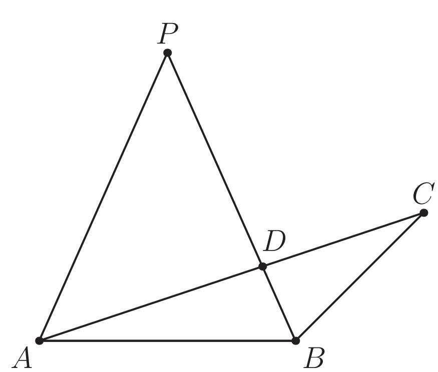
Q24. A rising number, such as 34689, is a positive integer each digit of which is larger than each of the digits to its left. There are $\binom{9}{5} = 126$ five-digit rising numbers. When these numbers are arranged from smallest to largest, the 97^{th} number in the list does not contain the digit

A) 4 **B)** 5 **C)** 6 **D)** 7 **E)** 8

Q25. Let $ABCD$ be a parallelogram and let $\overrightarrow{AA'}$, $\overrightarrow{BB'}$, $\overrightarrow{CC'}$, and $\overrightarrow{DD'}$ be parallel rays in space on the same side of the plane determined by $ABCD$. If $AA' = 10$, $BB' = 8$, $CC' = 18$, and $DD' = 22$ and M and N are the midpoints of $A'C'$ and $B'D'$, respectively, then $MN =$?

A) 0 **B)** 1 **C)** 2 **D)** 3 **E)** 4

Q26. Triangle ABC and point P in the same plane are given. Point P is equidistant from A and B, angle APB is twice angle ACB, and $\overline{AC}$ intersects $\overline{BP}$ at point D. If $PB = 3$ and $PD = 2$, then $AD \cdot CD =$



A) 5 **B)** 6 **C)** 7 **D)** 8 **E)** 9

Q27. Consider those functions f that satisfy $f(x+4)+f(x-4)=f(x)$ for all real x. Any such function is periodic, and there is a least common positive period p for all of them. Find p.

A) 8 **B)** 12 **C)** 16 **D)** 24 **E)** 32

Q28. How many ordered triples of integers (a,b,c) satisfy $|a+b|+c=19$ and $ab+|c|=97$?

A) 0 **B)** 4 **C)** 6 **D)** 10 **E)** 12

Q29. Call a positive real number special if it has a decimal representation that consists entirely of digits 0 and 7. For example, $\frac{700}{99}=7.\overline{07}=7.070707\cdots$ and 77.007 are special numbers. What is the smallest n such that 1 can be written as a sum of n special numbers?

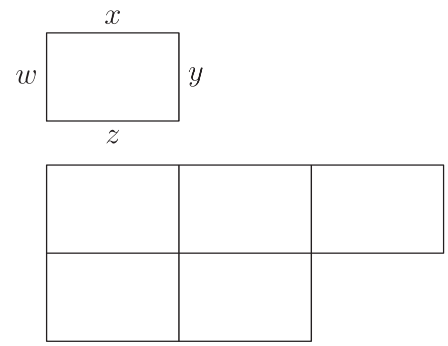
A) 7 **B)** 8 **C)** 9 **D)** 10

E) The number 1 cannot be represented as a sum of finitely many special numbers.

Q30. For positive integers n, denote $D(n)$ by the number of pairs of different adjacent digits in the binary (base two) representation of n. For example, $D(3)=D(11_2)=0$, $D(21)=D(10101_2)=4$, and $D(97)=D(1100001_2)=2$. For how many positive integers less than or equal 97 to does $D(n)=2$?

A) 16 **B)** 20 **C)** 26 **D)** 30 **E)** 35

Q1. Each of the sides of five congruent rectangles is labeled with an integer. In rectangle A, $w=4, x=1, y=6, z=9$. In rectangle B, $w=1, x=0, y=3, z=6$. In rectangle C, $w=3, x=8, y=5, z=2$. In rectangle D, $w=7, x=5, y=4, z=8$. In rectangle E, $w=9, x=2, y=7, z=0$. These five rectangles are placed, without rotating or reflecting, in position as below. Which of the rectangle is the top leftmost one?



A) A **B)** B **C)** C **D)** D **E)** E

Q2. Letters A, B, C, and D represent four different digits selected from $0, 1, 2, \ldots, 9$. If $(A+B)/(C+D)$ is an integer that is as large as possible, what is the value of $A+B$?

A) 13 **B)** 14 **C)** 15 **D)** 16 **E)** 17

Q3. If `a`,`b`, and `c` are digits for which

```
   7 a 2
-  4 8 b
--------
   c 7 3
```

then `a+b+c =`

A) 14 **B)** 15 **C)** 16 **D)** 17 **E)** 18

Q4. Define $[a, b, c]$ to mean $\frac{a+b}{c}$, where $c \neq 0$. What is the value of $[[60, 30, 90], [2, 1, 3], [10, 5, 15]]$?

A) 0 **B)** 0.5 **C)** 1 **D)** 1.5 **E)** 2

Q5. If $2^{1998} - 2^{1997} - 2^{1996} + 2^{1995} = k \cdot 2^{1995}$, what is the value of k?

A) 1 **B)** 2 **C)** 3 **D)** 4 **E)** 5

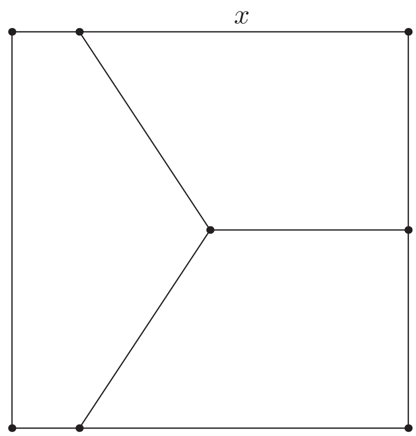
Q6. If 1998 is written as a product of two positive integers whose difference is as small as possible, then the difference is

A) 8 **B)** 15 **C)** 17 **D)** 47 **E)** 93

Q7. If $N > 1$, then $\sqrt[3]{N\sqrt[3]{N\sqrt[3]{N}}} =$

A) $N^{\frac{1}{27}}$ **B)** $N^{\frac{1}{9}}$ **C)** $N^{\frac{1}{3}}$ **D)** $N^{\frac{13}{27}}$ **E)** N

Q8. A square with sides of length 1 is divided into two congruent trapezoids and a pentagon, which have equal areas, by joining the center of the square with points on three of the sides, as shown. Find x, the length of the longer parallel side of each trapezoid.

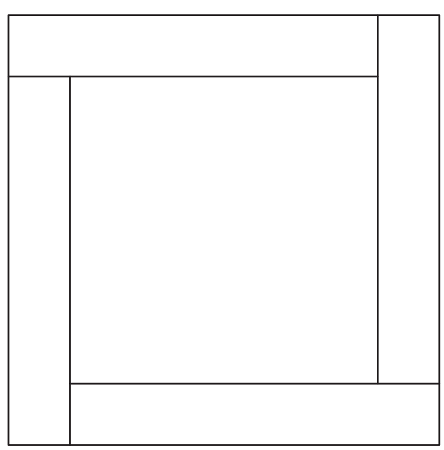


A) $\frac{3}{5}$ **B)** $\frac{2}{3}$ **C)** $\frac{3}{4}$ **D)** $\frac{5}{6}$ **E)** $\frac{7}{8}$

Q9. A speaker talked for sixty minutes to a full auditorium. Twenty percent of the audience heard the entire talk and ten percent slept through the entire talk. Half of the remainder heard one third of the talk and the other half heard two thirds of the talk. What was the average number of minutes of the talk heard by members of the audience?

A) 24 **B)** 27 **C)** 30 **D)** 33 **E)** 36

Q10. A large square is divided into a small square surrounded by four congruent rectangles as shown. The perimter of each of the congruent rectangles is 14. What is the area of the large square?



A) 49 **B)** 64 **C)** 100 **D)** 121 **E)** 196

Q11. Let R be a rectangle. How many circles in the plane of R have a diameter both of whose endpoints are vertices of R?

A) 1 **B)** 2 **C)** 4 **D)** 5 **E)** 6

Q12. How many different prime numbers are factors of N if $\log_2(\log_3(\log_5(\log_7 N))) = 11$?

A) 1 **B)** 2 **C)** 3 **D)** 4 **E)** 7

Q13. Walter rolls four standard six-sided dice and finds that the product of the numbers of the upper faces is 144. Which of he following could **not** be the sum of the upper four faces?

A) 14 **B)** 15 **C)** 16 **D)** 17 **E)** 18

Q14. A parabola has vertex of $(4, -5)$ and has two $x-$intercepts, one positive, and one negative. If this parabola is the graph of $y = ax^2 + bx + c$, which of $a, b,$ and c must be positive?

A) only a **B)** only b **C)** only c **D)** a and b only **E)** none

Q15. A regular hexagon and an equilateral triangle have equal areas. What is the ratio of the length of a side of the triangle to the length of a side of the hexagon?

A) $\sqrt{3}$ **B)** 2 **C)** $\sqrt{6}$ **D)** 3 **E)** 6

Q16. The figure shown is the union of a circle and two semicircles of diameters a and b, all of whose centers are collinear. The ratio of the area, of the shaded region to that of the unshaded region is

A) $\sqrt{\frac{a}{b}}$ **B)** $\frac{a}{b}$ **C)** $\frac{a^2}{b^2}$ **D)** $\frac{a+b}{2b}$ **E)** $\frac{a^2+2ab}{b^2+2ab}$

Q17. Let $f(x)$ be a function with the two properties:

a) for any two real numbers x and y, $f(x + y) = x + f(y)$, and

b) $f(0) = 2$.

What is the value of $f(1998)$?

A) 0 **B)** 2 **C)** 1996 **D)** 1998 **E)** 2000

Q18. A right circular cone of volume A, a right circular cylinder of volume M, and a sphere of volume C all have the same radius, and the common height of the cone and the cylinder is equal to the diameter of the sphere. Then

A) $A - M + C = 0$ **B)** $A + M = C$ **C)** $2A = M + C$ **D)** $A^2 - M^2 + C^2 = 0$ **E)** $2A + 2M = 3C$

Q19. How many triangles have area 10 and vertices at $(-5, 0), (5, 0)$ and $(5\cos\theta, 5\sin\theta)$ for some angle θ?

A) 0 **B)** 2 **C)** 4 **D)** 6 **E)** 8

Q20. Three cards, each with a positive integer written on it, are lying face-down on a table. Casey, Stacy, and Tracy are told that

a) the numbers are all different,

b) they sum to 13, and

c) they are in increasing order, left to right.

First, Casey looks at the number on the leftmost card and says, "I don't have enough information to determine the other two numbers." Then Tracy looks at the number on the rightmost card and says, "I don't have enough information to determine the other two numbers." Finally, Stacy looks at the number on the middle card and says, "I don't have enough information to determine the other two numbers."

Assume that each person knows that the other two reason perfectly and hears their comments. What number is on the middle card?

A) 2 **B)** 3 **C)** 4 **D)** 5

E) There is not enough information to determine the number.

Q21. In an h-meter race, Sunny is exactly d meters ahead of Windy when Sunny finishes the race. The next time they race, Sunny sportingly starts d meters behind Windy, who is at the starting line. Both runners run at the same constant speed as they did in the first race. How many meters ahead is Sunny when Sunny finishes the second race?

A) $\frac{d}{h}$ **B)** 0 **C)** $\frac{d^2}{h}$ **D)** $\frac{h^2}{d}$ **E)** $\frac{d^2}{h-d}$

Q22. What is the value of the expression

$$\frac{1}{\log_2 100!}+\frac{1}{\log_3 100!}+\frac{1}{\log_4 100!}+\cdots+\frac{1}{\log_{100} 100!}?$$

A) 0.01 **B)** 0.1 **C)** 1 **D)** 2 **E)** 10

Q23. The graphs of $x^2+y^2=4+12x+6y$ and $x^2+y^2=k+4x+12y$ intersect when k satisfies $a \le k \le b$, and for no other values of k. Find $b-a$.

A) 5 **B)** 68 **C)** 104 **D)** 140 **E)** 144

Q24. Call a 7-digit telephone number $d_1d_2d_3-d_4d_5d_6d_7$ "memorable" if the prefix sequence $d_1d_2d_3$ is exactly the same as either of the sequences $d_4d_5d_6$ or $d_5d_6d_7$ (possibly both). Assuming that each d_i can be any of the ten decimal digits $0, 1, 2, \ldots, 9$, the number of different memorable telephone numbers is

A) 19,810 **B)** 19,910 **C)** 19,990 **D)** 20,000 **E)** 20,100

Q25. A piece of graph paper is folded once so that $(0,2)$ is matched with $(4,0)$, and $(7,3)$ is matched with (m,n). Find $m+n$.

A) 6.7 **B)** 6.8 **C)** 6.9 **D)** 7.0 **E)** 8.0

Q26. In quadrilateral $ABCD$, it is given that $\angle A = 120^\circ$, angles B and D are right angles, $AB = 13$, and $AD = 46$. Then $AC =$

A) 60 **B)** 62 **C)** 64 **D)** 65 **E)** 72

Q27. A $9\times9\times9$ cube is composed of twenty-seven $3\times3\times3$ cubes. The big cube is ‘tunneled’ as follows: First, the six $3\times3\times3$ cubes which make up the center of each face as well as the center $3\times3\times3$ cube are removed. Second, each of the twenty remaining $3\times3\times3$ cubes is diminished in the same way. That is, the center facial unit cubes as well as each center cube are removed. The surface area of the final figure is:

Image:1998 AHSME num. 27.png

A) 384 **B)** 729 **C)** 864 **D)** 1024 **E)** 1056

Q28. In triangle ABC, angle C is a right angle and $CB > CA$. Point D is located on $\overline{BC}$ so that angle CAD is twice angle DAB. If $AC/AD = 2/3$, then $CD/BD = m/n$, where m and n are relatively prime positive integers. Find $m+n$.

A) 10 **B)** 14 **C)** 18 **D)** 22 **E)** 26

Q29. A point (x,y) in the plane is called a lattice point if both x and y are integers. The area of the largest square that contains exactly three lattice points in its interior is closest to

A) 4.0 **B)** 4.2 **C)** 4.5 **D)** 5.0 **E)** 5.6

Q30. For each positive integer n, let $a_n = \frac{(n+9)!}{(n-1)!}$ Let k denote the smallest positive integer for which the rightmost nonzero digit of a_k is odd. The rightmost nonzero digit of a_k is

A) 1 **B)** 3 **C)** 5 **D)** 7 **E)** 9

Q1. $1 - 2 + 3 - 4 + \cdots - 98 + 99 =$

A) -50 **B)** -49 **C)** 0 **D)** 49 **E)** 50

Q2. Which of the following statements is false?

A) All equilateral triangles are congruent to each other.

B) All equilateral triangles are convex.

C) All equilateral triangles are equiangular.

D) All equilateral triangles are regular polygons.

E) All equilateral triangles are similar to each other.

Q3. The number halfway between $1/8$ and $1/10$ is

A) $\frac{1}{80}$ **B)** $\frac{1}{40}$ **C)** $\frac{1}{18}$ **D)** $\frac{1}{9}$ **E)** $\frac{9}{80}$

Q4. Find the sum of all prime numbers between 1 and 100 that are simultaneously 1 greater than a multiple of 4 and 1 less than a multiple of 5.

A) 118 **B)** 137 **C)** 158 **D)** 187 **E)** 245

Q5. The marked price of a book was 30% less than the suggested retail price. Alice purchased the book for half the marked price at a Fiftieth Anniversary sale. What percent of the suggested retail price did Alice pay?

A) 25% **B)** 30% **C)** 35% **D)** 60% **E)** 65%

Q6. What is the sum of the digits of the decimal form of the product $2^{1999} \cdot 5^{2001}$?

A) 2 **B)** 4 **C)** 5 **D)** 7 **E)** 10

Q7. What is the largest number of acute angles that a convex hexagon can have?

A) 2 **B)** 3 **C)** 4 **D)** 5 **E)** 6

Q8. At the end of 1994, Walter was half as old as his grandmother. The sum of the years in which they were born was 3838. How old will Walter be at the end of 1999?

A) 48 **B)** 49 **C)** 53 **D)** 55 **E)** 101

Q9. Before Ashley started a three-hour drive, her car's odometer reading was 29792, a palindrome. (A palindrome is a number that reads the same way from left to right as it does from right to left). At her destination, the odometer reading was another palindrome. If Ashley never exceeded the speed limit of 75 miles per hour, which of the following was her greatest possible average speed?

A) $33\frac{1}{3}$ **B)** $53\frac{1}{3}$ **C)** $66\frac{2}{3}$ **D)** $70\frac{1}{3}$ **E)** $74\frac{1}{3}$

Q10. A sealed envelope contains a card with a single digit on it. Three of the following statements are true, and the other is false.

I) The digit is 1.

II) The digit is not 2.

III) The digit is 3.

IV) The digit is not 4.

Which one of the following must necessarily be correct?

A) I is true. **B)** I is false. **C)** II is true.

D) III is true. **E)** IV is false.

Q11. The student locker numbers at Olympic High are numbered consecutively beginning with locker number 1. The plastic digits used to number the lockers cost two cents apiece. Thus, it costs two cents to label locker number 9 and four centers to label locker number 10. If it costs $137.94 to label all the lockers, how many lockers are there at the school?

A) 2001 **B)** 2010 **C)** 2100 **D)** 2726 **E)** 6897

Q12. What is the maximum number of points of intersection of the graphs of two different fourth degree polynomial functions $y = p(x)$ and $y = q(x)$, each with leading coefficient 1?

A) 1 **B)** 2 **C)** 3 **D)** 4 **E)** 8

Q13. Define a sequence of real numbers a_1, a_2, a_3, $\ldots$ by $a_1 = 1$ and $a_{n+1}^3 = 99a_n^3$ for all $n \geq 1$. Then a_{100} equals

A) 33^{33} **B)** 33^{99} **C)** 99^{33} **D)** 99^{99} **E)** none of these

Q14. Four girls — Mary, Alina, Tina, and Hanna — sang songs in a concert as trios, with one girl sitting out each time. Hanna sang 7 songs, which was more than any other girl, and Mary sang 4 songs, which was fewer than any other girl. How many songs did these trios sing?

A) 7 **B)** 8 **C)** 9 **D)** 10 **E)** 11

Q15. Let x be a real number such that $\sec x - \tan x = 2$. Then $\sec x + \tan x =$

A) 0.1 **B)** 0.2 **C)** 0.3 **D)** 0.4 **E)** 0.5

Q16. What is the radius of a circle inscribed in a rhombus with diagonals of length 10 and 24?

A) 4 **B)** $\frac{58}{13}$ **C)** $\frac{60}{13}$ **D)** 5 **E)** 6

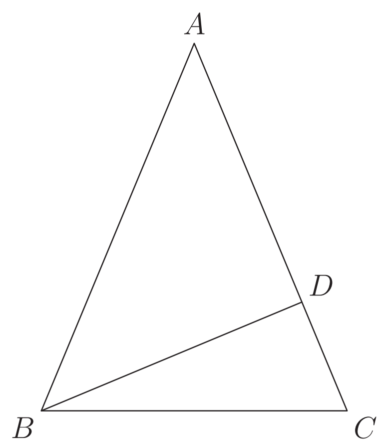
Q17. Let $P(x)$ be a polynomial such that when $P(x)$ is divided by $x - 19$, the remainder is 99, and when $P(x)$ is divided by $x - 99$, the remainder is 19. What is the remainder when $P(x)$ is divided by $(x - 19)(x - 99)$?

A) $-x + 80$ **B)** $x + 80$ **C)** $-x + 118$ **D)** $x + 118$ **E)** 0

Q18. How many zeros does $\mathrm{f}(x) = \cos(\log x)$ have on the interval $0 < x < 1$?

A) 0 **B)** 1 **C)** 2 **D)** 10 **E)** infinitely many

Q19. Consider all triangles ABC satisfying in the following conditions: $AB = AC$, D is a point on $\overline{AC}$ for which $\overline{BD} \perp \overline{AC}$, AC and CD are integers, and $BD^2 = 57$. Among all such triangles, the smallest possible value of AC is



A) 9 **B)** 10 **C)** 11 **D)** 12 **E)** 13

Q20. The sequence $a_1, a_2, a_3, \ldots$ statisfies $a_1 = 19, a_9 = 99$, and, for all $n \geq 3$, a_n is the arithmetic mean of the first $n - 1$ terms. Find a_2.

A) 29 **B)** 59 **C)** 79 **D)** 99 **E)** 179

Q21. A circle is circumscribed about a triangle with sides $20, 21$, and 29, thus dividing the interior of the circle into four regions. Let A, B, and C be the areas of the non-triangular regions, with C be the largest. Then

A) $A + B = C$ **B)** $A + B + 210 = C$ **C)** $A^2 + B^2 = C^2$

D) $20A + 21B = 29C$ **E)** $\frac{1}{A^2} + \frac{1}{B^2} = \frac{1}{C^2}$

Q22. The graphs of $y = -|x - a| + b$ and $y = |x - c| + d$ intersect at points $(2, 5)$ and $(8, 3)$. Find $a + c$.

A) 7 **B)** 8 **C)** 10 **D)** 13 **E)** 18

Q23. The equiangular convex hexagon $ABCDEF$ has $AB = 1, BC = 4, CD = 2$, and $DE = 4$. The area of the hexagon is

A) $\frac{15}{2}\sqrt{3}$ **B)** $9\sqrt{3}$ **C)** 16 **D)** $\frac{39}{4}\sqrt{3}$ **E)** $\frac{43}{4}\sqrt{3}$

Q24. Six points on a circle are given. Four of the chords joining pairs of the six points are selected at random. What is the probability that the four chords form a convex quadrilateral?

A) $\frac{1}{15}$ **B)** $\frac{1}{91}$ **C)** $\frac{1}{273}$ **D)** $\frac{1}{455}$ **E)** $\frac{1}{1365}$

Q25. There are unique integers $a_2, a_3, a_4, a_5, a_6, a_7$ such that

$$\frac{5}{7} = \frac{a_2}{2!} + \frac{a_3}{3!} + \frac{a_4}{4!} + \frac{a_5}{5!} + \frac{a_6}{6!} + \frac{a_7}{7!}$$

where $0 \leq a_i < i$ for $i = 2, 3, \ldots, 7$. Find $a_2 + a_3 + a_4 + a_5 + a_6 + a_7$.

A) 8 **B)** 9 **C)** 10 **D)** 11 **E)** 12

Q26. Three non-overlapping regular plane polygons, at least two of which are congruent, all have sides of length 1. The polygons meet at a point A in such a way that the sum of the three interior angles at A is $360°$. Thus the three polygons for a new polygon with A as an interior point. What is the largest possible perimeter that this polygon can have?

A) 12 **B)** 14 **C)** 18 **D)** 21 **E)** 24

Q27. In triangle ABC, $3\sin A + 4\cos B = 6$ and $4\sin B + 3\cos A = 1$. Then $\angle C$ in degrees is

A) 30 **B)** 60 **C)** 90 **D)** 120 **E)** 150

Q28. Let $x_1, x_2, \ldots, x_n$ be a sequence of integers such that

I) $-1 \leq x_i \leq 2$ for $i = 1, 2, \ldots n$

II) $x_1 + \cdots + x_n = 19$; and

III) $x_1^2 + x_2^2 + \cdots + x_n^2 = 99$.

Let m and M be the minimal and maximal possible values of $x_1^3 + \cdots + x_n^3$, respectively. Then $\frac{M}{m} =$

A) 3 **B)** 4 **C)** 5 **D)** 6 **E)** 7

Q29. A tetrahedron with four equilateral triangular faces has a sphere inscribed within it and a sphere circumscribed about it. For each of the four faces, there is a sphere tangent externally to the face at its center and to the circumscribed sphere. A point P is selected at random inside the circumscribed sphere. The probability that P lies inside one of the five small spheres is closest to

A) 0 **B)** 0.1 **C)** 0.2 **D)** 0.3 **E)** 0.4

Q30. The number of ordered pairs of integers (m, n) for which $mn \geq 0$ and $m^3 + n^3 + 99mn = 33^3$ is equal to

A) 2 **B)** 3 **C)** 33 **D)** 35 **E)** 99

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