

United Kingdom Mathematics Trust

Individual Challenge - Past Papers Collection
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## Junior Challenge

## www.CasperYC.club

## Last updated: September 22, 2023

## Instructions

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: $\mathbf{6 0}$ minutes.

No answers, or personal details, may be entered after the allowed time is over.
3. The use of blank or lined paper for rough working is allowed; squared paper, calculators and measuring instruments are forbidden.
4. Use a B or an HB non-propelling pencil. Mark at most one of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.
5. Do not expect to finish the whole paper in the time allowed. The questions in this paper have been arranged in approximate order of difficulty with the harder questions towards the end. You are not expected to complete all the questions during the time. You should bear this in mind when deciding which questions to tackle.
6. Scoring rules:

5 marks are awarded for each correct answer to Questions 1-15;
6 marks are awarded for each correct answer to Questions 16-25.
In this paper you will not lose marks for getting questions wrong.
7. Your Answer Sheet will be read by a machine. Do not write or doodle on the sheet except to mark your chosen options. The machine will read all black pencil markings even if they are in the wrong places. If you mark the sheet in the wrong place, or leave bits of eraser stuck to the page, the machine will interpret the mark in its own way.
8. The questions on this paper are designed to challenge you to think, not to guess. You will gain more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. This paper is about solving interesting problems, not about lucky guessing.

1. What is the value of $3202-2023$ ?
A 821
B 1001
C 1179
D 1221
E 1279
2. How many of the following five options are factors of 30 ?
A 1
B 2
C 3
D 4
E 5
3. What is the value of $\frac{1+2+3+4+5}{6+7+8+9+10}$ ?
A $\frac{1}{2}$
B $\frac{3}{8}$
C $\frac{7}{16}$
D $\frac{9}{20}$
E $\frac{1}{3}$
4. One of these is the largest two-digit positive integer that is divisible by the product of its digits. Which is it?
A 12
B 24
C 36
D 72
E 96
5. The record for travelling 100 m on a skateboard by a dog is 19.65 seconds. This was achieved by Jumpy in 2013. What was Jumpy's approximate average speed?
A $0.2 \mathrm{~m} / \mathrm{s}$
B $0.5 \mathrm{~m} / \mathrm{s}$
C $2 \mathrm{~m} / \mathrm{s}$
D $2.5 \mathrm{~m} / \mathrm{s}$
E $5 \mathrm{~m} / \mathrm{s}$
6. When this prime number square is completed, the eight circles contain eight different primes, and each of the four sides has total 43.
What is the sum of the five missing primes?
A 51
B 53
C 55
D 57
E 59

7. What is the difference between the largest two-digit multiple of 2 and the smallest three-digit multiple of 3 ?
A 5
B 4
C 3
D 2
E 1
8. How many of these six numbers are prime?
$0^{2}+1^{2} \quad 1^{2}+2^{2} \quad 2^{2}+3^{2} \quad 3^{2}+4^{2} \quad 4^{2}+5^{2} \quad 5^{2}+6^{2}$
A 1
B 2
C 3
D 4
E 5
9. Triangle $L M N$ is isosceles with $L M=L N$.

What is the value of $y$ ?
A 15
B 17
C 19
D 21
E 23


10．In the diagram，all distances shown are in cm ．The perimeter of the shape is 60 cm ．What is the area，in $\mathrm{cm}^{2}$ ，of the shape？
A 192
B 204
C 212
D 232
E 252


11．To save money，Scrooge is reusing tea bags．After a first＇decent＇cup of tea，he dries the bag and uses two such dried bags to make a new＇decent＇cup of tea．These bags are then dried again and four such bags now make a＇decent＇cup of tea．After that they are put on the compost heap．
How many＇decent＇cups of tea can Scrooge get out of a new box of 120 tea bags？
A 480
B 240
C 210
D 195
E 180

12．One afternoon，Brian the snail went for a slither at a constant speed．By $1: 50 \mathrm{pm}$ he had slithered 150 centimetres．By $2: 10$ pm he had slithered 210 centimetres．When did Brian start his slither？
A Noon
B 12：20 pm
C $12: 30 \mathrm{pm}$
D $12: 45 \mathrm{pm}$
E 1 pm

13．Four congruent rectangles are arranged as shown to form an inner square of area $20 \mathrm{~cm}^{2}$ and an outer square of area $64 \mathrm{~cm}^{2}$ ． What is the perimeter of one of the four congruent rectangles？
A 6 cm
B 8 cm
C 9.75 cm
D 16 cm
E 20 cm


14．In the addition shown，$x$ and $y$ represent different single digits．
What is the value of $x+y$ ？
A 10
B 11
C 12
D 13

E 14

| $+y y x$ |
| :--- |
| $1 x x 7$ |

15．My train was scheduled to leave at $17: 48$ and to arrive at my destination at $18: 25$ ．However，it started four minutes late，and the journey took twice as long as scheduled．
When did I arrive？
A 19：39
B 19：06
C 19：02
D 18：29
E 17：52

16．Amrita needs to select a new PIN．She decides it will be made up of four non－zero digits with the following properties：
i）The first two digits and the last two digits each make up a two－digit number which is a multiple of 11 ．
ii）The sum of all the digits is a multiple of 11 ．
How many different possibilities are there for Amrita＇s PIN？
A 1
B 2
C 4
D 8
E 16

17．Two numbers $p$ and $q$ are such that $0<p<q<1$ ．
Which is the largest of these expressions？
A $q-p$
B $p-q$
C $\frac{p+q}{2}$
D $\frac{p}{q}$
$\mathrm{E} \frac{q}{p}$

18．What is the sum of the four marked angles in the diagram？
A $540^{\circ}$
B $560^{\circ}$
C $570^{\circ}$
D $600^{\circ}$
E $720^{\circ}$


19．In a football match，Rangers beat Rovers 5 －4．The only time Rangers were ahead was after they scored the final goal．How many possible half－time scores were there？
A 9
B 10
C 15
D 16
E 25

20．Each cell in the crossnumber is to be filled with a single digit．

| Across | Down |
| :--- | :--- |
| 1．A cube | 1．A prime |
| 2．A square |  |


| 1 |  |
| :--- | :--- |
| 2 |  |

Which of these could be the sum of the four digits in the crossnumber？
A 17
B 16
C 15
D 14
E 13

21．Eleanor＇s Elephant Emporium has four types of elephant．There are twice as many grey elephants as pygmy elephants，three times as many white elephants as grey elephants and four times as many pink elephants as white elephants．There are 20 more white elephants than pygmy elephants．
How many elephants are in Eleanor＇s Emporium？
A 123
B 132
C 213
D 231
E 312

22．The positive integers from 1 to 9 inclusive are placed in the grid，one to a cell， so that the product of the three numbers in each row or column is as shown． What number should be placed in the bottom right－hand cell？
A 9
B 6
C 4
D 3
E 2


23．Regular pentagon $P Q R S T$ has centre $O$ ．Lines $P H, F I$ and $G J$ go through $O$ ． The six angles at $O$ are equal．
What is the size of angle $T G O$ ？
A $60^{\circ}$
B $72^{\circ}$
C $75^{\circ}$
D $76^{\circ}$
E $78^{\circ}$


24．Beatrix was born in this century．On her birthday this year，her age was equal to the sum of the digits of the year in which she was born．In which of these years will her age on her birthday be twice the sum of the digits of that year？
A 2027
B 2029
C 2031
D 2033
E 2035

25．Granny gave away her entire collection of antique spoons to three people．Her daughter received 8 more than a third of the total；her son received 8 more than a third of what was then left；finally her neighbour received 8 more than a third of what was then left．
What is the sum of the digits of the number of spoons which were in Granny＇s collection？
A 14
B 12
C 10
D 8
E 6


1．Which of these has the greatest value？
A $20+22$
B $202+2$
C $202 \times 2$
D $2 \times 0 \times 2 \times 2$
E $20 \times 22$

2．The number 5012 is reflected in the mirror－line shown． Onto which number is it reflected？
A 5102
B 2015
C 5012
D 2105
E 5105
your original
3．Think of any number．Add five；multiply by two；add ten；divide by two；subtract your original number；add three．What is the resulting number？
A 10
B 11
C 12
D 13
E 14

4．What is the value of $0.6+\frac{2}{5}$ ？
A 0.15
B 0.24
C 0.8
D 1
E 2.4

5．How many of the following take integer values？

$$
\frac{1}{1} \quad \frac{11}{1+1} \quad \frac{111}{1+1+1} \quad \frac{1111}{1+1+1+1} \quad \frac{11111}{1+1+1+1+1}
$$

A 0
B 1
C 2
D 3
E 4

6．The diagram shows the square $R S T U$ and two equilateral triangles，$P U T$ and $Q R U$ ．

What is the size of angle $Q P U$ ？
A $10^{\circ}$
B $15^{\circ}$
C $20^{\circ}$
D $25^{\circ}$
E $30^{\circ}$


7．Kiwi fruit contain roughly two and a half times as much vitamin C as the same weight of oranges． What weight of kiwi fruit contains approximately the same amount of vitamin C as 1 kg of oranges？
A 100 g
B 200 g
C 250 g
D 400 g
E 550 g

8．Today is Thursday．What day will it be in 100 days＇time？
A Tuesday
B Wednesday
C Thursday
D Friday
E Saturday

9．How many squares of any size can be seen in the diagram？
A 25
B 27
C 28
D 39
E 40


10．Half of a quarter of an eighth of a number is equal to $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}$ ．
What is the number？
A 14
B 28
C 42
D 56
E 64


11．Nine of the numbers $1,2,3,4,5,6,7,8,9,10$ are to be put in two groups so that the sum of the numbers in each group is a multiple of four．
What is the largest number that could be left out？
A 3
B 4
C 5
D 6
E 7

12．When my pot of paint is half full，it weighs 5.8 kg ．When my pot of paint is one quarter full，it weighs 3.1 kg ．What is the weight of the full pot？
A 8.9 kg
B 11.2 kg
C 11.6 kg
D 12 kg
E 12.4 kg

13．The diagram shows five squares whose side－lengths，in cm ，are $1,2,3,4$ and 5 ． What percentage of the area of the outer square is shaded？
A $25 \%$
B 30\％
C $36 \%$
D $40 \%$
E $42 \%$


14．A group of children stand evenly spaced around a circular ring and are numbered consecutively 1,2 ， 3 ，and so on．Number 13 is directly opposite number 35 ．How many children are there in the ring？
A 42
B 44
C 46
D 48
E 50

15．What is the value of $2 \div(4 \div(6 \div(8 \div 10)))$ ？
A $\frac{1}{960}$
B $\frac{1}{5}$
C $\frac{3}{8}$
D $\frac{1}{2}$
E $\frac{15}{4}$

16．The diagram shows a seven－sided polygon，$P Q R S T U V$ ．It is formed from two equilateral triangles $P Q W$ and $S T U$ of side－length 5 cm and 8 cm respectively．The two triangles overlap in an equilateral triangle of side－length 2 cm ．
What is the perimeter of PQRSTUV？
A 27 cm
B 30 cm
C 33 cm
D 36 cm
E 39 cm


17．Amrita and Beatrix play a game in which each player starts with 10 counters．In each round of the game，one player wins and is given 3 counters；and her opponent has 1 counter removed．At the end of the game，Amrita and Beatrix have 40 counters and 16 counters respectively．
How many rounds of the game did Amrita win？
A 10
B 11
C 12
D 13
E 14

18．The diagram shows a parallelogram．
What is the value of $y$ ？
A 22
B 24
C 25
D 28
E 30


19．At the start of the day I had three times as many apples as pears．By the end of the day，after eating five apples but no pears，I had twice as many pears as apples． How many pieces of fruit did I have at the start of the day？
A 4
B 8
C 12
D 16
E 20


20．During a particularly troublesome lesson，the following conversation occurs：

Pam：＂I always tell the truth．＂
Roger：＂Both Pam and Quentin are lying．＂

Quentin：＂Pam is lying．＂
Susan：＂Everyone is lying．＂

Terry：＂Everyone is telling the truth．＂
How many people are telling the truth？
A 0
B 1
C 2
D 3
E 4

21．Two lists of numbers are as shown below．

| List S： | 3 | 5 | 8 | 11 | 13 | 14 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| List T： | 2 | 5 | 6 | 10 | 12 | 13 |

Jenny decided she would move one number from List $S$ to List T and one number from List T to List S so that the sum of the numbers in the new List S is equal to the sum of the numbers in the new List T ． In how many ways could she do this？
A 1
B 2
C 3
D 4
E 5

22．A triangular pyramid with vertices $T, U, V$ and $Q$ is removed from the solid cube shown．
How many edges does the remaining solid have？
A 4
B 6
C 8
D 10
E 12


23．The price of a train ticket increased by $5 \%$ and then decreased by $20 \%$ in a special offer．It was then $£ 4$ less expensive than its original price．What was the original price of the ticket？
A $£ 8.60$
B $£ 13$
C £20．40
D $£ 25$
E $£ 26.40$

24．Flori＇s Flower shop contains fewer than 150 flowers．All the flowers are purple，yellow，red or white． The ratio of purple flowers to yellow flowers is $1: 2$ ，the ratio of yellow flowers to red flowers is $3: 4$ and the ratio of red flowers to white flowers is $5: 6$ ．
How many flowers are there in Flori＇s shop？
A 133
B 136
C 139
D 142
E 145

25．In the number pyramid shown，each cell above the bottom row contains the sum of the numbers in the two cells immediately below it．The sum of the numbers in the bottom row is 17 ．
What is the central number of the bottom row？
A 2
B 3
C 4
D 5
E 6


1．What is the value of $123-456+789$ ？
A 456
B 556
C 567
D 678
E 789

2．Brianna has $£ 20$ ，all in 5 p coins，and $£ 50$ ，all in 2 p coins．
How many coins does she have in total？
A 200
B 290
C 1000
D 2540
E 2900

3．What is the value of $1-2 \times 3+4 \div 5$ ？
A－4．2
B -2.8
C 0
D 0.2
E 4

4．How many of the following numbers are multiples of 11 ？

| 187 | 156 | 253 | 495 | 132 |
| :--- | :--- | :--- | :--- | :--- |

A 1
B 2
C 3
D 4
E 5

5．When I have walked $20 \%$ of the way to school，I have 1200 metres more to walk than when I have $20 \%$ of the walk remaining．
How far，in metres，is it from my home to my school？
A 1240
B 1440
C 1680
D 1800
E 2000

6．What is the value of $\left(2-\frac{1}{2}\right)\left(3-\frac{1}{3}\right)\left(4-\frac{1}{4}\right)$ ？
A 16
B 15
C 14
D 13
E 12

7．In the diagram shown，$P T=Q T=Q R$ ．
Also，$R T=R S$ and $\angle P T Q=36^{\circ}$ ．
What is $\angle P T S$ ？
A $72^{\circ}$
B $80^{\circ}$
C $90^{\circ}$
D $100^{\circ}$
E $108^{\circ}$


8．What is the value of $1-(2-(3-(4-5)))$ ？
A -5
B－3
C -1
D 1
E 3

9．Each cell in the crossnumber below contains a single non－zero digit．The answer to each clue is a two－digit number．

Clues

Across
1．A square
3．An odd square

Down
1．A square
2．A square


What is the value of $x$ ？
A 1
B 3
C 5
D 7
E 9

10．The diagram shows a rhombus formed by joining each vertex of a square to the midpoint of a side of the square．
What fraction of the area of the square has been shaded？

A $\frac{1}{2}$
B $\frac{1}{3}$
C $\frac{1}{4}$
D $\frac{1}{6}$
E $\frac{1}{8}$

11．A particular prism has ten faces．How many edges does it have？
A 20
B 21
C 24
D 27
E 30

12．The pupils in my class work very quickly．Jasleen answers four questions every 30 seconds and Ella answers five questions every 40 seconds．
Last week，Jasleen took exactly 1 hour to answer a large set of questions．
How many minutes more than Jasleen did Ella take to answer the same set of questions？
A 2
B $2 \frac{1}{2}$
C $3 \frac{1}{4}$
D 4
E $4 \frac{1}{2}$

13．Five line segments coincide at a point as shown．
What is the sum of the marked angles？
A $900^{\circ}$
B $720^{\circ}$
C $540^{\circ}$
D $360^{\circ}$
E $180^{\circ}$


14．I begin with a three－digit positive integer．I divide it by 9 and then subtract 9 from the answer．My final answer is also a three－digit integer．
How many different positive integers could I have begun with？
A 3
B 5
C 7
D 11
E 13

15．Alex has a pile of two pence coins．She swapped exactly half of them for the same number of 10 p coins．Now she has $£ 4.20$ ．How much money did Alex have initially？＂
A 42 p
B 84p
C $£ 1.12$
D $£ 1.40$
E $£ 1.68$

16．A cube has edge length 10 cm ．Starting at the vertices，dots are placed along every edge at 1 cm intervals．Part of this pattern is shown． How many dots will there be in total，once the pattern has been completed？
A 128
B 116
C 112
D 108
E 104


17．In 1770，Joseph－Louis Lagrange proved that every positive integer can be written as the sum of four squares．For example， $13=0^{2}+0^{2}+2^{2}+3^{2}$ ．
How many of the first 15 positive integers can be written as the sum of three squares？
A 11
B 12
C 13
D 14
E 15

18．Each of the numbers 1 to 9 is to be placed in a different cell of the grid shown so that the sum of the three numbers in each row is 15 ．Also，the sum of the two numbers in each shaded column is to be 15 ．


How many choices are there for the number to be placed in the central cell indicated by＊？
A 0
B 1
C 2
D 3
E 4

19．In my class，everyone studies French or German，but not both languages．One third of the girls and the same number of boys study German．Twice as many boys as girls study French． Which of these could be the total number of boys and girls in my class？
A 26
B 28
C 30
D 32
E 34

20．Each of the shapes shown has been made from four unit cubes．For each shape， Max takes eight copies of the shape and tries to fit them together to make a $2 \times 4 \times 4$ cuboid．


How many of the shapes can be used to make a cuboid of this size in this way？
A 0
B 1
C 2
D 3
E 4


21．Some fish，some dogs and some children are swimming in a bay．There are 40 legs in total，twice as many heads as tails and more dogs than fish． How many fish are in the bay？
A 1
B 2
C 3
D 4
E 5

22．The diagram shows four congruent rectangles，each of perimeter 20 cm ， surrounding a square of area $44 \mathrm{~cm}^{2}$ ． What is the area of each rectangle？
A $14 \mathrm{~cm}^{2}$
B $16 \mathrm{~cm}^{2}$
C $18 \mathrm{~cm}^{2}$
D $20 \mathrm{~cm}^{2}$
E $22 \mathrm{~cm}^{2}$


23．Four different positive integers $p, q, r, s$ satisfy the equation $(9-p)(9-q)(9-r)(9-s)=9$ ． What is the value of $p+q+r+s$ ？
A 20
B 24
C 28
D 32
E 36

24．In the diagram shown，$P Q=P R=Q S$ ．Line segments $P R$ and $Q S$ are perpendicular to each other．
What is the sum of $\angle P R Q$ and $\angle P S Q$ ？
A $90^{\circ}$
B $105^{\circ}$
C $120^{\circ}$
D $135^{\circ}$
E $150^{\circ}$


25．I choose four different integers．When I add all the pairs of these numbers in turn，the totals that I obtain are $23,26,29,32$ and 35 ，with one of these totals being repeated．
What is the largest of the four integers？
A 18
B 19
C 20
D 21
E 22


1．Exactly one of the following five numbers is not prime．Which is it？
A 101
B 103
C 107
D 109
E 111

2．What is the value of $2020 \div 20$ ？
A 10
B 11
C 100
D 101
E 111

3．Each of these figures is based on a rectangle whose centre is shown．


How many of the figures have rotational symmetry of order two？
A 1
B 2
C 3
D 4
E 5

4．How many centimetres are there in 66.6 metres？
A 66600
B 6660
C 666
D 66.6
E 66

5．Amrita thinks of a number．She doubles it，adds 9 ，divides her answer by 3 and finally subtracts 1 ． She obtains the same number she originally thought of． What was Amrita＇s number？
A 1
B 2
C 3
D 4
E 6

6．What is the value of $\frac{6}{12}-\frac{5}{12}+\frac{4}{12}-\frac{3}{12}+\frac{2}{12}-\frac{1}{12}$ ？
A $\frac{1}{2}$
B $\frac{1}{3}$
C $\frac{1}{4}$
D $\frac{1}{5}$
E $\frac{1}{6}$

7．Four different positive integers have a product of 110．What is the sum of the four integers？
A 19
B 22
C 24
D 25
E 28

8．Wesley has a grid of six cells．He wants to colour two of the cells black so that the two black cells share a vertex but not a side．In how many ways can he achieve this？
A 2
B 3
C 4
D 5
E 6


9．One half of one third of one quarter of one fifth of a number is 2 ．
What is the number？
A 240
B 120
C 60
D $\frac{1}{120}$
E $\frac{1}{240}$

10．How many of these equations have the solution $x=12$ ？

$$
x-2=10 \quad \frac{x}{2}=24 \quad 10-x=2 \quad 2 x-1=25
$$

A 4
B 3
C 2
D 1
E 0

11．This 3 by 3 grid shows nine $1 \mathrm{~cm} \times 1 \mathrm{~cm}$ squares and uses 24 cm of wire． What length of wire is required for a similar 20 by 20 grid？
A 400 cm
B 420 cm
C 441 cm
D 800 cm
E 840 cm


12．The diagram shows an equilateral triangle divided into four smaller equilateral triangles．One of these triangles has itself been divided into four smaller equilateral triangles．
What fraction of the area of the large triangle has been shaded？
A $\frac{1}{8}$
B $\frac{3}{16}$
C $\frac{1}{4}$
D $\frac{5}{16}$
E $\frac{3}{8}$


13．The mean of four positive integers is 5 ．The median of the four integers is 6 ． What is the mean of the largest and smallest of the integers？
A 3
B 4
C 5
D 6
E 8

14．In the diagram，angle $O L M$ is twice as large as angle $P O N$ ．What is the size of angle $O L M$ ？
A $102^{\circ}$
B $106^{\circ}$
C $108^{\circ}$
D $112^{\circ}$
E $124^{\circ}$


15．A group of 42 children all play tennis or football，or both sports．The same number play tennis as play just football．Twice as many play both tennis and football as play just tennis． How many of the children play football？
A 7
B 14
C 21
D 28
E 35

16．You are given the sequence of digits＂ 0625 ＂，and can insert a decimal point at the beginning，at the end，or at any of the other three positions．
Which of these numbers can you not make？
A $\frac{6}{25}$
B $\frac{5}{8}$
C $\frac{1}{16}$
D $\frac{25}{4}$
E $25^{2}$

17．In 1925，Zbigniew Morón published a rectangle that could be dissected into nine different sized squares as shown in the diagram．The lengths of the sides of these squares are $1,4,7,8,9,10,14,15$ and 18 ． What is the area of Morón＇s rectangle？
A 144
B 225
C 900
D 1024
E 1056


18．How many two－digit primes have both their digits non－prime？
A 6
B 5
C 4
D 3
E 2


19．In the table shown，the sum of each row is shown to the right of the row and the sum of each column is shown below the column． What is the value of $L$ ？
A 1
B 2
C 3
D 5
E 7

| $J$ | $K$ | $J$ |
| :---: | :---: | :---: |
|  | 5 |  |
| $K$ | $K$ | $L$ |
| 13 | 13 |  |
| $L$ | $J$ | $L$ |
|  | 15 |  |
| 11 | 7 | 15 |

20．Edmund makes a cube using eight small cubes．Samuel uses cubes of the same size as the small cubes to make a cuboid twice as long，three times as wide and four times as high as Edmund＇s cube． How many more cubes does Samuel use than Edmund？
A 9
B 24
C 64
D 184
E 190

21．The digits of both the two－digit numbers in the first calculation below have been reversed to give the two－digit numbers in the second calculation．The answers to the two calculations are the same．

$$
62 \times 13=806 \quad 26 \times 31=806
$$

For which one of the calculations below is the same thing true？
A $25 \times 36$
B $34 \times 42$
C $54 \times 56$
D $42 \times 48$
E $32 \times 43$

22．Harriet has a square piece of paper．She folds it in half to form a rectangle and then in half again to form a second rectangle（which is not a square）．The perimeter of the second rectangle is 30 cm ． What is the area of the original square？
A $36 \mathrm{~cm}^{2}$
B $64 \mathrm{~cm}^{2}$
C $81 \mathrm{~cm}^{2}$
D $100 \mathrm{~cm}^{2}$
E $144 \mathrm{~cm}^{2}$

23．There is more than one integer，greater than 1 ，which leaves a remainder of 1 when divided by each of the four smallest primes．
What is the difference between the two smallest such integers？
A 211
B 210
C 31
D 30
E 7

24．Susan is attending a talk at her son＇s school．There are 8 rows of 10 chairs where 54 parents are sitting． Susan notices that every parent is either sitting on their own or next to just one other person． What is the largest possible number of adjacent empty chairs in a single row at that talk？
A 3
B 4
C 5
D 7
E 8

25．In the diagram，$P Q R S, J Q K$ and $L R K$ are straight lines．
What is the size of the angle $J K L$ ？
A $34^{\circ}$
B $35^{\circ}$
C $36^{\circ}$
D $37^{\circ}$
E $38^{\circ}$


1．How many minutes is it from $23: 35$ today to $01: 15$ tomorrow？
A 100
B 110
C 120
D 130
E 140

2．Which of these is equal to $(0.1+0.2+0.3-0.4) \div 0.5$ ？
A 0.01
B 0.02
C 0.04
D 0.1
E 0.4

3．Sam has eaten three－quarters of the grapes．
What is the ratio of the number of grapes that remain to the number Sam has eaten？
A 1：3
B 1：4
C 1：5
D 1：6
E 1：7

4．Which of the following five shapes can be cut into four pieces by a single straight cut？
A

B

C

D

E


5．On Aoife＇s 16th birthday，Buster was three times her age．On Aoife＇s 21st birthday，how old was Buster？
A 32
B 48
C 53
D 63
E 64

6．Which of these is closest to 7 ？
A 7.09
В 6.918
C 7.17
D 6.7
E 7.085

7．The shortest street in the UK，Ebenezer Place in Wick，is 2.06 m long．The Trans－Canada Highway， one of the world＇s longest roads，is approximately 7821 km in length．
Approximately，how many times longer than the street is the highway？
A 4000000
B 400000
C 40000
D 4000
E 400

8．The diagram shows a kite $P G R F$ inside rhombus $P Q R S$ ．
Angle $P G Q=35^{\circ}$ ，angle $P F S=35^{\circ}$ ，
angle $P Q G=120^{\circ}$ and angle $P S F=120^{\circ}$ ．
What is the size of angle $F P G$ ？
A $10^{\circ}$
B $12^{\circ}$
C $15^{\circ}$
D $18^{\circ}$
E $20^{\circ}$


9．What is $50 \%$ of 18.3 plus $18.3 \%$ of 50 ？
A 9.15
B 18.3
C 27.15
D 59.15
E 68.3

10．What is the last digit of the smallest positive integer whose digits add to 2019 ？
A 1
B 4
C 6
D 8
E 9

11．Two players $X$ and $Y$ take alternate turns in a game，starting with the diagram alongside．
At each turn，one player writes one of 1,2 or 3 in an empty circle，so that no two circles connected by an edge contain the same number．A player loses when they cannot go．In each of the five diagrams below it is $Y$＇s turn．
In which of the diagrams can $Y$＇s move ensure that $X$ loses the game？

A

B

C

D

E


12．Jamal writes down a sequence of six integers．The rule he uses is，＂after the first three terms，each term is the sum of the three previous terms．＂His sequence is $-,-,-8,13,25$. What is his first term？
A 0
B 1
C 2
D 3
E 4

13．In how many different ways can you spell out JMC，starting at the centre，and moving to the next letter in a neighbouring square－horizontally，vertically，or diagonally－each time？
A 8
B 16
C 24
D 25
E 32

| C | C | C | C | C |
| :---: | :---: | :---: | :---: | :---: |
| C | M | M | M | C |
| C | M | J | M | C |
| C | M | M | M | C |
| C | C | C | C | C |

14．Each edge in the diagram has length 1 cm ．
What is the length of the longest path that can be followed along the edges， starting at a vertex and without revisiting any vertex？
A 7 cm
B 8 cm
C 9 cm
D 10 cm
E 11 cm


15．All four L－shapes shown in the diagram are to be placed in the 4 by 4 grid so that all sixteen cells are covered and there is no overlap．Each piece can be rotated or reflected before being placed and the black dot is visible from both sides．

How many of the 16 cells of the grid could contain the black dot？
A 4
B 7
C 8
D 12
E 16


16．Tamsin writes down three two－digit integers．One is square，one is prime and one is triangular． She uses the digits $3,4,5,6,7$ and 8 exactly once each．
Which prime does she write？
A 37
B 43
C 53
D 73
E 83

17．A rectangle is three times as long as it is high．The area of a square is twelve times the area of the rectangle．What is the ratio of the perimeter of the square to the perimeter of the rectangle？
A 12：1
B 6：1
C 4：1
D 3：1
E 2：1

18. What fraction of the integers from 1 to 8000 inclusive are cubes?
A $\frac{1}{1000}$
B $\frac{1}{800}$
C $\frac{1}{400}$
D $\frac{1}{200}$
E $\frac{1}{100}$
19. Each row, each column and each of the bold 2 by 3 rectangles in the grid has to contain each of the numbers $1,2,3,4,5$ and 6 (one number in each cell). What number should go in the cell marked $x$ ?
A 1
B 2
C 3
D 4
E 6

|  |  |  |  | $x$ | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  | 6 |  |
|  |  | 1 | 2 |  |  |
|  |  | 3 | 4 |  |  |
|  |  | 4 |  | 3 |  |
| 2 |  |  |  |  | 1 |

20. Emily writes down the largest two-digit prime such that each of its digits is prime.

Krish writes down the smallest two-digit prime such that each of its digits is prime.
Kirsten subtracts Krish's number from Emily's number.
What answer does Kirsten obtain?
A 14
B 20
C 36
D 45
E 50
21. The diagram shows a regular hexagon $P Q R S T U$, a square $P Q V W$ and an equilateral triangle $V X W$.
What is the size of angle $X V R$ ?
A $120^{\circ}$
B $125^{\circ}$
C $130^{\circ}$
D $135^{\circ}$
E $140^{\circ}$

22. In the multiplication shown alongside, $T, R, A$ and $P$ are all different digits. What is the value of $R$ ?
A 0
B 1
C 5
D 8
E 9
$\times \quad 9$
$P A R T$
23. The diagram shows two squares $J K L M$ and $P Q R S$.

The length of $J K$ is 6 cm and that of $P Q$ is 4 cm .
The vertex $K$ is the midpoint of side $R S$.
What is the area of the shaded region?
A $22 \mathrm{~cm}^{2}$
B $24 \mathrm{~cm}^{2}$
C $26 \mathrm{~cm}^{2}$
D $28 \mathrm{~cm}^{2}$
E $30 \mathrm{~cm}^{2}$
24. The diagram shows a regular heptagon.

Which of these expressions is equal to $p+q+r+s+t$ ?
A $180+q$
B $180+2 q$
C $360-q$
D 360
E $360+q$

25. The diagram shows the first fifteen positive integers arranged in a 'triangle'.

These numbers are to be rearranged so that the five integers along each 'edge' of the triangle have the same sum, unlike the example shown. When this is done, what is the greatest possible such sum?
A 38
B 42
C 48
D 52
E 54


1．What is the value of $(222+22) \div 2$ ？
A 111
B 112
C 122
D 133
E 233

2．A train carriage has 80 seats．On my journey I noticed that all the seats in my carriage were taken and 7 people were standing．
At Banbury， 9 people left the carriage， 28 people entered it and all seats were taken．
How many people now had no seat？
A 0
B 7
C 16
D 26
E 35

3．The diagram shows an equilateral triangle，a square，and one diagonal of the square．
What is the value of $x$ ？
A 105
B 110
C 115
D 120
E 135


4．The perimeter of the regular decagon $P$ is 8 times the perimeter of the regular octagon $Q$ ． Each edge of the regular octagon $Q$ is 10 cm long．
How long is each edge of the regular decagon $P$ ？
A 8 cm
B 10 cm
C 40 cm
D 60 cm
E 64 cm

5．My train left Southampton at 06：15 and arrived in Birmingham at 08：48 later that morning． How many minutes did the journey take？
A 153
B 193
C 233
D 1463
E 1501

6．The diagram shows a partially completed magic square，in which all rows，all columns and both main diagonals have the same total．
What is the value of $x+y$ ？
A 10
B 11
C 12
D 13
E 14

| 4 |  |  |
| :--- | :--- | :--- |
|  | 7 | $y$ |
| 6 | 5 | $x$ |

7．How many integers are greater than $20+18$ and also less than $20 \times 18$ ？
A 320
B 321
C 322
D 323
E 324

8．Gill scored a goal half way through the second quarter of a＇teachers versus pupils＇netball match．At that point，what fraction of the whole match remained to be played？
A $\frac{1}{4}$
B $\frac{3}{8}$
C $\frac{1}{2}$
D $\frac{5}{8}$
E $\frac{3}{4}$

9．The approximate cost of restoring the Flying Scotsman was $£ 4$ million．This was about 500 times the cost of building the steam engine in 1923.
Roughly what did the engine cost to build？
A £800
B £2000
C $£ 8000$
D £20 000
E £80 000

10．Adding four of the five fractions $\frac{1}{2}, \frac{1}{3}, \frac{1}{6}, \frac{1}{9}$ and $\frac{1}{18}$ gives a total of 1 ．
Which of the fractions is not used？
A $\frac{1}{2}$
B $\frac{1}{3}$
C $\frac{1}{6}$
D $\frac{1}{9}$
E $\frac{1}{18}$

11．How many digits are there in the correct answer to the calculation $123123123123 \div 123$ ？
A 4
B 6
C 8
D 10
E 12

12．The diagram shows a quadrilateral $P Q R S$ in which $P Q$ and $Q R$ have the same length．Also $P R$ bisects $\angle S P Q$ ，the ratio of $\angle S P R$ to $\angle P R S$ is $2: 3$ and $\angle P S R=110^{\circ}$ ．
How large is angle $P Q R$ ？
A $124^{\circ}$
B $120^{\circ}$
C $110^{\circ}$
D $90^{\circ}$
E $28^{\circ}$


13．The diagram shows a shape made from four $3 \mathrm{~cm} \times 3 \mathrm{~cm} \times 3 \mathrm{~cm}$ wooden cubes joined by their edges．
What，in $\mathrm{cm}^{2}$ ，is the surface area of the shape？
A 162
B 180
C 198
D 216
E 234


14．Billy has three times as many llamas as lambs．
Milly has twice as many lambs as llamas．
They have 17 animals in total．
How many of the animals are llamas？
A 5
B 6
C 7
D 8
E 9

15．Beatrix places copies of the L－shape shown on a $4 \times 4$ board so that each L－shape covers exactly three cells of the board．She is allowed to turn around or turn over an L－shape．
What is the largest number of L－shapes she can place on the board without overlaps？

A 2
B 3
C 4
D 5
E 6

16．How many pairs of digits $(p, q)$ are there so that the five－digit integer＇$p 869 q$＇is a multiple of 15 ？
A 2
B 3
C 4
D 5
E 6

17．The areas of the two rectangles in the diagram are $25 \mathrm{~cm}^{2}$ and $13 \mathrm{~cm}^{2}$ as indicated．
What is the value of $x$ ？
A 3
B 4
C 5
D 6
E 7


18．Between them，the two five－digit integers $M$ and $N$ contain all ten digits from 0 to 9 ． What is the least possible difference between $M$ and $N$ ？
A 123
B 247
C 427
D 472
E 742

19．Which one of these could be folded to make a cube？
A

B

C

D

E


20．A drawer contains ten identical yellow socks，eight identical blue socks and four identical pink socks．
Amrita picks socks from the drawer without looking．
What is the smallest number of socks she must pick to be sure that she has at least two pairs of matching socks？
A 5
B 6
C 8
D 11
E 13
21.
There are —＿vowels in this short sentence．

Which of the following options should replace＂ $\qquad$ - to make the sentence in the box true？
A twelve
B thirteen
C fourteen
D fifteen
E sixteen

22．In the triangles $P Q R$ and $S T U, \angle R P Q=2 \times \angle U S T, \angle P R Q=2 \times \angle S U T$ and $\angle R Q P=\frac{1}{5} \times \angle U T S$ ．
How large is $\angle U T S$ ？
A $90^{\circ}$
B $100^{\circ}$
C $120^{\circ}$
D $150^{\circ}$
E more information needed

23．Ali wants to fill the empty squares so that the number in each square after the fourth from the left is the sum of the

| 2 |  | 0 |  | 1 |  | 8 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | numbers in the four squares to its left．

What number should Ali write in the final square？
A 16
B 8
C 4
D 2
E 1

24．The shapes P and Q are formed from two and three identical rectangles，respectively．Their perimeters are 58 cm and 85 cm respectively．
What is the perimeter of one of the rectangles？

Q

A 30 cm
B 31 cm
C 32 cm
D 33 cm
E 34 cm

25．In the diagram $P Q$ and $Q R$ are sides of a regular $n$－sided polygon，$\angle S P Q=\angle S R Q=80^{\circ}$ ， $\angle P T R=70^{\circ}$ and $P T=S T=R T$ ． What is the value of $n$ ？
A 15
B 18
C 20
D 24
E 30



# Intermediate Challenge 

## www.CasperYC.club

## Last updated: September 22, 2023

## Instructions

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: $\mathbf{6 0}$ minutes.

No answers, or personal details, may be entered after the allowed time is over.
3. The use of blank or lined paper for rough working is allowed; squared paper, calculators and measuring instruments are forbidden.
4. Use a B or an HB non-propelling pencil. Mark at most one of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.
5. Do not expect to finish the whole paper in the time allowed. The questions in this paper have been arranged in approximate order of difficulty with the harder questions towards the end. You are not expected to complete all the questions during the time. You should bear this in mind when deciding which questions to tackle.
6. Scoring rules:

5 marks are awarded for each correct answer to Questions 1-15;
6 marks are awarded for each correct answer to Questions 16-25;
Each incorrect answer to Questions 16-20 loses 1 mark;
Each incorrect answer to Questions 21-25 loses 2 marks.
7. Your Answer Sheet will be read by a machine. Do not write or doodle on the sheet except to mark your chosen options. The machine will read all black pencil markings even if they are in the wrong places. If you mark the sheet in the wrong place, or leave bits of eraser stuck to the page, the machine will interpret the mark in its own way.
8. The questions on this paper are designed to challenge you to think, not to guess. You will gain more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. This paper is about solving interesting problems, not about lucky guessing.

1．Which of these numbers is neither a multiple of 3 ，nor a multiple of 4 ？
A 16
B 21
C 28
D 34
E 45

2．What is the area of this triangle？

A $6 \mathrm{~cm}^{2}$
B $7.5 \mathrm{~cm}^{2}$
C $8 \mathrm{~cm}^{2}$
D $10 \mathrm{~cm}^{2}$
E $12 \mathrm{~cm}^{2}$

3．What is the value of $1-(2-(3-4-(5-6)))$ ？
A－2
B -1
C 0
D 1
E 2

4．The diagram shows a square，its two diagonals and two line segments，each of which connects two midpoints of sides of the square．

What fraction of the area of the square is shaded？

A $\frac{1}{8}$
B $\frac{1}{10}$
C $\frac{1}{12}$
D $\frac{1}{16}$
E $\frac{1}{24}$

5．We know that $1+2+3+4=10$ ．It is also true that $1^{3}+2^{3}+3^{3}+4^{3}=10^{n}$ for some integer $n$ ． What is this integer？
A 1
B 2
C 3
D 4
E 5

6．To draw a 3 by 3 square grid you need 8 straight lines，as shown．
How many straight lines do you need to draw a $n$ by $n$ square grid？
A $n+5$
B $3 n-1$
C $n^{2}-1$
D $4(n-1)$
E $2(n+1)$


7．What is $0.015 \%$ of 60 million？
A 900
B 9000
C 90000
D 900000
E 9000000

8．$\sqrt{\sqrt{x}}=3$ ．What is the value of $x$ ？
A $\sqrt{\sqrt{3}}$
B $\sqrt{3}$
C 9
D 12
E 81

9．Merryn wrote down the numbers $2,0,2,3$ and one further number．
What was the median of her five numbers？
A 0
B 2
C 2.5
D 3
E more information required
10.

Across

## Down

1．A power of 6
1．A power of 5
2．A power of 4


Eight of the digits from 0 to 9 inclusive are used to fill the cells of the crossnumber． What is the sum of the two digits which are not used？
A 12
B 13
C 14
D 15


11．Jill was given a large jar of jam．She gave one sixth of the jam to Jan．Jill then gave one thirteenth of the remaining jam to Jas．Jill was left with 1 kg of jam．
What was the weight，in kg ，of the jam in Jill＇s jar at the start？
A 1.2
B 1.3
C 1.4
D 1.6
E 1.9

12．In the diagram，$P Q R S$ is a square，$P S T$ is an equilateral triangle and $S R U V W$ is a regular pentagon．

What is the size of angle WTS？
A $35^{\circ}$
B $36^{\circ}$
C $37^{\circ}$
D $38^{\circ}$
E $39^{\circ}$


13．The mean of $p$ and $q$ is 13 ；the mean of $q$ and $r$ is 16 ；the mean of $r$ and $p$ is 7 ． What is the mean of $p, q$ and $r$ ？
A 12
B 13
C 14
D 15
E 16

14．A regular octagon $P Q R S T U V W$ has sides of length 2 cm ．When I shade the rectangles $P Q T U$ and $R S V W$ ，four small triangles inside the octagon remain unshaded．What is the total area，in $\mathrm{cm}^{2}$ ，of these four triangles？
A 1
B 2
C 4
D 6
E 8

15．How many of the following polygons could exist？
A triangle with all three sides the same length，but three different interior angles．
A quadrilateral with all four sides the same length，but four different interior angles．
A pentagon with all five sides the same length，but five different interior angles．
A only the pentagon
B only the quadrilateral
C the quadrilateral and the pentagon
D all three
E none of them

16．The sum of the lengths of the three sides of a right－angled triangle is 16 cm ．The sum of the squares of the lengths of the three sides of the triangle is $98 \mathrm{~cm}^{2}$ ．
What is the area，in $\mathrm{cm}^{2}$ ，of the triangle？
A 8
B 10
C 12
D 14
E 16

17．A 3 by 2 rectangle is split into four congruent right－angled triangles，as shown in the left－hand diagram．
Those four triangles are rearranged to form a rhombus，as shown in the right－hand diagram．
What is the ratio of the perimeter of the rectangle to the perimeter of the rhombus？

A 3：2
B 2：1
C 1：1
D 1：2
E 2：3

18．How many squares are exactly four greater than a prime？
A 0
B 1
C 2
D 3
E 4

19．What is the positive difference between the numerator and the denominator when the expression shown is written as a single fraction in its simplest form？
A $2 n+2$
B $n+2$
C $n$
D 2
E 1
$\frac{n}{n+1-\frac{n+2}{n+3}}$
20．I roll two standard six－sided fair dice．At least one of the scores obtained on the dice is 3．What is the probability that both of the scores on the dice are 3 ？
A $\frac{1}{12}$
B $\frac{1}{11}$
C $\frac{1}{6}$
D $\frac{1}{3}$
E $\frac{1}{4}$

21．A semicircle of radius 3 units is drawn on one edge of a right－angled triangle， and a semicircle of radius 4 units is drawn on another edge．The semicircles intersect on the hypotenuse of the triangle，as shown．
What is the shaded area，in square units？

A $\frac{25 \pi}{2}-24$
B 12
C $\frac{25 \pi}{2}-6$
D $25 \pi-24$
E 24

22．The numbers $x$ and $y$ satisfy both of the equations

$$
23 x+977 y=2023 \quad \text { and } \quad 977 x+23 y=2977
$$

What is the value of $x^{2}-y^{2}$ ？
A 1
B 2
C 3
D 4
E 5

23．It is possible to choose，in two different ways，six different integers from 1 to 9 inclusive such that their product is a square．Let the two squares so obtained be $p^{2}$ and $q^{2}$ ，where $p$ and $q$ are both positive． What is the value of $p+q$ ？
A 72
B 84
C 96
D 108
E 120

24．A rectangle $P Q R S$ has side－lengths $a$ and $b$ ，with $a<b$ ．The rectangle PTUV has side－lengths $c$ and $d$ ，with $c<d$ ．Also，$a<d$ and $c<b$ ，as shown．The sides $R S$ and $T U$ cross at $X$ ． Which of these conditions guarantees that $Q, X$ and $V$ lie on a straight line？
A $\frac{a}{b}+\frac{c}{d}=1$
B $\frac{a}{c}+\frac{b}{d}=1$
C $\frac{a}{d}+\frac{c}{b}=1$
D $\frac{a}{c}+\frac{d}{b}=1$
E $\frac{c}{a}+\frac{b}{d}=1$


25．The diagram shows two unshaded circles which touch each other and also touch a larger circle．Chord $P Q$ of the larger circle is a tangent to both unshaded circles．The length of $P Q$ is 6 units．
What is the area，in square units，of the shaded region？

A $3 \pi$
B $\frac{7 \pi}{2}$
C $4 \pi$
D $\frac{9 \pi}{2}$
E $5 \pi$


1. How many hours is 6 minutes?
A 0.06
B 0.1
C 0.6
D 10
E 360
2. My recipe for apple crumble uses 100 g of flour, 50 g of butter and 50 g of sugar to make the crumble topping. When my family come for a meal, I have to use two and a half times each amount to provide enough crumble. In total, how much crumble topping do I then make?
A 0.5 kg
B 2 kg
C 2.5 kg
D 5 kg
E 50 kg
3. In the Caribbean, loggerhead turtles lay three million eggs in twenty thousand nests. On average, how many eggs are in each nest?
A 15
B 150
C 1500
D 15000
E 150000
4. Workers digging a tunnel for an underground railway complete 5 metres of tunnel on a typical day. Working every day, how long will it take them to dig a tunnel of length 2 kilometres?
A three months
B six months
$C$ just under a year
D just over a year
E nearly two years
5. Which of the following has the same value as $10006-8008$ ?
A $10007-8007$
B 100060-80080
C $10000-8002$
D 106-88
E 5003-4004
6. What is $20 \%$ of $3 \frac{3}{4}$ ?
A $\frac{123}{200}$
B $\frac{13}{20}$
C $\frac{7}{10}$
D $\frac{3}{4}$
E $\frac{4}{5}$
7. A function machine does the four operations shown in order.


Iris inputs a positive integer and the output is also a positive integer.
What is the smallest possible number which Iris could have input?
A 9
B 84
C 102
D 120
E 129
8. What is the difference between $40 \%$ of $50 \%$ of 60 and $50 \%$ of $60 \%$ of 70 ?
A 9
B 8
C 7
D 6
E 5
9. A number $x$ is greater than 2022. Which is the smallest of the following?
A $\frac{x}{2022}$
B $\frac{2022}{x-1}$
C $\frac{x+1}{2022}$
D $\frac{2022}{x}$
E $\frac{2022}{x+1}$
10. One hundred rectangles are arranged edge-to-edge in a continuation of the pattern shown.


Each rectangle measures 3 cm by 1 cm . What is the perimeter, in cm , of the completed shape?
A 800
B 700
C 602
D 600
E 502
11. The Universal Magazine of Knowledge and Pleasure (Vol. 1, 1747) asked the following question. "What number is that, whose quarter shall be 9 more than the whole?" What is the correct answer?
A 12
B 9
C 8
D -8
E -12


12．The shape shown is made up of three similar right－angled triangles．
The smallest triangle has two sides of side－length 2，as shown．
What is the area of the shape？

A 14
B $12+12 \sqrt{2}$
C 28
D $24+20 \sqrt{2}$
E 56

13．How many sets of three consecutive integers are there in which the sum of the three integers equals their product？
A 0
B 2
C 3
D 4
E 5

14．In a number pyramid，each cell above the bottom row contains the sum of the numbers in the two cells immediately below it．The three numbers on the second row are all equal，and are all integers．Which of these statements must be true？

A The bottom row contains at least one zero
B The third row contains at least one zero
C The top number is a multiple of three
D The top number is a multiple of four E None of the above

15．Reflection in the line $l$ transforms the point with coordinates $(5,3)$ into the point with coordinates $(1,-1)$ ．
What is the equation of the line $l$ ？
A $y=x-2$
B $y=1$
C $x=3$
D $y=2-x$
E $y=4-x$

16．What is half of $4^{2022}$ ？
A $4^{1011}$
B $2^{4044}$
C $4^{2021}$
D $2^{4043}$
E $2^{1011}$

17．The first figure shows four touching circles of radius 1 cm in a horizontal row，held together tightly by an outer band X ．


The second figure shows six touching circles of radius 1 cm ，again held tightly together by a surrounding band Y．
Which of the following statements is true？
A X is 2 cm longer than Y
B X is 1 cm longer than Y
C X and Y are the same length
D Y is 1 cm longer than X
$\mathrm{E} Y$ is 2 cm longer than X

18．Dick Turnip sold his horse，Slack Bess，for $£ 56$ ．The percentage profit he made was numerically the same as the cost，in pounds，of his horse．What was the cost of his horse？
A £36
B $£ 40$
C $£ 45$
D $£ 48$
E $£ 50$

19．A sector of a circle has radius 6 and arc length 10，as shown．
What is the area of the sector？

A 30
B 36
C 40
D 60
E 66

20．Aroon is asked to choose five integers so that the mode is 2 more than the median and the mean is 2 less than the median．What is the largest possible value of the range of Aroon＇s five integers？
A 2
B 5
C 12
D 15

E The largest possible range depends on the integers chosen
21．The diagram shows a shaded semicircle of diameter 4 ，from which a smaller semicircle has been removed．The two semicircles touch at exactly three points． What fraction of the larger semicircle is shaded？

A $\frac{2}{\pi}$
B $\frac{1}{2}$
C $\frac{\sqrt{2}}{3}$
D $\frac{\sqrt{2}}{2}$
E $\frac{3}{4 \pi}$

22．A rectangle with integer side－lengths is divided into four smaller rectangles，as shown．The perimeters of the largest and smallest of these smaller rectangles are 28 cm and 12 cm ．


Which of the following is a possible area of the original rectangle？
A $90 \mathrm{~cm}^{2}$
B $92 \mathrm{~cm}^{2}$
C $94 \mathrm{~cm}^{2}$
D $96 \mathrm{~cm}^{2}$
E $98 \mathrm{~cm}^{2}$

23．Two squares are drawn inside a regular hexagon with side－length 2 ，as shown． What is the area of the overlap of the two squares？

A 2
B $2-\sqrt{3}$
C $4-\sqrt{3}$
D $4-2 \sqrt{3}$
E $8-4 \sqrt{3}$

24．Pete＇s pies all cost an integer number of pounds．A cherry pie costs the same as two apple pies． A blueberry pie costs the same as two damson pies．A cherry pie and two damson pies cost the same as an apple pie and two blueberry pies．Paul buys one of each type of pie．
Which of the following could be the amount he spends？
A £16
B £18
C £20
D $£ 22$
E £24

25．Alvita is planning a garden patio to be made from identical square paving stones laid out in a rectangle measuring $x$ stones by $y$ stones．She finds that when she adds a border of width one stone around the patio，the area of the border is equal to the original area of the patio．
How many possible values for $x$ are there？
A 1
B 2
C 4
D 8
E 16

1．What is the value of $2021-2223+2425$ ？
A 2122
B 2223
C 2324
D 2425
E 2526

2．The day before the day before yesterday was two days after the day before my birthday．
Today is Thursday．On what day was my birthday？
A Sunday
B Monday
C Tuesday
D Wednesday
E Friday

3．What is the value of $2-(-2-2)-(-2-(-2-2))$ ？
A 0
B 2
C 4
D 6
E 8

4．The diagram shows three squares，$P Q R S, T U V W$ and $W X Y Z$ ． Angles $P U V$ and $Q Y X$ are $62^{\circ}$ and $74^{\circ}$ respectively． What is angle $V W X$ ？
A $44^{\circ}$
B $48^{\circ}$
C $60^{\circ}$
D $64^{\circ}$
E $68^{\circ}$


5．April，May and June have 90 sweets between them．May has three－quarters of the number of sweets that June has．April has two－thirds of the number of sweets that May has．
How many sweets does June have？
A 60
B 52
C 48
D 40
E 36

6．Kai has begun to list，in ascending order，the positive integers which are not factors of 240 ． What is the sixth number on Kai＇s list？
A 11
B 13
C 14
D 15
E 17

7．What is the value of $\left(4-\frac{1}{4}\right) \div\left(2-\frac{1}{2}\right)$ ？
A $1 \frac{1}{2}$
B 2
C $2 \frac{1}{2}$
D 3
E $4 \frac{1}{4}$

8．The diagram shows two 10 by 14 rectangles which are edge－to－edge and share a common vertex．It also shows the centre $O$ of one rectangle and the midpoint $M$ of one edge of the other．
What is the distance $O M$ ？

A 12
B 15
C 18
D 21
E 24

9．How many of the following statements are true？
A prime multiplied by a prime is always a prime．
A square multiplied by a square is always a square．
An odd number multiplied by an odd number is always an odd number．
An even number multiplied by an even number is always an even number．
A 0
B 1
C 2
D 3
E 4

10．The prime factor decomposition of 2021 is $43 \times 47$ ．
What is the value of $53 \times 57$ ？
A 2221
B 2521
C 2921
D 3021
E 3031

11．The line with equation $y=2 x+3$ is reflected in the $x$－axis．
Which of the following is the equation of the new line？
A $y=2 x-3$
B $y=-2 x+3$
C $x=2 y+3$
D $y=\frac{1}{2} x+3$
E $y=-2 x-3$

12．Andrew calculates that $40 \%$ of $50 \%$ of $x$ is equal to $20 \%$ of $30 \%$ of $y$ ，where $x \neq 0$ ．
Which of the following is true？
A $y=\frac{2 x}{3}$
B $y=\frac{4 x}{3}$
C $y=2 x$
D $y=\frac{8 x}{3}$
E $y=\frac{10 x}{3}$

13．What is the remainder when $12345 \times 54321$ is divided by 9 ？
A 0
B 1
C 2
D 3
E 4

14．The diagram shows a large square divided into squares of three different sizes． What percentage of the large square is shaded？
A 61\％
B 59\％
C 57\％
D 55\％
E 53\％


15．Patrick drives from $P$ to $Q$ at an average speed of 40 mph ．His drive back from $Q$ to $P$ is at an average speed of 45 mph and takes two minutes less．
How far，in miles，is it from P to Q ？
A 1.5
B 6
C 9
D 12
E 15

16．A semicircle is drawn on each side of a square，as shown．
The square has sides of length $2 \pi$ ．
What is the area of the resulting shape？
A $2 \pi^{2}(\pi+1)$
B $\pi^{2}(\pi+2)$
C $2 \pi^{2}(2 \pi+1)$
D $\pi^{2}(\pi+4)$
E $2 \pi^{2}(\pi+2)$

17．In the rectangle $P Q R S$ ，the side $P Q$ is of length 2 and the side $Q R$ is of length 4 ．Points $T$ and $U$ lie inside the rectangle so that $P Q T$ and $R S U$ are equilateral triangles．
What is the area of the quadrilateral $Q R U T$ ？
A $\frac{6-\sqrt{3}}{2}$
B $\frac{8}{3}$
C $4-2 \sqrt{3}$
D $4-\sqrt{3}$
E 3

18．Which of these is closest in size to 1 ？
A $0 . \dot{9} \dot{5}$
B $1.0 \dot{5}$
C $0 . \dot{9} 6 \dot{0}$
D $1.0 \dot{0} \dot{0}$
E $0.95^{\circ}$

19．The diagram shows two overlapping rectangles，each measuring $p$ by $q$ ．The area of overlap is exactly one－quarter of the total area of the figure． What is the ratio $p: q$ ？
A 5：2
B 4：1
C 3：1
D 2：1
E 3：2


20．Two straight lines have equations $y=p x+4$ and $p y=q x-7$ ，where $p$ and $q$ are constants．
The two lines meet at the point $(3,1)$ ．
What is the value of $q$ ？
A 1
B 2
C 3
D 4
E 5

21．The diagram shows two congruent equilateral triangles whose overlap is a hexagon． The areas of the smaller triangles，which are also equilateral，are $1,1,9,9,16$ and 16 ，as shown．
What is the area of the inner hexagon？
A 68
B 58
C 48
D 38
E 28


22．What is the result when we simplify the expression $\left(1+\frac{1}{x}\right)\left(1-\frac{2}{x+1}\right)\left(1+\frac{2}{x-1}\right)$ ？
A 1
B $\frac{1}{x(x+1)}$
C $\frac{1}{(x+1)(x-1)}$
D $\frac{1}{x(x+1)(x-1)}$
E $\frac{x+1}{x}$

23．The diagram shows a semicircle with centre $O$ and radius 2 and a semicircular arc with diameter $P R$ ．Angle $P O R$ is a right angle． What is the area of the shaded region？

A $\pi-2$
B 2
C $\pi$
D 3
E $2 \pi-2$

24．Sam writes on a white board the positive integers from 1 to 6 inclusive，once each．She then writes $p$ additional fives and $q$ sevens on the board．The mean of all the numbers on the board is then 5．3． What is the smallest possible value of $q$ ？
A 7
B 9
C 11
D 13
E 15

25．Thomas has constant speeds for both running and walking．When a down－escalator is moving，Thomas can run down it in 15 seconds or walk down it in 30 seconds．One day，when the escalator was broken （and stationary），it took Thomas 20 seconds to run down it．
How long，in seconds，would it take Thomas to walk down the broken escalator？
A 30
B 40
C 45
D 50
E 60

1．What is the value of $2-(3-4)-(5-6-7)$ ？
A 11
B 9
C 5
D－5
E－7

2．Which one of these is a multiple of 24 ？
A 200
B 300
C 400
D 500
E 600

3．What is the difference between $25 \%$ of $£ 37$ and $25 \%$ of $£ 17$ ？
A $£ 4.25$
B £5
C £6
D $£ 7.50$
E $£ 9.25$

4．What fraction of this diagram is shaded？
A $\frac{13}{32}$
B $\frac{1}{2}$
C $\frac{9}{16}$
D $\frac{5}{8}$
E $\frac{13}{16}$


5．Four of the following coordinate pairs are the corners of a square． Which is the odd one out？
A $(4,1)$
B $(2,4)$
C $(5,6)$
D $(3,5)$
$\mathrm{E}(7,3)$

6．Which of the following has the largest value？
A $2^{6}$
B $3^{5}$
C $4^{4}$
D $5^{3}$
E $6^{2}$

7．Kartik wants to shade three of the squares in this grid blue and Lucy wants to shade the remaining two squares red．There are ten possible finished
 grids．
In how many of the finished grids are Lucy＇s red squares next to each other？
A 3
B 4
C 5
D 6
E 8

8．One of these options gives the value of $17^{2}+19^{2}+23^{2}+29^{2}$ ．Which is it？
A 2004
B 2008
C 2012
D 2016
E 2020

9．Adam＇s house number is in exactly one of the following ranges．Which one？
A 123 to 213
B 132 to 231
C 123 to 312
D 231 to 312
E 312 to 321

10．What is the value of $\frac{2468 \times 2468}{2468+2468}$ ？
A 2
B 1234
C 2468
D 4936
E 6091024

11．I start at square＂ 1 ＂，and have to finish at square＂ 7 ＂，moving at each step to a higher numbered adjacent square．
How many possible routes are there？
A 7
B 9
C 10
D 11
E 13

| 2 | 4 | 6 |
| :--- | :--- | :--- |
| 1 | 3 | 5 |



12．Farmer Fatima rears chickens and goats．Today she returned from market and said，＂I sold 80 animals， and now there are 200 fewer legs on my farm than before！＂
How many goats did she sell？
A 15
B 20
C 25
D 30
E 35

13．What is half of $1.6 \times 10^{6}$ ？
A $8 \times 5^{6}$
B $4 \times 10^{6}$
C $8 \times 10^{5}$
D $8 \times 10^{2}$
E $1.6 \times 10^{3}$

14．The result of the calculation $9 \times 11 \times 13 \times 15 \times 17$ is the six－digit number＇ $3 n 8185$＇． What is the value of $n$ ？
A 2
B 4
C 6
D 8
E 0

15．Triangle $P Q R$ has been divided into twenty－five congruent right－angled triangles，as shown．The length of $R P$ is 2.4 cm ．
What is the length of $P Q$ ？
A 3 cm
B 3.2 cm
C 3.6 cm
D 4 cm
E 4.8 cm


16．As a decimal，what is the value of $\frac{1}{9}+\frac{1}{11}$ ？
A 0.10
B 0.20
C 0.2020
D 0.202020
E $0 . \dot{2} \dot{0}$

17．The Knave of Hearts stole some tarts．He ate half of them，and half a tart more．The Knave of Diamonds ate half of what was left，and half a tart more．Then the Knave of Clubs ate half of what remained，and half a tart more．This left just one tart for the Knave of Spades． How many tarts did the Knave of Hearts steal？
A 63
B 31
C 19
D 17
E 15

18．The diagram shows an isosceles right－angled triangle which has a hypotenuse of length $y$ ．The interior of the triangle is split up into identical squares and congruent isosceles right－angled triangles．
What is the total shaded area inside the triangle？
A $\frac{y^{2}}{2}$
B $\frac{y^{2}}{4}$
C $\frac{y^{2}}{8}$
D $\frac{y^{2}}{16}$
E $\frac{y^{2}}{32}$


19．The diagram shows two squares and four equal semicircles．The edges of the outer square have length 48 and the inner square joins the midpoints of the edges of the outer square．Each semicircle touches two edges of the outer square，and the diameter of each semicircle lies along an edge of the inner square．

What is the radius of each semicircle？
A 10
B 12
C 14
D 16
E 18

el

20．For any fixed value of $x$ ，which of the following four expressions has the largest value？

$$
(x+1)(x-1) \quad\left(x+\frac{1}{2}\right)\left(x-\frac{1}{2}\right) \quad\left(x+\frac{1}{3}\right)\left(x-\frac{1}{3}\right) \quad\left(x+\frac{1}{4}\right)\left(x-\frac{1}{4}\right)
$$

A $(x+1)(x-1)$
B $\left(x+\frac{1}{2}\right)\left(x-\frac{1}{2}\right)$
C $\left(x+\frac{1}{3}\right)\left(x-\frac{1}{3}\right)$
D $\left(x+\frac{1}{4}\right)\left(x-\frac{1}{4}\right)$

E it depends on the value of $x$

21．The diagram shows four semicircles，one with radius 2 cm ，touching the other three，which have radius 1 cm ．


What is the total area，in $\mathrm{cm}^{2}$ ，of the shaded regions？
A 1
B $\pi-2$
C $2 \pi-5$
D $\frac{3}{2}$
E $\frac{1}{2} \pi$

22．The diagram shows a regular pentagon and an irregular quadrilateral．
What is the sum of the three marked angles？
A $72^{\circ}$
B $90^{\circ}$
C $108^{\circ}$
D $126^{\circ}$
E $144^{\circ}$


23．Five congruent triangles，each of which is half a square，are placed together edge to edge in three different ways as shown to form shapes $\mathrm{P}, \mathrm{Q}$ and R ．


P


Q


R

Which of the following lists gives the shapes in ascending order of the lengths of their perimeters？
A P，Q，R
B $\mathrm{Q}, \mathrm{P}, \mathrm{R}$
C $\mathrm{R}, \mathrm{Q}, \mathrm{P}$
D R，P，Q
E P，R，Q

24．The positive integers $m$ and $n$ are such that $10 \times 2^{m}=2^{n}+2^{n+2}$ ．
What is the difference between $m$ and $n$ ？
A 1
B 2
C 3
D 4
E 5

25．The diagram shows six points $P, Q, R, S, T$ and $U$ equally spaced around a circle of radius 2 cm ．The inner circle has radius 1 cm ．The shaded region has three lines of symmetry．
Which of the following gives the area，in $\mathrm{cm}^{2}$ ，of the shaded region？
A $2 \pi+3$
B $3 \pi+2$
C $\frac{4 \pi+3}{2}$
D $3(\pi+2)$
E $4 \pi+3$



## Senior Challenge

## www.CasperYC.club

## Last updated: September 22, 2023

## InSTRUCTIONS

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: $\mathbf{9 0}$ minutes.

No answers, or personal details, may be entered after the allowed time is over.
3. The use of blank paper for rough working is allowed; squared paper, calculators and measuring instruments are forbidden.
4. Use a B or an HB non-propelling pencil. Mark A, B, C, D, E on the Answer Sheet for each question. Mark only one option, boldly, within the box.
5. Your Answer Sheet will be read by a machine. Do not write or doodle on the sheet except to mark your chosen options. The machine will read all markings, including bits of eraser stuck to the page, and interpret the mark in its own way.
6. Do not expect to finish the whole paper in the time allowed. The questions in this paper have been arranged in approximate order of difficulty with the harder questions towards the end. You are not expected to complete all the questions during the time. You should bear this in mind when deciding which questions to tackle.
7. Scoring rules: All candidates start with 25 marks; 0 marks are awarded for each question left unanswered; 4 marks are awarded for each correct answer; 1 mark is deducted for each incorrect answer (to discourage guessing).
8. The questions on this paper are designed to challenge you to think, not to guess. You will gain more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. This paper is about solving interesting problems, not about lucky guessing.

1．When the expression $\frac{\left(2^{2}-1\right) \times\left(3^{2}-1\right) \times\left(4^{2}-1\right) \times\left(5^{2}-1\right)}{(2 \times 3) \times(3 \times 4) \times(4 \times 5) \times(5 \times 6)}$ is simplified，which of the following is obtained？
A $\frac{1}{2}$
B $\frac{1}{3}$
C $\frac{1}{4}$
D $\frac{1}{5}$
E $\frac{1}{6}$

2．What is the smallest prime which is the sum of five different primes？
A 39
B 41
C 43
D 47
E 53

3．The figure shows a regular hexagon．
How many parallelograms are there in the figure？
A 2
B 4
C 6
D 8

E more than 8
4．The diagram shows two symmetrically placed squares with sides of length 2 and 5 ．
What is the ratio of the area of the small square to that of the shaded region？
A 7：24
B 1：3
C 8：25
D 8：21
E 2：5


5．What is the value of $\frac{1}{1.01}+\frac{1}{1.1}+\frac{1}{1}+\frac{1}{11}+\frac{1}{101}$ ？
A 2.9
B 2.99
C 3
D 3.01
E 3.1

6．What is the value of $\frac{4^{800}}{8^{400}}$ ？
A $\frac{1}{2^{400}}$
B $\frac{1}{2^{200}}$
C 1
D $2^{200}$
E $2^{400}$

7．In 2021 ，a first class postage stamp cost 85 p and a second class postage stamp cost 66 p．In order to spend an exact number of pounds and to buy at least one of each type，what is the smallest total number of stamps that should be purchased？
A 10
B 8
C 7
D 5
E 2

8．In the diagram，the outer hexagon is regular and has an area of 216.
What is the shaded area？
A 108
B 96
C 90
D 84
E 72


9．A light－nanosecond is the distance that a photon can travel at the speed of light in one billionth of a second．The speed of light is $3 \times 10^{8} \mathrm{~ms}^{-1}$ ．
How far is a light－nanosecond？
A 3 cm
B 30 cm
C 3 m
D 30 m
E 300 m

10．What is the value of $x$ in the equation $\frac{1+2 x+3 x^{2}}{3+2 x+x^{2}}=3$ ？
A－5
B－4
C -3
D－2
E－1

11．In the number triangle shown，each disc is to be filled with a positive integer． Each disc in the top or middle row contains the number which is the product of the two numbers immediately below．
What is the value of $n$ ？
A 1
B 2
C 3
D 6
E 33


12．What is the sum of the digits of the integer which is equal to $6666666^{2}-3333333^{2}$ ？
A 27
B 36
C 45
D 54
E 63

13．Three rugs have a combined area of $90 \mathrm{~m}^{2}$ ．When they are laid down to cover completely a floor of area $60 \mathrm{~m}^{2}$ ，the area which is covered by exactly two layers of rug is $12 \mathrm{~m}^{2}$ ．
What is the area of floor covered by exactly three layers of rug？
A $2 \mathrm{~m}^{2}$
B $6 \mathrm{~m}^{2}$
C $9 \mathrm{~m}^{2}$
D $10 \mathrm{~m}^{2}$
E $12 \mathrm{~m}^{2}$

14．The diagram shows a square，$K L M N$ ．A second square $P Q R S$ is drawn inside it，as shown in the diagram，where $P$ divides the side $K L$ in the ratio 1：2．Similarly，a third square $T U V W$ is drawn inside $P Q R S$ with $T$ dividing $P Q$ in the ratio $1: 2$ ．
What fraction of the area of $K L M N$ is shaded？
A $\frac{25}{81}$
B $\frac{16}{49}$
C $\frac{4}{9}$
D $\frac{40}{81}$
E $\frac{2}{3}$


15．The hare and the tortoise had a race over 100 m ，in which both maintained constant speeds．When the hare reached the finish line，it was 75 m in front of the tortoise．The hare immediately turned around and ran back towards the start line．

How far from the finish line did the hare and the tortoise meet？
A 54
B 60
C 64
D $66 \frac{2}{3}$
E 72

16．Which diagram could be a sketch of the curve $\sqrt{x}+\sqrt{y}=1$ ？
A

B

C

D

E


17．The shape shown is made by removing four equilateral triangles with side－length 1 from a regular octagon with side－length 1 ．

What is the area of the shape？
A $2-2 \sqrt{2}+\sqrt{3}$
B $2+2 \sqrt{2}-\sqrt{3}$
C $2+2 \sqrt{2}+\sqrt{3}$
D $3-2 \sqrt{2}-\sqrt{3}$
E $2-2 \sqrt{2}-\sqrt{3}$

18．The numbers $x$ and $y$ are such that $3^{x}+3^{y+1}=5 \sqrt{3}$ and $3^{x+1}+3^{y}=3 \sqrt{3}$ ．
What is the value of $3^{x}+3^{y}$ ？
A $\sqrt{3}$
B $2 \sqrt{3}$
C $3 \sqrt{3}$
D $4 \sqrt{3}$


19．How many pairs of real numbers $(x, y)$ satisfy the simultaneous equations $x^{2}-y=2022$ and $y^{2}-x=2022 ?$
A infinitely many
B 1
C 2
D 3
E 4

20．A square is inscribed inside a quadrant of a circle．The circle has radius 10. What is the area of the square？
A $25 \sqrt{2}$
B 36
C $12 \pi$
D 40
E $30 \sqrt{2}$


21．The perimeter of a logo is created from two vertical straight edges，two small semicircles with horizontal diameters and two large semicircles．Both of the straight edges and the diameters of the small semicircles have length 2 ．The logo has rotational symmetry as shown．
What is the shaded area？
A 4
B $4-\pi$
C 8
D $4+\pi$
E 12

22．How many pairs of integers $(x, y)$ satisfy the equation $\sqrt{x-\sqrt{x+23}}=2 \sqrt{2}-y$ ？
A 0
B 1
C 4
D 8
E infinitely many

23．Three squares $G Q O P, H J N O$ and $R K M N$ have vertices which sit on the sides of triangle FIL as shown．The squares have areas of 10,90 and 40 respectively．
What is the area of triangle FIL？
A 220.5
B $\frac{21}{5} \sqrt{10}$
C 252
D $\frac{21}{2} \sqrt{10}$
E 441

24．The numbers $x, y, p$ and $q$ are all integers．$x$ and $y$ are variable and $p$ and $q$ are constant and positive． The four integers are related by the equation $x y=p x+q y$ ． When $y$ takes its maximum possible value，which expression is equal to $y-x$ ？
A $p q-1$
B $(p-1)(q-1)$
C $(p+1)(q-1)$
D $(p-1)(q+1)$
$\mathrm{E}(p+1)(q+1)$

25．A drinks carton is formed by arranging four congruent triangles as shown．$Q P=R S=4 \mathrm{~cm}$ and $P R=P S=Q R=Q S=10 \mathrm{~cm}$ ．
What is the volume，in $\mathrm{cm}^{3}$ ，of the carton？
A $\frac{16}{3} \sqrt{23}$
B $\frac{4}{3} \sqrt{2}$
C $\frac{128}{25} \sqrt{6}$
D $\frac{13}{2} \sqrt{23}$
E $\frac{8}{3} \sqrt{6}$



1．Cicely had her $21^{\text {st }}$ birthday in 1939.
When did she have her $100^{\text {th }}$ birthday？
A 2020
B 2019
C 2018
D 2010
E 2008

2．The sequence，formed from the sequence of primes by rounding each to the nearest ten，begins 0,0 ， $10,10,10,10,20,20,20,30, \ldots$ ．

When continued，how many terms in this sequence are equal to 40 ？
A 1
B 2
C 3
D 4
E 5

3．The diagram shows two congruent regular pentagons and a triangle． The angles marked $x^{\circ}$ are equal．

What is the value of $x$ ？
A 24
B 30
C 36
D 40
E 45


4．The positive integer $k$ is a solution of the equation $(k \div 12) \div(15 \div k)=20$ ．
What is the sum of the digits of $k$ ？
A 15
B 12
C 9
D 6
E 3

5．The sum of four consecutive primes is itself prime．
What is the largest of the four primes？
A 37
B 29
C 19
D 13
E 7

6．Three points，$P, Q$ and $R$ are placed on the circumference of a circle with centre $O$ ．The arc lengths $P Q, Q R$ and $R P$ are in the ratio $1: 2: 3$ ．

In what ratio are the areas of the sectors $P O Q, Q O R$ and $R O P$ ？
A 1：1：1
B 1：2：3
C $1: \pi: \pi^{2}$
D 1：4：9
E 1：8：27

7．Which of these numbers is the largest？
A $2^{5000}$
B $3^{4000}$
C $4^{3000}$
D $5^{2000}$
E $6^{1000}$

8．What is the area of the region inside the quadrilateral $P Q R S$ ？
A 18
B 24
C 36
D 48
E more information needed


9．Alison has a set of ten fridge magnets showing the integers from 0 to 9 inclusive．
In how many different ways can she split the set into five pairs so that the sum of each pair is a multiple of 5 ？
A 1
B 2
C 3
D 4
E 5



10．In a survey，people were asked to name their favourite fruit pie． The pie chart shows the outcome．The angles shown are exact with no rounding．

What is the smallest number of people who could have been surveyed？
A 45
B 60
C 80
D 90
E 180


11．Alitta claims that if $p$ is an odd prime then $p^{2}-2$ is also an odd prime．
Which of the following values of $p$ is a counterexample to this claim？
A 3
B 5
C 7
D 9
E 11

12．For how many positive integers $N$ is the remainder 6 when 111 is divided by $N$ ？
A 5
B 4
C 3
D 2
E 1

13．Which of these is the mean of the other four？
A $\sqrt{2}$
B $\sqrt{18}$
C $\sqrt{200}$
D $\sqrt{32}$
E $\sqrt{8}$

14．What is the smallest number of rectangles，each measuring 2 cm by 3 cm ，which are needed to fit together without overlap to form a rectangle whose sides are in the ratio $5: 4$ ？
A 10
B 15
C 20
D 30
E 60

15．Three dice，each showing numbers 1 to 6 ，are coloured red，blue and yellow respectively．Each of the dice is rolled once．The total of the numbers rolled is 10 ．In how many different ways can this happen？
A 36
B 30
C 27
D 24
E 21

16．An array of 25 equally spaced dots is drawn in a square grid as shown．Point $O$ is in the bottom left corner．Linda wants to draw a straight line through the diagram which passes through $O$ and exactly one other point．

How many such lines can Linda draw？
A 4
B 6
C 8
D 12
E 24
$O^{\bullet}$

17．A circle of radius $r$ and a right－angled isosceles triangle are drawn such that one of the shorter sides of the triangle is a diameter of the circle． What is the shaded area？
A $\sqrt{2} r$
B $r^{2}$
C $2 \pi r$
D $\frac{\pi r^{2}}{4}$
E $(\sqrt{2}-1) \pi r^{2}$



18．The number 840 can be written as $\frac{p!}{q!}$ ，where $p$ and $q$ are positive integers less than 10 ．
What is the value of $p+q$ ？
Note that，$n!=1 \times 2 \times 3 \times \cdots \times(n-1) \times n$ ．
A 8
B 9
C 10
D 12
E 15

19．The diagram shows two overlapping triangles：triangle $F G H$ with interior angles $60^{\circ}, 30^{\circ}$ and $90^{\circ}$ and triangle $E G H$ which is a right－angled isosceles triangle．

What is the ratio of the area of triangle $I F G$ to the area of triangle IEH？

A 1： 1
B $1: \sqrt{2}$
C $1: \sqrt{3}$
D 1：2
E 1：3

20．Laura and Dina have a running race．Laura runs at constant speed and Dina runs $n$ times as fast where $n>1$ ．Laura starts $s \mathrm{~m}$ in front of Dina．

What distance，in metres，does Dina run before she overtakes Laura？
A $\frac{n s}{n-1}$
B $n s$
C $\frac{s}{n-1}$
D $\frac{n s}{n+1}$
E $\frac{s}{n}$

21．The numbers $m$ and $k$ satisfy the equations $2^{m}+2^{k}=p$ and $2^{m}-2^{k}=q$ ．
What is the value of $2^{m+k}$ in terms of $p$ and $q$ ？
A $\frac{p^{2}-q^{2}}{4}$
B $\frac{p q}{2}$
C $p+q$
D $\frac{(p-q)^{2}}{4}$
E $\frac{p+q}{p-q}$

22．A triangle with interior angles $60^{\circ}, 45^{\circ}$ and $75^{\circ}$ is inscribed in a circle of radius 2 ．
What is the area of the triangle？
A $2 \sqrt{3}$
B 4
C $6+\sqrt{3}$
D $6 \sqrt{3}$
E $3+\sqrt{3}$

23．Let $x$ be a real number．What is the minimum value of $\left(x^{2}-4 x+3\right)\left(x^{2}+4 x+3\right)$ ？
A -16
B－9
C 0
D 9
E 16

24．Saba，Rayan and Derin are cooperating to complete a task．They each work at a constant rate independent of whoever else is working on the task．When all three work together，it takes 5 minutes to complete the task．When Saba is working with Derin，the task takes 7 minutes to complete．When Rayan is working with Derin，the task takes 15 minutes to complete．

How many minutes does it take for Derin to complete the task on his own？
A 21
B 28
C 35
D 48
E 105

25．Five line segments of length $2,2,2,1$ and 3 connect two corners of a square as shown in the diagram．

What is the shaded area？
A 8
B 9
C 10
D 11
E 12


1．What is the value of $\frac{2020}{20 \times 20}$ ？
A 10.1
B 5.5
C 5.1
D 5.05
E 0.55

2．What is the remainder when $1234 \times 5678$ is divided by 5 ？
A 0
B 1
C 2
D 3
E 4

3．A shape is made from five unit cubes，as shown．
What is the surface area of the shape？
A 22
B 24
C 26
D 28
E 30


4．The numbers $p, q, r$ and $s$ satisfy the equations $p=2, p \times q=20, p \times q \times r=202$ and $p \times q \times r \times s=2020$ ．
What is the value of $p+q+r+s$ ？
A 32
B 32.1
C 33
D 33.1
E 34

5．What is $\sqrt{123454321}$ ？
A 1111111
B 111111
C 11111
D 1111
E 111

6．There are fewer than 30 students in the A－level mathematics class．One half of them play the piano， one quarter play hockey and one seventh are in the school play．
How many of the students play hockey？
A 3
B 4
C 5
D 6
E 7

7．Official UK accident statistics showed that there were 225 accidents involving teapots in one year． However，in the following year there were 47 such accidents．

What was the approximate percentage reduction in recorded accidents involving teapots from the first year to the second？
A $50 \%$
B $60 \%$
C $70 \%$
D $80 \%$
E 90\％

8．What is the largest prime factor of $106^{2}-15^{2}$ ？
A 3
B 7
C 11
D 13
E 17

9．In 2018，a racing driver was allowed to use the Drag Reduction System provided that the car was within 1 second of the car ahead．Suppose that two cars were 1 second apart，each travelling at 180 $\mathrm{km} / \mathrm{h}$（in the same direction！）．
How many metres apart were they？
A 100
B 50
C 10
D 5
E 1

10．Six friends Pat，Qasim，Roman，Sam，Tara and Uma，stand in a line for a photograph．There are three people standing between Pat and Qasim，two between Qasim and Roman and one between Roman and Sam．Sam is not at either end of the line．
How many people are standing between Tara and Uma？
A 4
B 3
C 2
D 1
E 0

11．Two congruent pentagons are each formed by removing a right－angled isosceles triangle from a square of side－length 1 ．The two pentagons are then fitted together as shown．

What is the length of the perimeter of the octagon formed？
A 4
B $4+2 \sqrt{2}$
C 5
D $6-2 \sqrt{2}$
E 6


12．A three－piece suit consists of a jacket，a pair of trousers and a waistcoat．Two jackets and three pairs of trousers cost $£ 380$ ．A pair of trousers costs the same as two waistcoats．

What is the cost of a three－piece suit？
A £150
B £190
C $£ 200$
D £228

E more information needed
13．The number $16!\div 2^{k}$ is an odd integer．Note that $n!=1 \times 2 \times 3 \times \cdots \times(n-1) \times n$ ．
What is the value of $k$ ？
A 9
B 11
C 13
D 15
E 17

14．Diane has five identical blue disks，two identical red disks and one yellow disk． She wants to place them on the grid opposite so that each cell contains exactly one disk．The two red disks are not to be placed in cells that share a common

|  |  |  |  |
| :--- | :--- | :--- | :--- |
|  |  |  |  | edge．

How many different－looking completed grids can she produce？
A 96
B 108
C 144
D 180
E 216

15．The shaded area shown in the diagram consists of the interior of a circle of radius 3 together with the area between the circle and two tangents to the circle．The angle between the tangents at the point where they meet is $60^{\circ}$ ．

What is the shaded area？
A $6 \pi+9 \sqrt{3}$
B $15 \sqrt{3}$
C $9 \pi$
D $9 \pi+4 \sqrt{3}$
E $6 \pi+\frac{9 \sqrt{3}}{4}$


16．Which diagram represents the set of all points $(x, y)$ satisfying $y^{2}-2 y=x^{2}+2 x$ ？
A

B

C

D

E


17．The positive integers $m, n$ and $p$ satisfy the equation $3 m+\frac{3}{n+\frac{1}{p}}=17$ ． What is the value of $p$ ？
A 2
B 3
C 4
D 6
E 9


18．Two circles $C_{1}$ and $C_{2}$ have their centres at the point $(3,4)$ and touch a third circle，$C_{3}$ ．The centre of $C_{3}$ is at the point $(0,0)$ and its radius is 2.
What is the sum of the radii of the two circles $C_{1}$ and $C_{2}$ ？
A 6
B 7
C 8
D 9
E 10

19．The letters $p, q, r, s$ and $t$ represent different positive single－digit numbers such that $p-q=r$ and $r-s=t$ ．

How many different values could $t$ have？
A 6
B 5
C 4
D 3
E 2

20．The real numbers $x$ and $y$ satisfy the equations $4^{y}=\frac{1}{8(\sqrt{2})^{x+2}}$ and $9^{x} \times 3^{y}=3 \sqrt{3}$ ． What is the value of $5^{x+y}$ ？
A $5 \sqrt{5}$
B 5
C $\sqrt{5}$
D $\frac{1}{5}$
E $\frac{1}{\sqrt{5}}$

21．When written out in full，the number $\left(10^{2020}+2020\right)^{2}$ has 4041 digits．
What is the sum of the digits of this 4041－digit number？
A 9
B 17
C 25
D 2048
E 4041

22．A square with perimeter 4 cm can be cut into two congruent right－angled triangles and two congruent trapezia as shown in the first diagram in such a way that the four pieces can be rearranged to form the rectangle shown in the second diagram．


What is the perimeter，in centimetres，of this rectangle？
A $2 \sqrt{5}$
B $4 \sqrt{2}$
C 5
D $4 \sqrt{3}$
E $3 \sqrt{7}$

23．A function $f$ satisfies $y^{3} f(x)=x^{3} f(y)$ and $f(3) \neq 0$ ．What is the value of $\frac{f(20)-f(2)}{f(3)}$ ？
A 6
B 20
C 216
D 296
E 2023

24．In the diagram shown，$M$ is the mid－point of $P Q$ ．The line $P S$ bisects $\angle R P Q$ and intersects $R Q$ at $S$ ．The line $S T$ is parallel to $P R$ and intersects $P Q$ at T．The length of $P Q$ is 12 and the length of $M T$ is 1 ．The angle $S Q T$ is $120^{\circ}$ ．

What is the length of $S Q$ ？

A 2
B 3
C 3.5
D 4
E 5

25．A regular $m$－gon，a regular $n$－gon and a regular $p$－gon share a vertex and pairwise share edges，as shown in the diagram．
What is the largest possible value of $p$ ？
A 6
B 20
C 42
D 50
E 100

1．What is the value of $123^{2}-23^{2}$ ？
A 10000
B 10409
C 12323
D 14600
E 15658

2．What is the value of $(2019-(2000-(10-9)))-(2000-(10-(9-2019)))$ ？
A 4040
B 40
C－ 400
D－4002
E－4020

3．Used in measuring the width of a wire，one mil is equal to one thousandth of an inch．An inch is about 2.5 cm ．

Which of these is approximately equal to one mil？
A $\frac{1}{40} \mathrm{~mm}$
B $\frac{1}{25} \mathrm{~mm}$
C $\frac{1}{4} \mathrm{~mm}$
D 25 mm
E 40 mm

4．For how many positive integer values of $n$ is $n^{2}+2 n$ prime？
A 0
B 1
C 2
D 3
E more than 3

5．Olive Green wishes to colour all the circles in the diagram so that，for each circle， there is exactly one circle of the same colour joined to it．
What is the smallest number of colours that Olive needs to complete this task？
A 1
B 2
C 3
D 4
E 5


6．Each of the factors of 100 is to be placed in a 3 by 3 grid，one per cell，in such a way that the products of the three numbers in each row，column and diagonal are all equal．The positions of the numbers $1,2,50$ and $x$ are shown in the diagram．

What is the value of $x$ ？

| $x$ | 1 | 50 |
| :--- | :--- | :--- |
|  |  |  |
| 2 |  |  |

A 4
B 5
C 10
D 20
E 25

7．Lucy is asked to choose $p, q, r$ and $s$ to be the numbers $1,2,3$ and 4 ，in some order，so as to make the value of $\frac{p}{q}+\frac{r}{s}$ as small as possible．
What is the smallest value Lucy can achieve in this way？
A $\frac{7}{12}$
B $\frac{2}{3}$
C $\frac{3}{4}$
D $\frac{5}{6}$
E $\frac{11}{12}$

8．The number $x$ is the solution to the equation $3^{\left(3^{x}\right)}=333$ ．
Which of the following is true？
A $0<x<1$
B $1<x<2$
C $2<x<3$
D $3<x<4$
E $4<x<5$

9．A square of paper is folded in half four times to obtain a smaller square．Then a corner is removed as shown．

Which of the following could be the paper after it is unfolded？

A $\quad 0$
B

C

D

E



10．Which of the following five values of $n$ is a counterexample to the statement in the box below？
For a positive integer $n$ ，at least one of $6 n-1$ and $6 n+1$ is prime．
A 10
B 19
C 20
D 21
E 30

11．For how many integer values of $k$ is $\sqrt{200-\sqrt{k}}$ also an integer？
A 11
B 13
C 15
D 17
E 20

12．A circle with radius 1 touches the sides of a rhombus，as shown．Each of the smaller angles between the sides of the rhombus is $60^{\circ}$ ． What is the area of the rhombus？
A 6
B 4
C $2 \sqrt{3}$
D $3 \sqrt{3}$
E $\frac{8 \sqrt{3}}{3}$


13．Anish has a number of small congruent square tiles to use in a mosaic．When he forms the tiles into a square of side $n$ ，he has 64 tiles left over．When he tries to form the tiles into a square of side $n+1$ ，he has 25 too few．

How many tiles does Anish have？
A 89
B 1935
C 1980
D 2000
E 2019

14．One of the following is the largest square that is a factor of $10!$ ．Which one？
Note that，$n!=1 \times 2 \times 3 \times \cdots \times(n-1) \times n$ ．
A（4！）$)^{2}$
B $(5!)^{2}$
C $(6!)^{2}$
D $(7!)^{2}$
$\mathrm{E}(8!)^{2}$

15．The highest common factors of all the pairs chosen from the positive integers $Q, R$ and $S$ are three different primes．
What is the smallest possible value of $Q+R+S$ ？
A 41
B 31
C 30
D 21
E 10

16．The numbers $x, y$ and $z$ satisfy the equations $9 x+3 y-5 z=-4$ and $5 x+2 y-2 z=13$ ．
What is the mean of $x, y$ and $z$ ？
A 10
B 11
C 12
D 13
E 14

17．Jeroen writes a list of 2019 consecutive integers．The sum of his integers is 2019 ．
What is the product of all the integers in Jeroen＇s list？
A $2019^{2}$
B $\frac{2019 \times 2020}{2}$
C $2^{2019}$
D 2019
E 0

18．Alison folds a square piece of paper in half along the dashed line shown in the diagram．After opening the paper out again，she then folds one of the corners onto the dashed line．

What is the value of $\alpha$ ？
A 45
B 60
C 65
D 70
E 75

19. Which of the following could be the graph of $y^{2}=\sin \left(x^{2}\right)$ ?
A

B

C

D

E

20. The "heart" shown in the diagram is formed from an equilateral triangle $A B C$ and two congruent semicircles on $A B$. The two semicircles meet at the point $P$. The point $O$ is the centre of one of the semicircles. On the semicircle with centre $O$, lies a point $X$. The lines $X O$ and $X P$ are extended to meet $A C$ at $Y$ and $Z$ respectively. The lines $X Y$ and $X Z$ are of equal length.
What is $\angle Z X Y$ ?
A $20^{\circ}$
B $25^{\circ}$
C $30^{\circ}$
D $40^{\circ}$
E $45^{\circ}$
21. In a square garden $P Q R T$ of side 10 m , a ladybird sets off from $Q$ and moves along edge $Q R$ at 30 cm per minute. At the same time, a spider sets off from $R$ and moves along edge $R T$ at 40 cm per minute. What will be the shortest distance between them, in metres?
A 5
B 6
C $5 \sqrt{2}$
D 8
E 10
22. A function $f$ satisfies the equation $(n-2019) f(n)-f(2019-n)=2019$ for every integer $n$. What is the value of $f(2019)$ ?
A 0
B 1
C $2018 \times 2019$
D $2019^{2}$
E $2019 \times 2020$
23. The edge-length of the solid cube shown is 2 . A single plane cut goes through the points $Y, T, V$ and $W$ which are midpoints of the edges of the cube, as shown. What is the area of the cross-section?
A $\sqrt{3}$
B $3 \sqrt{3}$
C 6
D $6 \sqrt{2}$
E 8

24. The numbers $x, y$ and $z$ are given by $x=\sqrt{12-3 \sqrt{7}}-\sqrt{12+3 \sqrt{7}}, y=\sqrt{7-4 \sqrt{3}}-\sqrt{7+4 \sqrt{3}}$ and $z=\sqrt{2+\sqrt{3}}-\sqrt{2-\sqrt{3}}$.

What is the value of $x y z$ ?
A 1
B -6
C -8
D 18
E 12
25. Two circles of radius 1 are such that the centre of each circle lies on the other circle. A square is inscribed in the space between the circles. What is the area of the square?
A $2-\frac{\sqrt{7}}{2}$
B $2+\frac{\sqrt{7}}{2}$
C $4-\sqrt{5}$
D 1
E $\frac{\sqrt{5}}{5}$


1．When the following are evaluated，how many of the answers are odd numbers？

$$
1^{2}, 2^{3}, 3^{4}, 4^{5}, 5^{6}
$$

A 1
B 2
C 3
D 4
E 5

2．The positive integer 2018 is the product of two primes．
What is the sum of these two primes？
A 1001
В 1010
C 1011
D 1100
E 1101

3．Which of the following shows the digit 6 after it has been rotated clockwise through $135^{\circ}$ ？
A
B
c 9
D
E

4．Which of the following is not a multiple of 5？
A $2019^{2}-2014^{2}$
B $2019^{2} \times 10^{2}$
C $2020^{2} \div 101^{2}$
D $2010^{2}-2005^{2}$
E $2015^{2} \div 5^{2}$

5．Which of the following numbers is the largest？
A $\frac{397}{101}$
B $\frac{487}{121}$
C $\frac{596}{153}$
D $\frac{678}{173}$
E $\frac{796}{203}$

6．Which of the following is equal to $25 \times 15 \times 9 \times 5.4 \times 3.24$ ？
A $3^{9}$
B $3^{10}$
C $3^{11}$
D $3^{14}$
E $3^{17}$

7．The circles $P, Q$ and $R$ are all tangent to each other．Their centres all lie on a diameter of $P$ ，as shown in the figure．
What is the value of $\frac{\text { circumference of } Q+\text { circumference of } R}{\text { circumference of } P}$ ？
A 1
B $\frac{1}{2}$
C $\frac{1}{3}$
D $\frac{1}{4}$
E more information needed


8．What are the last two digits of $7^{2018}$ ？
A 07
B 49
C 43
D 01
E 18

9．The diagram shows a rectangle $A E F J$ inside a regular decagon $A B C D E F G H I J$ ．
What is the ratio of the area of the rectangle to the area of the decagon？
A $2: 5$
B 1：4
C 3：5
D 3：10
E 3：20


10．On a training ride，Laura averages speeds of $12 \mathrm{~km} / \mathrm{h}$ for 5 minutes，then $15 \mathrm{~km} / \mathrm{h}$ for 10 minutes and finally $18 \mathrm{~km} / \mathrm{h}$ for 15 minutes．
What was her average speed over the whole ride？
A $13 \mathrm{~km} / \mathrm{h}$
B $14 \mathrm{~km} / \mathrm{h}$
C $15 \mathrm{~km} / \mathrm{h}$
D $16 \mathrm{~km} / \mathrm{h}$
E $17 \mathrm{~km} / \mathrm{h}$

11．How many of the following four equations has a graph that does not pass through the origin？

$$
y=x^{4}+1 \quad y=x^{4}+x \quad y=x^{4}+x^{2} \quad y=x^{4}+x^{3}
$$

A 0
B 1
C 2
D 3
E 4

12．A regular tetrahedron is a polyhedron with four faces，each of which is an equilateral triangle，as shown．A solid regular tetrahedron is cut into two pieces by a single plane cut．
Which of the following could not be the shape of the section formed by the cut？
A a pentagon
B a square
C a rectangle that is not a square
D a trapezium
E a triangle that is not equilateral


13．The lines $y=x$ and $y=m x-4$ intersect at the point $P$ ．
What is the sum of the positive integer values of $m$ for which the coordinates of $P$ are also positive integers？
A 3
B 5
C 7
D 8
E 10

14．The following twelve integers are written in ascending order：

$$
1, x, x, x, y, y, y, y, y, 8,9,11 .
$$

The mean of these twelve integers is 7 ．What is the median？
A 6
B 7
C 7.5
D 8
E 9

15．A square is inscribed in a circle of radius 1 ．An isosceles triangle is inscribed in the square as shown．
What is the ratio of the area of this triangle to the area of the shaded region？
A $\pi: \sqrt{2}$
B $\pi: 1$
C 1： 4
D 1：$\pi-2 \quad$ E 2：$\pi$


16．The numbers $p, q, r$ and $s$ satisfy the following equations：

$$
p+2 q+3 r+4 s=k \quad 4 p=3 q=2 r=s
$$

What is the smallest value of $k$ for which $p, q, r$ and $s$ are all positive integers？
A 20
B 24
C 25
D 77
E 154

17．Bethany has 11 pound coins and some 20 p coins and some 50 p coins in her purse．The mean value of the coins is 52 pence．
Which could not be the number of coins in the purse？
A 35
B 40
C 50
D 65
E 95

18．$P, Q$ and $R$ are the three angles of a triangle，when each has been rounded to the nearest degree．
Which of the following is the complete list of possible values of $P+Q+R$ ？
A $179^{\circ}, 180^{\circ}$ or $181^{\circ}$
B $180^{\circ}, 181^{\circ}$ or $182^{\circ}$
C $178^{\circ}, 179^{\circ}$ or $180^{\circ}$
D $180^{\circ}$
E $178^{\circ}, 179^{\circ}, 180^{\circ}, 181^{\circ}$ or $182^{\circ}$

19．How many pairs of numbers $(m, n)$ are there such that the following statement is true？
＇A regular $m$－sided polygon has an exterior angle of size $n^{\circ}$ and
a regular $n$－sided polygon has an exterior angle of size $m^{\circ}$.
A 24
B 22
C 20
D 18
E 16

20．The diagram shows a semicircle of radius 1 inside an isosceles triangle．The diameter of the semicircle lies along the＇base＇of the triangle，and the angle of the triangle opposite the＇base＇is equal to $2 \theta$ ．Each of the two equal sides of the triangle is tangent to the semicircle．
What is the area of the triangle？

A $\frac{1}{2} \tan 2 \theta$
B $\sin \theta \cos \theta$
C $\sin \theta+\cos \theta$
D $\frac{1}{2} \cos 2 \theta$
E $\frac{1}{\sin \theta \cos \theta}$

21．The graph of $y=\frac{1}{x}$ is reflected in the line $y=1$ ．The resulting image is reflected in the line $y=-x$ ． What is the equation of the final graph？
A $y=\frac{-1}{(x+2)}$
B $y=\frac{1}{(x-1)}$
C $y=\frac{1}{(x-2)}$
D $y=\frac{-1}{(x-1)}$
E $y=\frac{-1}{(x-2)}$

22．The diagram shows two overlapping triangles；an isosceles triangle with an angle of $120^{\circ}$ and an equilateral triangle with area 36．Two of the vertices of the equilateral triangle are midpoints of the equal sides of the isosceles triangle．
What is the total area of the shaded regions（inside the isosceles triangle but outside the equilateral triangle）？

A 24
B 26
C 28
D 30
E 32

23．For particular real numbers $a$ and $b$ ，the function $f$ is defined by $f(x)=a x+b$ ，and is such that $f(f(f(x)))=27 x-52$ ．
Which of the following formulas defines the function $g$ such that，for all values of $x, g(f(x))=x$ ？
A $\frac{1}{3} x-4$
B $\frac{1}{3} x+\frac{4}{3}$
C $4 x-3$
D $\frac{1}{3} x-\frac{4}{3}$
E $3 x-4$

24．The diagram shows a circle with centre $O$ which lies in a horizontal plane．The diameter $A B$ has length 4 ．Point $P$ lies vertically above $O$ and $P O=2 \sqrt{2}$ ．Point $C$ lies on the semicircular arc $A B$ such that the ratio of the lengths of the arcs $A C$ and $C B$ is $2: 1$ ．
What is the shortest distance from $A$ to $P C$ ？
A $\sqrt{2}$
B $\sqrt{3}$
C 2
D $2 \sqrt{2}$
E 3


25．A semicircle is inscribed in a quarter circle as shown．
What fraction of the quarter circle is shaded？
A $\frac{1}{3}$
B $\frac{1}{\sqrt{3}}$
C $\frac{2}{3}$
D $\frac{\sqrt{3}}{2}$
E $\frac{1}{\sqrt{2}}$


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