

United Kingdom Mathematics Trust

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Junior Challenge

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INSTRUCTIONS

- 1. Do not open the paper until the invigilator tells you to do so.
- Time allowed: 60 minutes.
 No answers, or personal details, may be entered after the allowed time is over.
- 3. The use of blank or lined paper for rough working is allowed; squared paper, calculators and measuring instruments are forbidden.
- 4. Use a B or an HB non-propelling pencil. Mark at most one of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.
- 5. **Do not expect to finish the whole paper in the time allowed.** The questions in this paper have been arranged in approximate order of difficulty with the harder questions towards the end. You are not expected to complete all the questions during the time. You should bear this in mind when deciding which questions to tackle.
- 6. Scoring rules:
 5 marks are awarded for each correct answer to Questions 1-15;
 6 marks are awarded for each correct answer to Questions 16-25. In this paper you will not lose marks for getting questions wrong.
- 7. Your Answer Sheet will be read by a machine. **Do not write or doodle on the sheet except to mark your chosen options.** The machine will read all black pencil markings even if they are in the wrong places. If you mark the sheet in the wrong place, or leave bits of eraser stuck to the page, the machine will interpret the mark in its own way.
- 8. The questions on this paper are designed to challenge you to think, not to guess. You will gain more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. This paper is about solving interesting problems, not about lucky guessing.



| A 821 | B 1001 | C 1179 | D 122 | 1 F | E 1279 |
|--|---|---|---|---|---|
| 11 021 | D 1001 | | | - I | - 1-17 |
| How many of | the following five of | options are factors | of 30? | | |
| A 1 | B 2 | C 3 | D 4 | E | Ε 5 |
| What is the va | alue of $\frac{1+2+3+}{6+7+8+}$ | $\frac{-4+5}{9+10}$? | | | |
| A $\frac{1}{2}$ | B $\frac{3}{8}$ | C $\frac{7}{16}$ | D $\frac{9}{20}$ | Ε | $E \frac{1}{3}$ |
| One of these i Which is it? | is the largest two-di | git positive integer | that is divisible b | y the product o | f its digits. |
| A 12 | B 24 | C 36 | D 72 | E | E 96 |
| The record fo Jumpy in 201 | or travelling 100m of 3. What was Jumpy | on a skateboard by 's approximate ave | a dog is 19.65 so prage speed? | econds. This w | vas achieved |
| A 0.2 m/s | B 0.5 m/s | C 2 m/s | D 2.5 | m/s E | E 5 m/s |
| When this pri different prim What is the su | me number square les, and each of the im of the five missing | is completed, the four sides has total ng primes? | eight circles cont 43. | ain eight (| |
| When this pridifferent prim What is the su A 51 What is the di | ime number square les, and each of the im of the five missin B 53 fference between the | is completed, the four sides has total ng primes? C 55 D e largest two-digit r | eight circles cont 43. 57 E 5 nultiple of 2 and t | ain eight (59 (he smallest thre | 132 |
| When this pridifferent prim What is the su A 51 What is the di of 3? A 5 | ime number square les, and each of the um of the five missin B 53 fference between the B 4 | is completed, the four sides has total ng primes? C 55 D e largest two-digit r C 3 | eight circles cont 43. 57 E 5 nultiple of 2 and t D 2 | ain eight (59 (he smallest thre E | 132 $-29 - (29 -$ |
| When this pridifferent prim What is the su A 51 What is the di of 3? A 5 How many of | ime number square les, and each of the im of the five missin B 53 fference between the B 4 | is completed, the four sides has total ng primes? C 55 D e largest two-digit r C 3 are prime? | eight circles conta 43. 57 E 5 nultiple of 2 and t D 2 | ain eight (59 (he smallest thre E | 13 - 2 $-29 - 29 - 29 - 29 - 29 - 29 - 29 - 29$ |
| When this pridifferent prim What is the su A 51 What is the di of 3? A 5 How many of $0^2 + 1^2$ | time number square ties, and each of the tim of the five missin B 53 fference between the B 4 these six numbers a $1^2 + 2^2$ | is completed, the four sides has total ng primes? C 55 D e largest two-digit r C 3 are prime? $2^2 + 3^2$ | eight circles conta 43. 57 E 5 nultiple of 2 and t D 2 $3^2 + 4^2$ | ain eight (59 (he smallest three $4^2 + 5^2$ | 13 - 2 $-29 - 29 - 29 - 29 - 29 - 29 - 29 - 29$ |
| When this pridifferent prim What is the su A 51 What is the di of 3? A 5 How many of $0^2 + 1^2$ A 1 | time number square ties, and each of the sum of the five missin B 53 fference between the B 4 these six numbers a $1^2 + 2^2$ B 2 | is completed, the four sides has total ng primes? C 55 D e largest two-digit r C 3 are prime? $2^2 + 3^2$ C 3 | eight circles conta 43. 57 E 5 multiple of 2 and t D 2 $3^2 + 4^2$ D 4 | ain eight (59 (he smallest three $\frac{4^2 + 5^2}{E}$ | 13 - 29 - 29 - 29 - 29 - 29 - 29 - 29 - 2 |
| When this pridifferent prim What is the su A 51 What is the di of 3? A 5 How many of $0^2 + 1^2$ A 1 Triangle <i>LM1</i> | time number square times, and each of the sum of the five missin B 53 fference between the B 4 these six numbers a $1^2 + 2^2$ B 2 V is isosceles with R | is completed, the four sides has total ng primes? C 55 D e largest two-digit r C 3 are prime? $2^2 + 3^2$ C 3 LM = LN. | eight circles conta 43. 57 E 5 multiple of 2 and t D 2 $3^2 + 4^2$ D 4 | ain eight (59 (he smallest three $4^2 + 5^2$ E | 13 - 2 $-29 - 29 - 29 - 29 - 29 - 29 - 29 - 29$ |
| When this pridifferent prim What is the su A 51 What is the di of 3? A 5 How many of $0^2 + 1^2$ A 1 Triangle <i>LMI</i> What is the value | time number square times, and each of the sum of the five missin B 53 fference between the B 4 these six numbers a $1^2 + 2^2$ B 2 V is isosceles with a alue of y? | is completed, the four sides has total ng primes? C 55 D e largest two-digit r C 3 are prime? $2^2 + 3^2$ C 3 LM = LN. | eight circles conta 43. 57 E 5 nultiple of 2 and t D 2 $3^2 + 4^2$ D 4 | ain eight (59 (he smallest three $4^2 + 5^2$ E | 13 - 29 - 29 - 29 - 29 - 29 - 29 - 29 - 2 |

10. In the diagram, all distances shown are in cm. The perimeter of the shape
is 60 cm. What is the area, in cm², of the shape?
A 192A 192B 204C 212D 232E 252A 192B 204C 212D 232E 252A 18

11. To save money, Scrooge is reusing tea bags. After a first 'decent' cup of tea, he dries the bag and uses two such dried bags to make a new 'decent' cup of tea. These bags are then dried again and four such bags now make a 'decent' cup of tea. After that they are put on the compost heap. How many 'decent' cups of tea can Scrooge get out of a new box of 120 tea bags?

A 480 B 240 C 210 D 195 E 180

12. One afternoon, Brian the snail went for a slither at a constant speed. By 1:50 pm he had slithered 150 centimetres. By 2:10 pm he had slithered 210 centimetres. When did Brian start his slither?

| A Noon B 12:20 pm C 12:30 pm D 12:45 pm E 1 p | pm |
|---|----|
|---|----|

| 13 | Four congruent re | ectangles are arr | anged as shown | to form an inner | square of area | | |
|---|-------------------|--------------------|------------------|------------------|----------------|------|--|
| 13. | | | | | square of area | | |
| $20 \mathrm{cm}^2$ and an outer square of area 64 cm ² . | | | | | | | |
| | What is the perin | neter of one of th | ne four congruer | nt rectangles? | | | |
| | A 6 cm | B 8 cm | C 9.75 cm | D 16 cm | E 20 cm | | |
| | | | | | | | |
| | | | | | | | |

| 14. In the additio | on shown, x and y | represent differen | t single digits. | | 77 |
|---------------------------|-----------------------|--------------------|------------------|------|---------|
| What is the v | alue of $x + y$? | | | | 6 y x |
| A 10 | B 11 | C 12 | D 13 | E 14 | + y y x |
| | | | | | 1 x x 7 |

15. My train was scheduled to leave at 17:48 and to arrive at my destination at 18:25. However, it started four minutes late, and the journey took twice as long as scheduled. When did I arrive?

A 19:39 B 19:06 C 19:02 D 18:29 E 17:52

- **16.** Amrita needs to select a new PIN. She decides it will be made up of four non-zero digits with the following properties:
 - i) The first two digits and the last two digits each make up a two-digit number which is a multiple of 11.
 - ii) The sum of all the digits is a multiple of 11.

How many different possibilities are there for Amrita's PIN?

- A 1 B 2 C 4 D 8 E 16
- **17.** Two numbers p and q are such that 0 . Which is the largest of these expressions?
 - A q p B p q C $\frac{p + q}{2}$

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 $D = \frac{p}{2}$

 $E = \frac{q}{2}$

p

| 18. What is the sum of the four marked angles in the diagram? | | | | | | |
|--|--|---|--|--|--|---|
| | A 540° | B 560° | C 570° | D 600° | E 720° | |
| 19. | In a football m scored the fina | natch, Rangers l goal. How m | beat Rovers any possible | 5 - 4. The or half-time score | nly time Rangers were res were there? | e ahead was after they |
| | A 9 | B 10 | | C 15 | D 16 | E 25 |
| 20. | Each cell in the | e crossnumber | is to be fille | d with a single | digit. | 1 |
| | | A | cross | Down | | |
| | | 1. 2. | A cube A square | 1. A prime | | 2 |
| | Which of these | e could be the s | sum of the fo | our digits in the | e crossnumber? | |
| | A 17 | B 16 | C 15 | D 1 | 4 E 13 | |
| 22. | elephants as w How many elephants A 123 The positive in so that the proo | hite elephants. phants are in E B 132 tegers from 1 t duct of the thre should be place | There are 2 leanor's Employed of 9 inclusive of 9 inclusive and in the both | 0 more white e porium? C 213 are placed in t n each row or o tom right-hand | D 231 The grid, one to a cell, column is as shown. | E 312 E 312 18 105 |
| | A 9 | B 6 | C 4 | D 3 | E 2 | ? 192 56 180 36 |
| 23. | Regular pentag The six angles What is the siz | gon <i>PQRST</i> ha at <i>O</i> are equal e of angle <i>TG</i> (| as centre <i>O</i> . 1 <i>O</i> ? | Lines PH, FI | and GJ go through O | P $Q \xrightarrow{J} \xrightarrow{F} T$ |
| | A 60° | B 72° | C 75° | D 76° | е 78° | $I \bigcirc G$ R H S |
| 24. | Beatrix was bo of the year in v sum of the dig | orn in this centu which she was its of that year | ury. On her b born. In whi ? | wirthday this ye ich of these ye | ar, her age was equal t ars will her age on her | o the sum of the digits birthday be twice the |
| | A 2027 | B 202 | 9 | C 2031 | D 2033 | E 2035 |
| 25. | Granny gave a | way her entire | collection of | of antique spo | ons to three people. I | Her daughter received |

8 Standy gave away her entire conection of antique spoons to three people. Her daughter received 8 more than a third of the total; her son received 8 more than a third of what was then left; finally her neighbour received 8 more than a third of what was then left.
What is the sum of the digits of the number of another which were in Creater's collection?

What is the sum of the digits of the number of spoons which were in Granny's collection?

A 14 B 12 C 10 D 8 E 6

UKMT

非淡泊无以明志,非宁静无以致远。

| 1. | which of these | has the greatest | uiue. | | |
|----------------|---|---|--|--|---|
| | A 20+22 | B 202 + | $2 \qquad C 202 \times 2$ | D $2 \times 0 \times 2 \times$ | $2 E 20 \times 22$ |
| 2. | The number 50 Onto which num | 12 is reflected in mber is it reflecte | the mirror-line shown. d? | | |
| | A 5102 | B 2015 | C 5012 D 2105 | E 5105 | |
| 3. | Think of any n number; add th | number. Add five ree. What is the r | e; multiply by two; addressulting number? | d ten; divide by two; s | subtract your original |
| | A 10 | B 11 | C 12 | D 13 | E 14 |
| 4. | What is the val | ue of $0.6 + \frac{2}{5}$? | | | |
| | A 0.15 | B 0.24 | C 0.8 | D 1 | E 2.4 |
| 5. | How many of the | he following take | integer values? | | |
| | | 1 11 | 111 111 | 11 1111 | |
| | | 1 1+1 | 1+1+1 1+1+ | 1+1 1+1+1+ | 1 + 1 |
| | A 0 | B 1 | C 2 | D 3 | E 4 |
| 6. | The diagram sh and <i>QRU</i> . | nows the square R | STU and two equilater | al triangles, <i>PUT</i> | P |
| | | | | | |
| | What is the size | e of angle <i>QPU</i> ? | | | |
| | What is the size A 10° | e of angle <i>QPU</i> ? B 15° | C 20° D 25° | E 30° | Q U T R S |
| 7. | What is the size A 10° Kiwi fruit conta What weight of | e of angle <i>QPU</i> ? B 15° ain roughly two a kiwi fruit contai | C 20° D 25° nd a half times as much ns approximately the sa | E 30° n vitamin C as the same amount of vitamin | $Q \qquad U \qquad T \\ R \qquad S$ e weight of oranges. C as 1 kg of oranges? |
| 7. | What is the size A 10° Kiwi fruit conta What weight of A 100 g | e of angle <i>QPU</i> ? B 15° ain roughly two a kiwi fruit contai B 200 g | C 20° D 25° nd a half times as much ns approximately the sa C 250 g | E 30° n vitamin C as the same amount of vitamin D 400 g | $Q \qquad U \qquad T \\ R \qquad S$ e weight of oranges. C as 1 kg of oranges? E 550 g |
| 7. 8. | What is the size A 10° Kiwi fruit conta What weight of A 100 g Today is Thurse | e of angle <i>QPU</i> ? B 15° ain roughly two a kiwi fruit contai B 200 g day. What day wi | C 20° D 25° nd a half times as much ns approximately the sa C 250 g 11 it be in 100 days' tim | E 30° n vitamin C as the same ame amount of vitamin D 400 g e? | $Q \qquad U \qquad T \\ R \qquad S$ e weight of oranges. C as 1 kg of oranges? E 550 g |
| 7. 8. | What is the size A 10° Kiwi fruit conta What weight of A 100 g Today is Thurso A Tuesday | e of angle <i>QPU</i> ? B 15° ain roughly two a kiwi fruit contai B 200 g day. What day wi B Wedne | C 20° D 25° nd a half times as much ns approximately the sa C 250 g Il it be in 100 days' tim esday C Thursday | E 30° n vitamin C as the same amount of vitamin D 400 g e? D Friday | $Q \qquad U \qquad T \\ R \qquad S$ e weight of oranges. C as 1 kg of oranges? E 550 g E Saturday |
| 7. 8. 9. | What is the size A 10° Kiwi fruit conta What weight of A 100 g Today is Thurse A Tuesday How many squa | e of angle <i>QPU</i> ? B 15° ain roughly two a kiwi fruit contai B 200 g day. What day wi B Wedne ares of any size c | C 20° D 25° nd a half times as much ns approximately the sa C 250 g Il it be in 100 days' tim esday C Thursday an be seen in the diagra | E 30° n vitamin C as the same amount of vitamin D 400 g e? D Friday am? | $Q \qquad U \qquad T \\ R \qquad S$ e weight of oranges. C as 1 kg of oranges? E 550 g E Saturday |
| 7. 8. 9. | What is the size A 10° Kiwi fruit conta What weight of A 100 g Today is Thurse A Tuesday How many squa A 25 | e of angle <i>QPU</i> ? B 15° ain roughly two a kiwi fruit contai B 200 g day. What day wi B Wedne ares of any size c B 27 | C 20° D 25° nd a half times as much ns approximately the sa C 250 g Il it be in 100 days' tim esday C Thursday an be seen in the diagra C 28 D 3 | E 30° n vitamin C as the same ume amount of vitamin D 400 g e? D Friday um? 39 E 40 | $Q \qquad \qquad U \qquad T \\ R \qquad S$ e weight of oranges. C as 1 kg of oranges? E 550 g E Saturday |
| 7. 8. 9. | What is the size A 10° Kiwi fruit conta What weight of A 100 g Today is Thurse A Tuesday How many square A 25 Half of a quartee What is the nur | e of angle <i>QPU</i> ? B 15° ain roughly two a kiwi fruit contai B 200 g day. What day wi B Wedne ares of any size c B 27 er of an eighth of nber? | C 20° D 25° Ind a half times as much ns approximately the sa C 250 g Il it be in 100 days' times esday C Thursday an be seen in the diagra C 28 D 3 a number is equal to $\frac{1}{2}$ | E 30° h vitamin C as the same amount of vitamin D 400 g le? D Friday am? 39 E 40 $+\frac{1}{4}+\frac{1}{8}.$ | $Q \qquad U \qquad T \\ R \qquad S$ e weight of oranges. C as 1 kg of oranges? E 550 g E Saturday $\overline{U} \qquad \overline{U} \qquad $ |

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| 11. | Nine of the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 are to be put in two groups so that the sum of the numbers in each group is a multiple of four. What is the largest number that could be left out? | | | | | | |
|-----|---|---|---|--|---|-----------------------------------|--|
| | A 3 | B 4 | C 5 | D 6 | I | E 7 | |
| 12. | When my pot of p 3.1 kg. What is th | aint is half full, i e weight of the f | t weighs 5.8 kg. Whe | en my pot of pa | aint is one quarte | er full, it weighs | |
| | A 8.9 kg | B 11.2 kg | C 11.6 kg | D 12 | 2 kg I | E 12.4 kg | |
| 13. | The diagram show What percentage of | vs five squares woof the area of the | hose side-lengths, ir outer square is shace | cm, are 1, 2, led? | 3, 4 and 5. | | |
| | A 25% | B 30% | C 36% I | D 40% | E 42% | | |
| 14. | A group of childred 3, and so on. Nun | en stand evenly s aber 13 is direct | spaced around a circuly opposite number 3 | lar ring and a 5. How many | re numbered cor children are the | nsecutively 1, 2, re in the ring? | |
| | A 42 | B 44 | C 46 | D 48 | 3 I | E 50 | |
| 15. | What is the value | of $2 \div (4 \div (6 \div$ | (8 ÷ 10)))? | | | | |
| | A $\frac{1}{960}$ | $B \frac{1}{5}$ | C $\frac{3}{8}$ | D $\frac{1}{2}$ | Ι | $E \frac{15}{4}$ | |
| 16. | The diagram show from two equilate and 8 cm respecti triangle of side-le What is the perim | rs a seven-sided p ral triangles <i>PQ</i> vely. The two tr ngth 2 cm. eter of <i>PQRST</i> | oolygon, <i>PQRSTUV</i> W and <i>STU</i> of side riangles overlap in a | . It is formed -length 5 cm n equilateral | $5 \mathrm{cm}$ | U 2 cm 8 cm | |
| | A 27 cm E 39 cm | B 30 cm | C 33 cm | D 36 cm | Q R / S | | |
| 17. | Amrita and Beatr game, one player of the game, Amr How many rounds | ix play a game i wins and is give ita and Beatrix h s of the game did | n which each player n 3 counters; and her nave 40 counters and l Amrita win? | starts with 10 opponent has 16 counters re | counters. In ea 1 counter remo espectively. | ch round of the ved. At the end | |
| | A 10 | B 11 | C 12 | D 13 | 3 1 | Ξ 14 | |

- **18.** The diagram shows a parallelogram. What is the value of *y*?
 - A 22 B 24 C 25 D 28 E 30
- **19.** At the start of the day I had three times as many apples as pears. By the end of the day, after eating five apples but no pears, I had twice as many pears as apples. How many pieces of fruit did I have at the start of the day?

A 4 B 8 C 12 D 16

E 20

 $(3x-40)^{\circ} (4y-50)^{\circ}$ $(2x-30)^{\circ}$



非淡泊无以明志,非宁静无以致远。

20. During a particularly troublesome lesson, the following conversation occurs:
Pam: "I always tell the truth."
Roger: "Both Pam and Quentin are lying."
Terry: "Everyone is telling the truth."

How many people are telling the truth?

| A 0 | B 1 | C 2 | D 3 | E 4 |
|-----|-----|-----|-----|-----|
| | | | | |

21. Two lists of numbers are as shown below.

| List S: | 3 | 5 | 8 | 11 | 13 | 14 |
|---------|---|---|---|----|----|----|
| List T: | 2 | 5 | 6 | 10 | 12 | 13 |

Jenny decided she would move one number from List S to List T and one number from List T to List S so that the sum of the numbers in the new List S is equal to the sum of the numbers in the new List T. In how many ways could she do this?

| A 1 | B 2 | C 3 | D 4 | E 5 |
|-------|-----|-----|-----|-----|
| 1 L L | | 0 0 | | |

22. A triangular pyramid with vertices T, U, V and Q is removed from the solid cube shown.How many addres does the remaining solid have?

How many edges does the remaining solid have?

| A 4 | B 6 | C 8 | D 10 | E 12 |
|-----|-----|-----|------|------|
| | | | | |



23. The price of a train ticket increased by 5% and then decreased by 20% in a special offer. It was then £4 less expensive than its original price. What was the original price of the ticket?

| A £8.60 B £13 | C £20.40 | D £25 | E £26.40 |
|---------------|----------|-------|----------|
|---------------|----------|-------|----------|

24. Flori's Flower shop contains fewer than 150 flowers. All the flowers are purple, yellow, red or white. The ratio of purple flowers to yellow flowers is 1 : 2, the ratio of yellow flowers to red flowers is 3 : 4 and the ratio of red flowers to white flowers is 5 : 6.How many flowers are there in Elori's shop?

How many flowers are there in Flori's shop?

| 133 | B 136 | C 139 | D 142 | E 145 |
|-----|-------|-------|-------|-------|
| | | | | |

25. In the number pyramid shown, each cell above the bottom row contains the sum of the numbers in the two cells immediately below it. The sum of the numbers in the bottom row is 17. What is the central number of the bottom row?





Α

| UKMT | | | ior Mathematical Chal | Page 11 of 248 | |
|------|--|--|------------------------------------|--------------------|-----------------------|
| 1. | . What is the va | lue of 123 – 456 + 7 | 789? | | |
| | A 456 | B 556 | C 567 | D 678 | E 789 |
| 2. | . Brianna has £ How many co | 20, all in 5p coins, a ins does she have in | nd £50, all in 2p coins. total? | | |
| | A 200 | B 290 | C 1000 | D 2540 | E 2900 |
| 3. | . What is the va | lue of $1 - 2 \times 3 + 4$ | ÷ 5? | | |
| | A -4.2 | B -2.8 | C 0 | D 0.2 | E 4 |
| 4. | . How many of | the following number | ers are multiples of 11 | ? | |
| | | 187 | 156 253 | 495 132 | |
| | A 1 | B 2 | C 3 | D 4 | E 5 |
| 5. | When I have v 20% of the wa How far, in mo | walked 20% of the w lk remaining. etres, is it from my h | way to school, I have 1 | 200 metres more to | walk than when I have |
| | A 1240 | B 1440 | C 1680 | D 1800 | E 2000 |
| 6. | . What is the va | lue of $(2 - \frac{1}{2})(3 - \frac{1}{3})$ | $)(4-\frac{1}{4})?$ | | |
| | A 16 | B 15 | C 14 | D 13 | E 12 |
| 7. | In the diagram Also, $RT = R$. What is $\angle PTS$ | shown, $PT = QT =$ S and $\angle PTQ = 36^{\circ}$. | = <i>QR</i> . | T | |
| | A 72° E 108° | B 80° C 9 | 90° D 100° | P Q | R " S |
| 8. | . What is the va | lue of $1 - (2 - (3 - $ | (4-5)))? | | |
| | A -5 | B -3 | C -1 | D 1 | E 3 |

9. Each cell in the crossnumber below contains a single non-zero digit. The answer to each clue is a two-digit number.

| | Clues | | 1 | 2 |
|----------------------|--|------------------------|-----|---|
| | Across | Down | | |
| | A square An odd squar | 1. A square2. A square | 3 | x |
| What is the value of | of x ? | | | |
| A 1 | B 3 | C 5 | D 7 | |

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E 9



| 10. | The diagram shows a rhombus formed by joining each vertex of a square to the midpoint of a side of the square. What fraction of the area of the square has been shaded? | | | | | | |
|-----|---|---|---|---|-------------------------------------|--|--|
| | A $\frac{1}{2}$ | B $\frac{1}{3}$ | $C \frac{1}{4}$ | D $\frac{1}{6}$ | $E \frac{1}{8}$ | | |
| 11. | A particular pris | m has ten faces. | How many edges does | s it have? | | | |
| | A 20 | B 21 | C 24 | D 27 | E 30 | | |
| 12. | The pupils in my answers five ques Last week, Jaslee How many minu | class work very stions every 40 en took exactly 1 tes more than Ja | y quickly. Jasleen answ seconds. hour to answer a large sleen did Ella take to a | ers four questions every e set of questions. Inswer the same set of c | y 30 seconds and Ella questions? | | |
| | A 2 | B $2\frac{1}{2}$ | C $3\frac{1}{4}$ | D 4 | E $4\frac{1}{2}$ | | |
| 13. | Five line segmen What is the sum A 900° | ts coincide at a of the marked a B 720° | point as shown. ngles? C 540° D 30 | 50° E 180° | | | |
| 14. | I begin with a the final answer is al How many differ | ree-digit positiv so a three-digit rent positive inte | e integer. I divide it by integer. gers could I have begu | y 9 and then subtract 9 n with? | from the answer. My | | |
| | A 3 | B 5 | C 7 | D 11 | E 13 | | |
| 15. | Alex has a pile o coins. Now she h | of two pence coi nas £4.20. How | ns. She swapped exac much money did Alex | tly half of them for the have initially?" | same number of 10p | | |
| | A 42p | B 84p | C £1.12 | D £1.40 | E £1.68 | | |
| 16. | A cube has edge every edge at 1 c How many dots v A 128 | e length 10 cm. m intervals. Par will there be in t B 116 | Starting at the vertices t of this pattern is show otal, once the pattern h C 112 D 10 | s, dots are placed along vn. aas been completed? 08 E 104 | g 1cm | | |
| 17. | In 1770, Joseph- squares. For example, where the second se | Louis Lagrange mple, $13 = 0^2 +$ e first 15 positive | proved that every posi $0^2 + 2^2 + 3^2$. e integers can be writte | tive integer can be writt on as the sum of <i>three</i> so | ten as the sum of four quares? | | |
| | A 11 | B 12 | C 13 | D 14 | E 15 | | |

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18. Each of the numbers 1 to 9 is to be placed in a different cell of the grid shown

so that the sum of the three numbers in each row is 15. Also, the sum of the two numbers in each shaded column is to be 15. How many choices are there for the number to be placed in the central cell indicated by *? A 0 **C** 2 E 4 **B** 1 D 3 19. In my class, everyone studies French or German, but not both languages. One third of the girls and the same number of boys study German. Twice as many boys as girls study French. Which of these could be the total number of boys and girls in my class? B 28 C 30 A 26 D 32 E 34 **20.** Each of the shapes shown has been made from four unit cubes. For each shape, Max takes eight copies of the shape and tries to fit them together to make a $2 \times 4 \times 4$ cuboid. How many of the shapes can be used to make a cuboid of this size in this way? C 2 D 3 E 4 A 0 **B** 1 **21.** Some fish, some dogs and some children are swimming in a bay. There are 40 legs in total, twice as many heads as tails and more dogs than fish. How many fish are in the bay? **B** 2 C 3 A 1 D 4 E 5 22. The diagram shows four congruent rectangles, each of perimeter 20 cm, surrounding a square of area 44 cm^2 . What is the area of each rectangle? $44\,\mathrm{cm}^2$ C 18 cm^2 D 20 cm^2 E 22 cm^2 A 14 cm^2 B 16 cm^2 **23.** Four different positive integers p, q, r, s satisfy the equation (9 - p)(9 - q)(9 - r)(9 - s) = 9. What is the value of p + q + r + s? A 20 B 24 C 28 D 32 E 36 24. In the diagram shown, PQ = PR = QS. Line segments PR and QS are perpendicular to each other. What is the sum of $\angle PRQ$ and $\angle PSQ$? B 105° C 120° A 90° D 135° E 150° Q R 25. I choose four different integers. When I add all the pairs of these numbers in turn, the totals that I obtain are 23, 26, 29, 32 and 35, with one of these totals being repeated. What is the largest of the four integers? A 18 B 19 C 20 D 21 E 22 www.CasperYC.Club/ukmt 非淡泊无以明志,非宁静无以致远。

| 1. | Exactly one of the | e following five nu | mbers is <i>not</i> prime. | Which is it? | |
|-----|---|--|--|--|--------------------------|
| | A 101 | B 103 | C 107 | D 109 | E 111 |
| 2. | What is the value | of 2020 ÷ 20 ? | | | |
| | A 10 | B 11 | C 100 | D 101 | E 111 |
| 3. | Each of these figu | res is based on a | rectangle whose centr | re is shown. | |
| | • | | • | | - |
| | How many of the | figures have rotat | ional symmetry of or | der two? | |
| | A 1 | B 2 | C 3 | D 4 | E 5 |
| 4. | How many centim | netres are there in | 66.6 metres? | | |
| | A 66600 | B 6660 | C 666 | D 66.6 | E 66 |
| 5. | Amrita thinks of a She obtains the sa What was Amrita | a number. She do ume number she o 's number? | oubles it, adds 9, divi riginally thought of. | des her answer by 3 | and finally subtracts 1. |
| | A 1 | B 2 | C 3 | D 4 | E 6 |
| 6. | What is the value | of $\frac{6}{12} - \frac{5}{12} + \frac{4}{12}$ | $-\frac{3}{12} + \frac{2}{12} - \frac{1}{12}?$ | | |
| | A $\frac{1}{2}$ | $B \frac{1}{3}$ | C $\frac{1}{4}$ | D $\frac{1}{5}$ | $E \frac{1}{6}$ |
| 7. | Four different pos | itive integers have | e a product of 110. W | hat is the sum of the | four integers? |
| | A 19 | B 22 | C 24 | D 25 | E 28 |
| 8. | Wesley has a grid of two black cells sh this? | of six cells. He wa are a vertex but n | nts to colour two of thot a side. In how man | ne cells black so that t ny ways can he achie | he ve |
| | A 2 | B 3 0 | C 4 D 5 | E 6 | |
| 9. | One half of one the What is the number | nird of one quarter er? | of one fifth of a nun | nber is 2. | |
| | A 240 | B 120 | C 60 | D $\frac{1}{120}$ | $E \frac{1}{240}$ |
| 10. | How many of thes | se equations have | the solution $x = 12$? | | |
| | | x - 2 = 1 | $0 \frac{x}{2} = 24 10 - x = 24$ | = 2 2x - 1 = 25 | |
| | A 4 | В 3 | C 2 | D 1 | E 0 |

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- 11. This 3 by 3 grid shows nine $1 \text{ cm} \times 1 \text{ cm}$ squares and uses 24 cm of wire. What length of wire is required for a similar 20 by 20 grid? B 420 cm C 441 cm D 800 cm A 400 cm E 840 cm **12.** The diagram shows an equilateral triangle divided into four smaller equilateral triangles. One of these triangles has itself been divided into four smaller equilateral triangles. What fraction of the area of the large triangle has been shaded? A $\frac{1}{8}$ B $\frac{3}{16}$ C $\frac{1}{4}$ D $\frac{5}{16}$ $E \frac{3}{8}$ 13. The mean of four positive integers is 5. The median of the four integers is 6. What is the mean of the largest and smallest of the integers? A 3 **B** 4 C 5 D 6 E 8 14. In the diagram, angle OLM is twice as large as angle PON. What is the size of angle OLM? $J - \frac{124^{\circ}}{124^{\circ}}$ A 102° B 106° C 108° D 112° E 124° **15.** A group of 42 children all play tennis or football, or both sports. The same number play tennis as play just football. Twice as many play both tennis and football as play just tennis. How many of the children play football? A 7 **B** 14 C 21 D 28 E 35
- 16. You are given the sequence of digits "0625", and can insert a decimal point at the beginning, at the end, or at any of the other three positions.Which of these numbers can you *not* make?

 $C \frac{1}{16}$

D $\frac{25}{4}$

- A $\frac{6}{25}$ B $\frac{5}{8}$
- **17.** In 1925, Zbigniew Morón published a rectangle that could be dissected into nine different sized squares as shown in the diagram. The lengths of the sides of these squares are 1, 4, 7, 8, 9, 10, 14, 15 and 18. What is the area of Morón's rectangle?
 - A 144 B 225 C 900 D 1024 E 1056



 $E 25^2$

18. How many two-digit primes have both their digits non-prime?

A 6 B 5 C 4 D 3

E 2



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5

13

15

L

L

15

| 10 |) In the table shown, the sum of each row is shown to the right of the row and | | | | | | |
|-----|--|---------------------------|---------|-----|-----|----|---|
| 19. | the sum of each column is shown helew the column | | | | | | Κ |
| | What is the value of L? | | | | | | K |
| | A 1 | B 2 | C_{3} | D 5 | F 7 | L | J |
| | 11 1 | \mathbf{D} \mathbf{Z} | C 5 | D J | L / | 11 | 7 |

20. Edmund makes a cube using eight small cubes. Samuel uses cubes of the same size as the small cubes to make a cuboid twice as long, three times as wide and four times as high as Edmund's cube. How many more cubes does Samuel use than Edmund?

A 9 B 24 C 64 D 184 E 190

21. The digits of both the two-digit numbers in the first calculation below have been reversed to give the two-digit numbers in the second calculation. The answers to the two calculations are the same.

 $62 \times 13 = 806$ $26 \times 31 = 806$

For which one of the calculations below is the same thing true?

A 25×36 B 34×42 C 54×56 D 42×48 E 32×43

22. Harriet has a square piece of paper. She folds it in half to form a rectangle and then in half again to form a second rectangle (which is not a square). The perimeter of the second rectangle is 30 cm. What is the area of the original square?

A 36 cm^2 B 64 cm^2 C 81 cm^2 D 100 cm^2 E 144 cm^2

23. There is more than one integer, greater than 1, which leaves a remainder of 1 when divided by each of the four smallest primes.What is the difference between the two smallest such integers?

What is the difference between the two smallest such integers?

A 211 B 210 C 31 D 30 E 7

24. Susan is attending a talk at her son's school. There are 8 rows of 10 chairs where 54 parents are sitting. Susan notices that every parent is either sitting on their own or next to just one other person. What is the largest possible number of adjacent empty chairs in a single row at that talk?

A 3 B 4 C 5 D 7 E 8

25. In the diagram, *PQRS*, *JQK* and *LRK* are straight lines.

What is the size of the angle *JKL*?

A 34° B 35° C 36° D 37° E 38°

K S



| 1. | How many | minutes is | it from 23:3 | 5 today to | 01:15 ton | norrow? | | | | |
|------|---|---|---|---|--------------------------|-----------------------|-----------------------|------------|----------|-------------|
| | A 100 | | B 110 | (| C 120 | | D 130 | | E 140 | |
| 2. | Which of th | ese is equ | al to $(0.1 + 0.1)$ | 0.2 + 0.3 - | $-0.4) \div 0.3$ | 5? | | | | |
| | A 0.01 | | B 0.02 | (| C 0.04 | | D 0.1 | | E 0.4 | |
| 3. | Sam has eat | en three-q | uarters of th | e grapes. | | | | | | |
| | What is the | ratio of th | e number of | grapes th | at remain t | o the nu | mber Sam | has eater | 1? | |
| | A 1:3 | | B 1:4 | (| C 1:5 | | D 1:6 | | E 1:7 | |
| 4. | Which of th | e followin | g five shapes | s can be c | ut into fou | pieces | by a single | straight | cut? | |
| | A | | B | (| | | D | | E |]]] |
| 5. | On Aoife's Buster? | 16th birth | day, Buster | was three | times her | age. On | Aoife's 2 | 1st birthd | ay, how | old was |
| | A 32 | | B 48 | (| C 53 | | D 63 | | E 64 | |
| 6. | Which of th | ese is clos | est to 7? | | | | | | | |
| | A 7.09 | | B 6.918 | (| 2 7.17 | | D 6.7 | | E 7.085 | 5 |
| 7. | The shortest one of the w | t street in t vorld's lon | he UK, <i>Eber</i> gest roads, is | <i>ezer Plac</i> approxir | e in Wick, nately 782 | is 2.061 1 km in 1 | n long. Th length. | e Trans-C | Canada H | lighway, |
| | Approximat | ely, how n | nany times lo | onger thar | the street | is the hi | ghway? | | | |
| | A 40000 | 000 | B 400 000 | C | C 40 000 | | D 4000 | | E 400 | |
| 8. | The diagram Angle <i>PGQ</i> angle <i>PQG</i> | h shows a l $r = 35^{\circ}$, an $r = 120^{\circ}$ and | tite $PGRF$ in agle $PFS = 3$ and angle PSF | nside rhom 85° , $T = 120^{\circ}$. | nbus PQR. | S. | | S 120° | 35° F | |
| | What is the | size of ang | gle FPG? | | | Р | \langle | | | > R |
| | A 10° | B 12° | C 15° | D 18° | E 20° |) | | 120° Q | 35° G | |
| 9. | What is 50% | % of 18.3 1 | olus 18.3% o | f 50? | | | | | | |
| | A 9.15 | 1 | B 18.3 | (| 27.15 | | D 59.15 | | E 68.3 | |
| 10. | What is the | last digit o | of the smalle | st positive | e integer w | hose dig | gits add to 2 | 2019? | | |
| | A 1 | - | B 4 | (| C 6 | - | D 8 | | E 9 | |
| ww.C | CasperYC.Cl | ub/ukmt | 非淡 | 泊无以明 | 志,非宁 萧 | •无以致 | 远。 | | | |

2

E 4

M M M C

M J M

 $C \mid C$

С

С

CM

C | C | C | C | C

CCC

Μ

M C

С

11. Two players *X* and *Y* take alternate turns in a game, starting with the diagram alongside.

At each turn, one player writes one of 1, 2 or 3 in an empty circle, so that no two circles connected by an edge contain the same number. A player loses when they cannot go. In each of the five diagrams below it is Y's turn.

In which of the diagrams can *Y*'s move ensure that *X* loses the game?



12. Jamal writes down a sequence of six integers. The rule he uses is, "after the first three terms, each term is the sum of the three previous terms." His sequence is —, —, 8, 13, 25.

What is his first term?

- A 0 B 1 C 2 D 3
- **13.** In how many different ways can you spell out JMC, starting at the centre, and moving to the next letter in a neighbouring square horizontally, vertically, or diagonally each time?
 - A 8 B 16 C 24 D 25 E 32
- **14.** Each edge in the diagram has length 1 cm.

What is the length of the longest path that can be followed along the edges, starting at a vertex and without revisiting any vertex?

| A 7 cm | B 8 cm | C 9 cm | D 10 cm | E 11 cm |
|--------|--------|--------|---------|---------|
|--------|--------|--------|---------|---------|

15. All four L-shapes shown in the diagram are to be placed in the 4 by 4 grid so that all sixteen cells are covered and there is no overlap. Each piece can be rotated or reflected before being placed and the black dot is visible from both sides.

How many of the 16 cells of the grid could contain the black dot?

A 4 B 7 C 8 D 12 E 16

16. Tamsin writes down three two-digit integers. One is square, one is prime and one is triangular. She uses the digits 3, 4, 5, 6, 7 and 8 exactly once each.

Which prime does she write?

- A 37 B 43 C 53 D 73 E 83
- **17.** A rectangle is three times as long as it is high. The area of a square is twelve times the area of the rectangle. What is the ratio of the perimeter of the square to the perimeter of the rectangle?

A 12:1 B 6:1 C 4:1 D 3:1



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18. What fraction of the integers from 1 to 8000 inclusive are cubes?

A
$$\frac{1}{1000}$$
 B $\frac{1}{800}$ C $\frac{1}{400}$ D $\frac{1}{200}$

19. Each row, each column and each of the bold 2 by 3 rectangles in the grid has to contain each of the numbers 1, 2, 3, 4, 5 and 6 (one number in each cell). What number should go in the cell marked *x*?

| A 1 | B 2 | C 3 | D 4 | E 6 |
|-----|-----|-----|-----|-----|
| | | | | |

20. Emily writes down the largest two-digit prime such that each of its digits is prime. Krish writes down the smallest two-digit prime such that each of its digits is prime. Kirsten subtracts Krish's number from Emily's number. What answer does Kirsten obtain?

A 14 B 20 C 36 D 45

21. The diagram shows a regular hexagon PQRSTU, a square PQVW and an equilateral triangle VXW.

What is the size of angle *XVR*?

| A 120° | B 125° | C 130° | D 135° | E 140° |
|--------|--------|--------|--------|--------|
| | | | | |

22. In the multiplication shown alongside, *T*, *R*, *A* and *P* are all different digits. What is the value of *R*?

A 0 B 1 C 5 D 8

23. The diagram shows two squares *JKLM* and *PQRS*. The length of *JK* is 6 cm and that of *PQ* is 4 cm. The vertex *K* is the midpoint of side *RS*. What is the area of the shaded region?

- A 22 cm^2 B 24 cm^2 C 26 cm^2 D 28 cm^2 E 30 cm^2
- **24.** The diagram shows a regular heptagon.

Which of these expressions is equal to p + q + r + s + t?

| A $180 + q$ | B $180 + 2q$ | C 360 – q | D 360 |
|------------------|--------------|-----------|-------|
| E 360 + <i>q</i> | | | |

25. The diagram shows the first fifteen positive integers arranged in a 'triangle'. These numbers are to be rearranged so that the five integers along each 'edge' of the triangle have the same sum, unlike the example shown. When this is done, what is the greatest possible such sum?



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х 5 6 2 1 3 4 4 3

 $E \frac{1}{100}$



E 50



E 9





| UKMT | | Junior | Mathematical Cha | llenge 2018 | Page 20 of 2 | 248 |
|-------|--|--|--|---|---|-----|
| 1. | What is the value of A 111 | of (222 + 22) ÷ B 112 | C 122 | D 133 | E 233 | |
| 2. | A train carriage ha taken and 7 people At Banbury, 9 peop How many people | s 80 seats. On were standing ple left the car | my journey I notic ; riage, 28 people en | ed that all the seats tered it and all seats | in my carriage were s were taken. | |
| | A 0 | B 7 | C 16 | D 26 | E 35 | |
| 3. | The diagram shows of the square. What is the value of A 105 B 110 | s an equilatera of <i>x</i> ? C 115 D | l triangle, a square 120 E 135 | , and one diagonal | <i>x</i> ° | > |
| 4. | The perimeter of the Each edge of the re How long is each e A 8 cm | ne regular deca egular octagon edge of the reg B 10 cm | Q is 10 cm long. Q is 10 cm long. Q is 40 cm | ne perimeter of the ro D 60 cm | egular octagon <i>Q</i> . E 64 cm | |
| 5. | My train left South How many minutes A 153 | ampton at 06: s did the journ B 193 | 15 and arrived in B ey take? C 233 | Birmingham at 08:48 D 1463 | 8 later that morning. E 1501 | |
| 6. | The diagram shows rows, all columns a What is the value of A 10 B 11 | s a partially co and both main of $x + y$? C 12 | mpleted magic squ diagonals have the D 13 | are, in which all same total. E 14 | 4 | |
| 7. | How many integers A 320 | s are greater th B 321 | an 20 + 18 and al C 322 | so less than 20 × 1 D 323 | 8? E 324 | |
| 8. | Gill scored a goal I match. At that point A $\frac{1}{4}$ | half way through t, what fraction B $\frac{3}{8}$ | gh the second quar n of the whole mat $C \frac{1}{2}$ | ter of a 'teachers ve tech remained to be p D $\frac{5}{8}$ | ersus pupils' netball blayed? E $\frac{3}{4}$ | |
| 9. | The approximate c times the cost of bu Roughly what did t A £800 | ost of restoring uilding the stea the engine cost B £2000 | g the Flying Scotsn um engine in 1923. t to build? C £8000 | nan was £4 million. D £20 000 | This was about 500 E £80 000 | |
| 10. | Adding four of the | five fractions | $\frac{1}{2}, \frac{1}{3}, \frac{1}{6}, \frac{1}{9} \text{ and } \frac{1}{18}$ | gives a total of 1. | | |
| | A $\frac{1}{2}$ | B $\frac{1}{3}$ | C $\frac{1}{6}$ | D $\frac{1}{9}$ | $E \frac{1}{18}$ | |
| www.C | asperYC.Club/ukmt | 书山有 | 下路勤为径,学海无 | 涯苦作舟。 | | ĸ |

书山有路勤为径,学海无涯苦作舟。

| 11. | How many digits are there in the correct answer to the calculation 123 123 123 123 ÷ 123?A 4B 6C 8D 10E 12 |
|-------|---|
| 12. | The diagram shows a quadrilateral <i>PQRS</i> in which <i>PQ</i> and <i>QR</i> have the same length. Also <i>PR</i> bisects $\angle SPQ$, the ratio of $\angle SPR$ to $\angle PRS$ is 2 : 3 and $\angle PSR = 110^{\circ}$. How large is angle <i>PQR</i> ? A 1240 D 1200 C 1100 D 000 D 200 |
| | A 124° B 120° C 110° D 90° E 28° |
| 13. | The diagram shows a shape made from four 3 cm × 3 cm × 3 cm wooden cubes joined by their edges. What, in cm ² , is the surface area of the shape? A 162 B 180 C 198 D 216 E 234 |
| 14. | Billy has three times as many llamas as lambs.Milly has twice as many lambs as llamas.They have 17 animals in total.How many of the animals are llamas?A 5B 6C 7D 8E 9 |
| 15. | Beatrix places copies of the L-shape shown on a 4 × 4 board so that each L-shape covers exactly three cells of the board. She is allowed to turn around or turn over an L-shape. What is the largest number of L-shapes she can place on the board without overlaps? A 2 B 3 C 4 D 5 E 6 |
| 16. | How many pairs of digits (p, q) are there so that the five-digit integer ' $p869q$ ' is a multiple of 15? |
| 17. | A 2 B 3 C 4 D 5 E 6 The areas of the two rectangles in the diagram are 25 cm ² and 13 cm ² as indicated. What is the value of x? A 3 B 4 C 5 D 6 E 7 25 cm^2 13 cm^2 25 cm^2 3 cm y |
| 18. | Between them, the two five-digit integers M and N contain all ten digits from 0 to 9. What is the least possible difference between M and N ? |
| www.C | A 125 B 24/ C 42/ D 472 E 742 CasperYC.Club/ukmt 非淡泊无以明志,非宁静无以致远。 |

E 13

19. Which one of these could be folded to make a cube?



20. A drawer contains ten identical yellow socks, eight identical blue socks and four identical pink socks.

C 8

Amrita picks socks from the drawer without looking.

B 6

What is the smallest number of socks she must pick to be sure that she has at least two pairs of matching socks?

D 11

A 5

21. - vowels in this short sentence. There are – Which of the following options should replace "-----" to make the sentence in the box true? B thirteen C fourteen D fifteen A twelve E sixteen In the triangles PQR and STU, $\angle RPQ = 2 \times \angle UST$, $\angle PRQ = 2 \times \angle SUT$ and 22. $\angle RQP = \frac{1}{5} \times \angle UTS.$ How large is $\angle UTS$? A 90° B 100° C 120° E more information needed D 150° 23. Ali wants to fill the empty squares so that the number in 2 0 1 8 each square after the fourth from the left is the sum of the numbers in the four squares to its left. What number should Ali write in the final square? A 16 **B** 8 C 4 D 2 E 1 24. The shapes P and Q are formed from two and three identical O Р rectangles, respectively. Their perimeters are 58 cm and 85 cm respectively. What is the perimeter of one of the rectangles? A 30 cm B 31 cm C 32 cm D 33 cm E 34 cm In the diagram PQ and QR are sides of a regular 25. *n*-sided polygon, $\angle SPQ = \angle SRQ = 80^\circ$, R $\angle PTR = 70^{\circ} \text{ and } PT = ST = RT.$ 80° What is the value of *n*? 70° A 15 B 18 C 20 D 24 E 30 Q 80°



Intermediate Challenge

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Last updated: September 22, 2023

INSTRUCTIONS

- 1. Do not open the paper until the invigilator tells you to do so.
- Time allowed: 60 minutes.
 No answers, or personal details, may be entered after the allowed time is over.
- 3. The use of blank or lined paper for rough working is allowed; squared paper, calculators and measuring instruments are forbidden.
- 4. Use a B or an HB non-propelling pencil. Mark at most one of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.
- 5. **Do not expect to finish the whole paper in the time allowed.** The questions in this paper have been arranged in approximate order of difficulty with the harder questions towards the end. You are not expected to complete all the questions during the time. You should bear this in mind when deciding which questions to tackle.
- 6. Scoring rules:

5 marks are awarded for each correct answer to Questions 1-15; 6 marks are awarded for each correct answer to Questions 16-25; Each incorrect answer to Questions 16-20 loses 1 mark; Each incorrect answer to Questions 21-25 loses 2 marks.

- 7. Your Answer Sheet will be read by a machine. **Do not write or doodle on the sheet except to mark your chosen options.** The machine will read all black pencil markings even if they are in the wrong places. If you mark the sheet in the wrong place, or leave bits of eraser stuck to the page, the machine will interpret the mark in its own way.
- 8. The questions on this paper are designed to challenge you to think, not to guess. You will gain more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. This paper is about solving interesting problems, not about lucky guessing.



| A 16 | | B 21 | C 28 | D 3/ | F 45 |
|---|----------------------------------|---------------------------------------|-----------------------------------|----------------------------|----------------------------|
| A 10 | 1 6.1 | D 21 | C 28 | D 34 | E 43 |
| 2. What is t | he area of th | is triangle? | | | 4 cm 5 cm |
| A 6 c | m^2 | B $7.5 \mathrm{cm}^2$ | $C \ 8 \ cm^2$ | $D 10 \text{ cm}^2$ | $E 12 \text{ cm}^2$ |
| 3. What is t | he value of 1 | - (2 - (3 - 4 - | -(5-6)))? | | |
| A -2 | | B -1 | C 0 | D 1 | E 2 |
| 4. The diag connects | ram shows a two midpoir | square, its two nts of sides of th | diagonals and two e square. | line segments, each | of which |
| What fra | ction of the a | rea of the squar | e is shaded? | | |
| A $\frac{1}{8}$ | | B $\frac{1}{10}$ | C $\frac{1}{12}$ | D $\frac{1}{16}$ | $E \frac{1}{24}$ |
| 5. We know What is t | that 1 + 2 + this integer? | 3 + 4 = 10. It is | also true that 1^3 + | $2^3 + 3^3 + 4^3 = 10^n$ f | or some integer <i>n</i> . |
| A 1 | | B 2 | C 3 | D 4 | E 5 |
| 6. To draw | a 3 by 3 squa | re grid you need | l 8 straight lines, a | s shown. | |
| How ma | ny straight lir | nes do vou need | to draw a <i>n</i> by <i>n</i> so | uare grid? | |
| A <i>n</i> + | 5 B | 3n - 1 C | $2 n^2 - 1$ D | 4(n-1) E 2(n | + 1) |
| 7 What is (|) 015% of 60 |) million? | | | |
| A 900 |) | B 9000 | C 90,000 | D 900000 | E 9000000 |
| 8 $\sqrt{\sqrt{x}} = 3$ | What is the | $\frac{2}{2}$ value of r^2 | | _ / 00 000 | _ / 000 000 |
| $\int \int \int \int \partial f = \int \partial f $ | $\overline{\sqrt{2}}$ | $D \sqrt{2}$ | C 0 | D 12 | E 91 |
| 9. Merryn y | s vrote down f | ы уз he numbers 2 0 | 2 3 and one furth | D 12 | E 01 |
| What wa | s the median | of her five num | bers? | ier number. | |
| A 0 E mo | re informatic | B 2 on required | C | 2.5 | D 3 |
| 0. | Across 1. A powe 2. A powe | Down ar of 5 1. A ar of 4 | n power of 6 | 2 | 1 |
| Eight of | the digits fro | m 0 to 9 inclusi | ve are used to fill t | he cells of the crossn | umber. |
| What is t | he cum of th | a two digits whi | ch are not used? | | |

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| 11. | Jill was given a l the remaining ja What was the we | arge jar of jam. m to Jas. Jill w eight, in kg, of | She gave one as left with 1 k the jam in Jill' | sixth of the sg of jam. s jar at the s | jam to Jan. Jill th tart? | en gave one thirteenth of |
|------|--|---|--|--|--|--|
| | A 1.2 | B 1.3 | С | 1.4 | D 1.6 | E 1.9 |
| 12. | In the diagram, <i>SRUVW</i> is a reg | <i>PQRS</i> is a sequence of the se | quare, <i>PST</i> is | s an equilat | eral triangle and | $Q \qquad P$ |
| | What is the size | of angle WTS? | | | | $R \longrightarrow S$ |
| | A 35° | B 36° | C 37° | D 38° | E 39° | |
| 13. | The mean of p a What is the mean | nd q is 13; the p of p , q and r ? | mean of q and | <i>r</i> is 16; the | mean of r and p i | s 7. |
| | A 12 | B 13 | С | 14 | D 15 | E 16 |
| 14. | A regular octage <i>RSVW</i> , four smathese four triang | on <i>PQRSTUVV</i> all triangles ins les? | V has sides of ide the octago | length 2 cm n remain un | . When I shade th shaded. What is t | the rectangles $PQTU$ and the total area, in cm^2 , of |
| | A 1 | B 2 | С | 4 | D 6 | E 8 |
| 15. | How many of the | e following poly | gons could ex | tist? | | |
| | A triangle with a A quadrilateral v A pentagon with | ll three sides th vith all four sid all five sides th | e same length es the same len ne same length | , but three d ngth, but fou , but five dif | ifferent interior ar r different interio ferent interior ang | ngles. r angles. gles. |
| | A only the pe C the quadril E none of the | ntagon ateral and the p em | entagon | B o D a | nly the quadrilater 1 three | ral |
| 16. | The sum of the l of the lengths of What is the area, | engths of the th the three sides $\sin \text{cm}^2$, of the | ree sides of a of the triangle triangle? | right-angled is 98 cm ² . | triangle is 16 cm. | . The sum of the squares |
| | A 8 | B 10 | С | 12 | D 14 | E 16 |
| 17. | A 3 by 2 rectant triangles, as show Those four triang in the right-hand What is the ratio of the rhombus? | ngle is split in wn in the left-ha gles are rearrang diagram. of the perimeter | to four congr and diagram. ged to form a r | uent right-a hombus, as le to the per | shown $2 \\ \frac{1}{2}$ | |
| | A 3:2 | B 2:1 | С | 1:1 | D 1:2 | E 2:3 |
| 18. | How many squar | res are exactly f | our greater that | an a prime? | | |
| | A 0 | B 1 | С | 2 | D 3 | E 4 |
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- **19.** What is the positive difference between the numerator and the denominator when the expression shown is written as a single fraction in its simplest form? A 2n+2 B n+2 C n D 2 E 1 $n+1-\frac{n+2}{n+3}$
- **20.** I roll two standard six-sided fair dice. At least one of the scores obtained on the dice is 3. What is the probability that both of the scores on the dice are 3?
 - A $\frac{1}{12}$ B $\frac{1}{11}$ C $\frac{1}{6}$ D $\frac{1}{3}$

21. A semicircle of radius 3 units is drawn on one edge of a right-angled triangle, and a semicircle of radius 4 units is drawn on another edge. The semicircles intersect on the hypotenuse of the triangle, as shown. What is the shaded area, in square units?

A
$$\frac{25\pi}{2} - 24$$
 B 12 C $\frac{25\pi}{2} - 6$ D $25\pi - 24$ E 24

22. The numbers *x* and *y* satisfy both of the equations

23x + 977y = 2023 and 977x + 23y = 2977.

What is the value of $x^2 - y^2$?

- A 1 B 2 C 3 D 4 E 5
- **23.** It is possible to choose, in two different ways, six different integers from 1 to 9 inclusive such that their product is a square. Let the two squares so obtained be p^2 and q^2 , where p and q are both positive. What is the value of p + q?

24. A rectangle *PQRS* has side-lengths *a* and *b*, with a < b. The rectangle *PTUV* has side-lengths *c* and *d*, with c < d. Also, a < d and c < b, as shown. The sides *RS* and *TU* cross at *X*. Which of these conditions guarantees that *Q*, *X* and *V* lie on a straight line?



25. The diagram shows two unshaded circles which touch each other and also touch a larger circle. Chord PQ of the larger circle is a tangent to both unshaded circles. The length of PQ is 6 units.

What is the area, in square units, of the shaded region?

A 3π B $\frac{7\pi}{2}$ C 4π D $\frac{9\pi}{2}$ E 5π



 $E \frac{1}{4}$





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|------|---|--|--|---|--------------------------------------|
| 1. | How many hours is 6 | minutes? | | | |
| | A 0.06 | B 0.1 | C 0.6 | D 10 | E 360 |
| 2. | My recipe for apple cr topping. When my fan enough crumble. In to | umble uses 100 g of nily come for a meal otal, how much crum | flour, 50 g of butter a , I have to use two an ble topping do I ther | and 50 g of sugar to 1 Id a half times each a 1 make? | make the crumble mount to provide |
| | A 0.5 kg | B 2 kg | C 2.5 kg | D 5 kg | E 50 kg |
| 3. | In the Caribbean, logg On average, how many | gerhead turtles lay th y eggs are in each ne | ree million eggs in t est? | wenty thousand nest | S. |
| | A 15 | B 150 | C 1500 | D 15000 | E 150000 |
| 4. | Workers digging a tun Working every day, ho | nel for an undergrou w long will it take th | and railway complete them to dig a tunnel of | e 5 metres of tunnel of length 2 kilometre | on a typical day. os? |
| | A three months E nearly two years | B six months | C just ur | nder a year D ju | ust over a year |
| 5. | Which of the followin | g has the same value | e as 10006 - 8008? | | |
| | A 10007 - 8007 E 5003 - 4004 | B 100060 - 8 | 30 080 C 10 000 |) – 8002 D 1 | 06 – 88 |
| 6. | What is 20% of $3\frac{3}{4}$? | | | | |
| | A $\frac{123}{200}$ | B $\frac{13}{20}$ | C $\frac{7}{10}$ | D $\frac{3}{4}$ | $E \frac{4}{5}$ |
| 7. | A function machine de | oes the four operatio | ns shown in order. | | |
| | | | -10 ÷3 × | -10 | |
| | Iris inputs a positive in What is the smallest p | nteger and the output | t is also a positive in the lris could have ing | teger. out? | |
| | A 9 | B 84 | C 102 | D 120 | E 129 |
| 8. | What is the difference | between 40% of 50 | % of 60 and 50% of | 60% of 70? | |
| | A 9 | B 8 | C 7 | D 6 | E 5 |
| 9. | A number x is greater | than 2022. Which is | s the smallest of the | following? | |
| | A $\frac{x}{2022}$ | B $\frac{2022}{x-1}$ | C $\frac{x+1}{2022}$ | D $\frac{2022}{x}$ | $E \frac{2022}{x+1}$ |
| 10. | One hundred rectangle | es are arranged edge | -to-edge in a continu | ation of the pattern | shown. |
| | | | | | |
| | Each rectangle measur | res 3 cm by 1 cm. W | hat is the perimeter, | in cm, of the compl | eted shape? |
| | A 800 | B 700 | C 602 | D 600 | E 502 |
| 11. | The Universal Magazi "What number is that, What is the correct an | ne of Knowledge an whose quarter shall swer? | d Pleasure (Vol. 1, 1 be 9 more than the v | 747) asked the follo whole?" | wing question. |
| | A 12 | B 9 | C 8 | D -8 | E -12 |

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12. The shape shown is made up of three similar right-angled triangles. The smallest triangle has two sides of side-length 2, as shown.

What is the area of the shape?

B $12 + 12\sqrt{2}$ C 28 A 14

- 13. How many sets of three consecutive integers are there in which the sum of the three integers equals their product ?
 - A 0 B 2 C 3

14. In a number pyramid, each cell above the bottom row contains the sum of the numbers in the two cells immediately below it. The three numbers on the second row are all equal, and are all integers. Which of these statements must be true?

- A The bottom row contains at least one zero
- C The top number is a multiple of three
- E None of the above
- 15. Reflection in the line l transforms the point with coordinates (5, 3) into the point with coordinates (1, -1).

What is the equation of the line *l*?

A y = x - 2 B y = 1 C x = 3 D y = 2 - x E y = 4 - x

 $C 4^{2021}$

16. What is half of 4^{2022} ?

A 4¹⁰¹¹

17. The first figure shows four touching circles of radius 1 cm in a horizontal row, held together tightly by an outer band X.

B 2^{4044}

The second figure shows six touching circles of radius 1 cm, again held tightly together by a surrounding band Y. Which of the following statements is true?

| А | X i | s 2 | cm | longer | than | Y |
|---|-----|-----|----|--------|------|---|
|---|-----|-----|----|--------|------|---|

- C X and Y are the same length
- E Y is 2 cm longer than X
- 18. Dick Turnip sold his horse, Slack Bess, for £56. The percentage profit he made was numerically the same as the cost, in pounds, of his horse. What was the cost of his horse?
 - A £36 B £40 C £45 D £48 E £50

B The third row contains at least one zero

D The top number is a multiple of four

D 24 + $20\sqrt{2}$

D 4

D 2⁴⁰⁴³

B X is 1 cm longer than Y D Y is 1 cm longer than X

E 2¹⁰¹¹





E 56

E 5 third row second row -



How many possible values for x are there?

A 1 B 2 C 4 D 8 E 16



| 1. | What is the | value of 2021 | - 2223 + 2423 | 5? | | | |
|----|--|--|--|--|---|-------------------------------|--|
| | A 2122 | В | 2223 | C 2324 | D | 2425 | E 2526 |
| 2. | The day bef Today is Th | Fore the day be ursday. On wi | efore yesterday hat day was my | was two day / birthday? | vs after the day | y before my bir | thday. |
| | A Sunda | y B | Monday | C Tuesda | ny D | Wednesday | E Friday |
| 3. | What is the | value of $2 - ($ | (-2-2) - (-2) | -(-2-2)) |)? | | |
| | A 0 | В | 2 | C 4 | D | 6 | E 8 |
| 4. | The diagram Angles <i>PUV</i> What is ang A 44° | n shows three V and QYX ar le VWX? B 48° | squares, <i>PQR</i> e 62° and 74° C 60° | S, <i>TUVW</i> arrespectively. D 64° | nd <i>WXYZ</i> . E 68° | S U 62° P | R V V V X 74° V Q |
| 5. | April, May that June ha How many | and June have is. April has t sweets does Ju | 90 sweets bet wo-thirds of th ane have? | ween them. e number of | May has three sweets that N | e-quarters of the Aay has. | e number of sweets |
| | A 60 | В | 52 | C 48 | D | 40 | E 36 |
| 6. | Kai has beg What is the | un to list, in a sixth number | scending order on Kai's list? | , the positive | e integers whi | ich are <i>not</i> facto | ors of 240. |
| | A 11 | В | 13 | C 14 | D | 15 | E 17 |
| 7. | What is the | value of (4 – | $(\frac{1}{4}) \div (2 - \frac{1}{2})?$ | | | | |
| | A $1\frac{1}{2}$ | В | 2 | C $2\frac{1}{2}$ | D | 3 | E $4\frac{1}{4}$ |
| 8. | 3. The diagram shows two 10 by 14 rectangles which are edge-to-edge and share a common vertex. It also shows the centre <i>O</i> of one rectangle and the midpoint <i>M</i> of one edge of the other. What is the distance <i>OM</i> ? | | | | | | |
| | A 12 | B 15 | C 18 | 3 I | D 21 | E 24 | |
| 9. | How many | of the following | ng statements a | are true? | | | |
| | | A prime mu A square mu An odd num | ltiplied by a pr ltiplied by a so ber multiplied | ime is alway quare is alwa by an odd n | rs a prime. ays a square. umber is alwa | ays an odd num | ber. |

An even number multiplied by an even number is always an even number.

D 3

C 2

E 4

A 0

B 1

10. The prime factor decomposition of 2021 is 43×47 .

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What is the value of 53×57 ? A 2221 B 2521 C 2921 D 3021 E 3031 11. The line with equation y = 2x + 3 is reflected in the x-axis. Which of the following is the equation of the new line? A y = 2x - 3 B y = -2x + 3 C x = 2y + 3 D $y = \frac{1}{2}x + 3$ E y = -2x - 312. And rew calculates that 40% of 50% of x is equal to 20% of 30% of y, where $x \neq 0$. Which of the following is true? A $y = \frac{2x}{3}$ B $y = \frac{4x}{3}$ C y = 2x D $y = \frac{8x}{3}$ E $y = \frac{10x}{3}$ **13.** What is the remainder when 12345×54321 is divided by 9? C 2 E 4 A 0 **B** 1 D 3 14. The diagram shows a large square divided into squares of three different sizes. What percentage of the large square is shaded? B 59% C 57% D 55% E 53% A 61% 15. Patrick drives from P to Q at an average speed of 40 mph. His drive back from Q to P is at an average speed of 45 mph and takes two minutes less. How far, in miles, is it from P to Q? A 1.5 C 9 B 6 D 12 E 15 **16.** A semicircle is drawn on each side of a square, as shown. The square has sides of length 2π . What is the area of the resulting shape? A $2\pi^2(\pi + 1)$ B $\pi^2(\pi + 2)$ C $2\pi^2(2\pi + 1)$ D $\pi^2(\pi + 4)$ 2π E $2\pi^2(\pi + 2)$ 17. In the rectangle PQRS, the side PQ is of length 2 and the side QR is of length 4. Points T and U lie inside the rectangle so that *PQT* and *RSU* are equilateral triangles. What is the area of the quadrilateral *QRUT*? A $\frac{6-\sqrt{3}}{2}$ B $\frac{8}{3}$ C $4 - 2\sqrt{3}$ D $4 - \sqrt{3}$ E 3 **18.** Which of these is closest in size to 1? A 0.95 B 1.05 C 0.960 D 1.040 E 0.95 **19.** The diagram shows two overlapping rectangles, each measuring p by q. The area of overlap is exactly one-quarter of the total area of the figure. What is the ratio p: q? q A 5:2 B 4:1 C 3 : 1 D 2 : 1 E 3:2

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16

0

2

E $2\pi - 2$

- **20.** Two straight lines have equations y = px + 4 and py = qx 7, where *p* and *q* are constants. The two lines meet at the point (3, 1). What is the value of *q*?
 - A 1 B 2 C 3 D 4 E 5

21. The diagram shows two congruent equilateral triangles whose overlap is a hexagon. The areas of the smaller triangles, which are also equilateral, are 1, 1, 9, 9, 16 and 16, as shown.

What is the area of the inner hexagon?

- A 68 B 58 C 48 D 38 E 28
- **22.** What is the result when we simplify the expression $\left(1 + \frac{1}{x}\right)\left(1 \frac{2}{x+1}\right)\left(1 + \frac{2}{x-1}\right)$?
 - A 1 B $\frac{1}{x(x+1)}$ C $\frac{1}{(x+1)(x-1)}$ D $\frac{1}{x(x+1)(x-1)}$ E $\frac{x+1}{x}$
- 23. The diagram shows a semicircle with centre O and radius 2 and a semicircular arc with diameter PR. Angle POR is a right angle. What is the area of the shaded region?

A $\pi - 2$ B 2

24. Sam writes on a white board the positive integers from 1 to 6 inclusive, once each. She then writes p additional fives and q sevens on the board. The mean of all the numbers on the board is then 5.3. What is the smallest possible value of q?

D 3

A 7 B 9 C 11 D 13 E 15

 $C \pi$

25. Thomas has constant speeds for both running and walking. When a down-escalator is moving, Thomas can run down it in 15 seconds or walk down it in 30 seconds. One day, when the escalator was broken (and stationary), it took Thomas 20 seconds to run down it. How long, in seconds, would it take Thomas to walk down the broken escalator?

A 30 B 40 C 45 D 50 E 60



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|--|--|---|---|----------------------------------|------------------|
| 1. What is the valu | ue of $2 - (3 - 4)$ | -(5-6-7) | ? | | |
| A 11 | B 9 | С | 5 | D -5 | E -7 |
| 2. Which one of th | nese is a multipl | e of 24? | | | |
| A 200 | B 300 | С | 400 | D 500 | E 600 |
| 3. What is the diffe | erence between | 25% of £37 a | nd 25% of £1 | 7? | |
| A £4.25 | B £5 | C | £6 | D £7.50 | E £9.25 |
| 4. What fraction of | f this diagram is | s shaded? | | | |
| A $\frac{13}{32}$ | $B \frac{1}{2}$ | $C \frac{9}{16}$ | D $\frac{5}{8}$ | E $\frac{13}{16}$ | |
| 5. Four of the following Which is the odd | owing coordinat d one out? | e pairs are the | corners of a | square. | |
| A (4, 1) | B (2, 4) | C | (5, 6) | D (3, 5) | E (7, 3) |
| 6. Which of the fo | llowing has the | largest value? | | | |
| A 2 ⁶ | B 3 ⁵ | С | 4 ⁴ | D 5 ³ | E 6 ² |
| 7. Kartik wants to a to shade the rem grids.In how many or other? | shade three of th naining two squ f the finished g | e squares in th ares red. The rids are Lucy | nis grid blue an ere are ten pos 's red squares | nd Lucy wants ssible finished | |
| A 3 | B 4 | C 5 | D 6 | E 8 | |
| 8. One of these on | tions gives the | value of $17^2 +$ | $19^2 + 23^2 + 23^2$ | 29 ² . Which is it? | |
| A 2004 | В 2008 | С | 2012 | D 2016 | E 2020 |
| 9. Adam's house n | number is in exa | ctly one of the | e following ra | nges. Which one? | |
| A 123 to 213 | B 132 to | 231 C | 123 to 312 | D 231 to 312 | E 312 to 321 |
| 10. What is the valu | the of $\frac{2468 \times 24}{2468 + 24}$ | $\frac{68}{68}$? | | | |
| A 2 | B 1234 | C | 2468 | D 4936 | E 6091024 |
| 11. I start at square higher numbere How many poss | e "1", and have d adjacent squa sible routes are t | to finish at sc re. here? | quare "7", mo | wing at each step to a | 2 4 6 1 3 5 7 |
| A 7 | B 9 | C 10 | D 11 | E 13 | |
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| 12. | Farmer Fatima rea and now there are How many goats | ars chickens and 200 fewer legs did she sell? | goats. Today she return on my farm than before | ed from market and s e!" | aid, "I sold 80 animals, |
|-----|--|---|--|-----------------------------|--------------------------|
| | A 15 | B 20 | C 25 | D 30 | E 35 |
| 13. | What is half of 1. | 6×10^{6} ? | | | |
| | A 8×5^6 | B 4×10^6 | C 8×10^5 | D 8×10^2 | E 1.6×10^3 |
| 14. | The result of the of What is the value | calculation 9×1 of <i>n</i> ? | $11 \times 13 \times 15 \times 17$ is the | e six-digit number '3n | 18185'. |
| | A 2 | B 4 | C 6 | D 8 | E 0 |
| 15. | Triangle <i>PQR</i> har right-angled trian | as been divideo gles, as shown. | l into twenty-five con The length of <i>RP</i> is 2.4 | gruent l cm. | R |
| | What is the length | n of <i>PQ</i> ? | | | |
| | A 3 cm E 4.8 cm | B 3.2 cm | C 3.6 cm D 4 cm | n P | Q |
| 16. | As a decimal, wh | at is the value of | $f \frac{1}{9} + \frac{1}{11}?$ | | |
| | A 0.10 | В 0.20 | C 0.2020 | D 0.202020 | E 0.20 |
| | | | | | |

17. The Knave of Hearts stole some tarts. He ate half of them, and half a tart more. The Knave of Diamonds ate half of what was left, and half a tart more. Then the Knave of Clubs ate half of what remained, and half a tart more. This left just one tart for the Knave of Spades. How many tarts did the Knave of Hearts steal?

- A 63 B 31 C 19 D 17 E 15
- 18. The diagram shows an isosceles right-angled triangle which has a hypotenuse of length *y*. The interior of the triangle is split up into identical squares and congruent isosceles right-angled triangles.What is the total shaded area inside the triangle?

What is the total shaded area inside the triangle?

- A $\frac{y^2}{2}$ B $\frac{y^2}{4}$ C $\frac{y^2}{8}$ D $\frac{y^2}{16}$ E $\frac{y^2}{32}$
- **19.** The diagram shows two squares and four equal semicircles. The edges of the outer square have length 48 and the inner square joins the midpoints of the edges of the outer square. Each semicircle touches two edges of the outer square, and the diameter of each semicircle lies along an edge of the inner square.

What is the radius of each semicircle?

A 10 B 12 C 14 D 16 E 18





20. For any fixed value of *x*, which of the following four expressions has the largest value?

21. The diagram shows four semicircles, one with radius 2 cm, touching the other three, which have radius 1 cm.



E $\frac{1}{2}\pi$

What is the total area, in cm^2 , of the shaded regions?

A 1 B $\pi - 2$ C $2\pi - 5$ D $\frac{3}{2}$

- **22.** The diagram shows a regular pentagon and an irregular quadrilateral.What is the sum of the three marked angles?A 72° B 90° C 108° D 126° E 144°
- **23.** Five congruent triangles, each of which is half a square, are placed together edge to edge in three different ways as shown to form shapes P, Q and R.



Which of the following lists gives the shapes in ascending order of the lengths of their perimeters? A P, Q, R B Q, P, R C R, Q, P D R, P, Q E P, R, Q

- **24.** The positive integers *m* and *n* are such that $10 \times 2^m = 2^n + 2^{n+2}$. What is the difference between *m* and *n*?
 - A 1 B 2 C 3 D 4 E 5

25. The diagram shows six points *P*, *Q*, *R*, *S*, *T* and *U* equally spaced around a circle of radius 2 cm. The inner circle has radius 1 cm. The shaded region has three lines of symmetry.

Which of the following gives the area, in cm², of the shaded region?

A $2\pi + 3$ B $3\pi + 2$ C $\frac{4\pi + 3}{2}$ D $3(\pi + 2)$ E $4\pi + 3$

 $U = \frac{P}{T} = \frac{Q}{S}$

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Senior Challenge

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Last updated: September 22, 2023

INSTRUCTIONS

- 1. Do not open the paper until the invigilator tells you to do so.
- Time allowed: 90 minutes.
 No answers, or personal details, may be entered after the allowed time is over.
- 3. The use of blank paper for rough working is allowed; squared paper, calculators and measuring instruments are forbidden.
- 4. Use a B or an HB non-propelling pencil. Mark A, B, C, D, E on the Answer Sheet for each question. Mark only one option, boldly, within the box.
- 5. Your Answer Sheet will be read by a machine. **Do not write or doodle on the sheet except to mark your chosen options.** The machine will read all markings, including bits of eraser stuck to the page, and interpret the mark in its own way.
- 6. **Do not expect to finish the whole paper in the time allowed.** The questions in this paper have been arranged in approximate order of difficulty with the harder questions towards the end. You are not expected to complete all the questions during the time. You should bear this in mind when deciding which questions to tackle.
- 7. Scoring rules: All candidates start with 25 marks; 0 marks are awarded for each question left unanswered; 4 marks are awarded for each correct answer; 1 mark is deducted for each incorrect answer (to discourage guessing).
- 8. The questions on this paper are designed to challenge you to think, not to guess. You will gain more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. This paper is about solving interesting problems, not about lucky guessing.





| 1. | When the expression is obtained? | $n \frac{(2^2 - 1) \times (2 \times 3)}{(2 \times 3) \times 2}$ | $(3^2 - 1) \times (4^2 - 1) \times (3 \times 4) \times (4 \times 5) \times $ | $\frac{(5^2 - 1)}{5 \times 6}$ is simplified, | which of the following |
|-----|--|---|--|--|--|
| | A $\frac{1}{2}$ | B $\frac{1}{3}$ | $C \frac{1}{4}$ | D $\frac{1}{5}$ | $E \frac{1}{6}$ |
| 2. | What is the smallest | t prime which | is the sum of five dif | ferent primes? | |
| | A 39 | B 41 | C 43 | D 47 | E 53 |
| 3. | The figure shows a r | regular hexago | on. | | |
| | How many parallelo | ograms are the | ere in the figure? | | |
| | A 2 E more than 8 | B 4 | C 6 | D 8 | |
| 4. | The diagram shows | two symmetric | cally placed squares v | with sides of length 2 an | d 5. |
| | What is the ratio of | the area of the | e small square to that | of the shaded region? | |
| | A 7:24 I | 3 1:3 | C 8:25 D | 8:21 E 2:5 | |
| 5. | What is the value of | $\frac{1}{1.01} + \frac{1}{1.1} + \frac{1}{1.1}$ | $-\frac{1}{1}+\frac{1}{11}+\frac{1}{101}$? | | |
| | A 2.9 | B 2.99 | C 3 | D 3.01 | E 3.1 |
| 6. | What is the value of | $\frac{4^{800}}{8^{400}}$? | | | |
| | A $\frac{1}{2^{400}}$ | B $\frac{1}{2^{200}}$ | C 1 | D 2 ²⁰⁰ | $E 2^{400}$ |
| 7. | In 2021, a first clas to spend an exact number of stamps th | s postage star umber of pour nat should be j | np cost 85p and a se nds and to buy at lea purchased? | cond class postage star st one of each type, wh | mp cost 66p. In order hat is the smallest total |
| | A 10 | B 8 | C 7 | D 5 | E 2 |
| 8. | In the diagram, the | outer hexagon | is regular and has ar | area of 216. | |
| | What is the shaded a | area? | | | |
| | A 108 H | 3 96 | C 90 D | 84 E 72 | |
| 9. | A light-nanosecond second. The speed of | is the distanc of light is $3 \times$ | that a photon can tr $10^8 \mathrm{ms}^{-1}$. | ravel at the speed of lig | ht in one billionth of a |
| | How far is a light-na | anosecond? | | | |
| | A 3 cm | B 30 cm | C 3 m | D 30 m | E 300 m |
| 10. | What is the value of | f x in the equa | tion $\frac{1+2x+3x^2}{2+2x+x^2} = 3$ | 3? | |
| | A -5 | B -4 | $3 + 2x + x^2$ C -3 | D -2 | E -1 |
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非淡泊无以明志,非宁静无以致远。

| UKN | ſT | Senio | r Mathematical Ch | allenge 2022 | Page 168 of 248 |
|-----|---|---|---|--|---|
| 11. | In the number tri Each disc in the to of the two number | er. et <u>2022</u> | | | |
| | What is the value | e of <i>n</i> ? | | | |
| | A 1 | B 2 C | 3 D 6 | E 33 | |
| 12. | What is the sum | of the digits of the | integer which is ea | qual to $6666666^2 - 33^2$ | 333333 ² ? |
| | A 27 | B 36 | C 45 | D 54 | E 63 |
| 13. | Three rugs have area 60 m^2 , the at | a combined area or rea which is covere | f 90 m ² . When the ed by exactly two la | y are laid down to con ayers of rug is 12 m ² . | ver completely a floor of |
| | What is the area | of floor covered by | exactly three layer | rs of rug? | |
| | $A 2 m^2$ | $B 6 m^2$ | $C 9 m^2$ | $D 10 m^2$ | $E 12 m^2$ |
| 14. | The diagram show inside it, as show ratio 1 : 2. Simila T dividing PQ in | ws a square, <i>KLMN</i> on in the diagram, w arly, a third square 7 of the ratio 1 : 2. | V. A second square where P divides the TUVW is drawn in | <i>PQRS</i> is drawn e side <i>KL</i> in the side <i>PQRS</i> with | $K \xrightarrow{P} L$ $W \xrightarrow{T} Q$ |
| | What fraction of | the area of KLMN | is shaded? | | |

A $\frac{25}{81}$ B $\frac{16}{49}$ C $\frac{4}{9}$ D $\frac{40}{81}$ E $\frac{2}{3}$

15. The hare and the tortoise had a race over 100 m, in which both maintained constant speeds. When the hare reached the finish line, it was 75 m in front of the tortoise. The hare immediately turned around and ran back towards the start line.

How far from the finish line did the hare and the tortoise meet?

- A 54 B 60 C 64 D $66\frac{2}{3}$ E 72
- **16.** Which diagram could be a sketch of the curve $\sqrt{x} + \sqrt{y} = 1$?



17. The shape shown is made by removing four equilateral triangles with side-length 1 from a regular octagon with side-length 1.

What is the area of the shape?

A $2 - 2\sqrt{2} + \sqrt{3}$ B $2 + 2\sqrt{2} - \sqrt{3}$ C $2 + 2\sqrt{2} + \sqrt{3}$ D $3 - 2\sqrt{2} - \sqrt{3}$ E $2 - 2\sqrt{2} - \sqrt{3}$

18. The numbers x and y are such that $3^x + 3^{y+1} = 5\sqrt{3}$ and $3^{x+1} + 3^y = 3\sqrt{3}$.

What is the value of $3^x + 3^y$?

A $\sqrt{3}$ B $2\sqrt{3}$ C $3\sqrt{3}$ D $4\sqrt{3}$

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R

E

M

N



19. How many pairs of real numbers (x, y) satisfy the simultaneous equations $x^2 - y = 2022$ and $y^2 - x = 2022$?

A infinitely manyB 1C 2D 3E 4

20. A square is inscribed inside a quadrant of a circle. The circle has radius 10.

What is the area of the square?

A $25\sqrt{2}$ B 36 C 12π D 40 E $30\sqrt{2}$

21. The perimeter of a logo is created from two vertical straight edges, two small semicircles with horizontal diameters and two large semicircles. Both of the straight edges and the diameters of the small semicircles have length 2. The logo has rotational symmetry as shown.

What is the shaded area?

A 4 B $4 - \pi$ C 8 D $4 + \pi$ E 12

22. How many pairs of integers (x, y) satisfy the equation $\sqrt{x - \sqrt{x + 23}} = 2\sqrt{2} - y$? A 0 B 1 C 4 D 8 E infinitely many

23. Three squares *GQOP*, *HJNO* and *RKMN* have vertices which sit on the sides of triangle *FIL* as shown. The squares have areas of 10, 90 and 40 respectively.

What is the area of triangle *FIL*?

- A 220.5 B $\frac{21}{5}\sqrt{10}$ C 252 D $\frac{21}{2}\sqrt{10}$ E 441
- 24. The numbers x, y, p and q are all integers. x and y are variable and p and q are constant and positive. The four integers are related by the equation xy = px + qy.

When y takes its maximum possible value, which expression is equal to y - x?

A pq-1E (p+1)(q+1)B (p-1)(q-1)C (p+1)(q-1)D (p-1)(q+1)

25. A drinks carton is formed by arranging four congruent triangles as shown. QP = RS = 4 cm and PR = PS = QR = QS = 10 cm.

What is the volume, in cm³, of the carton?

A $\frac{16}{3}\sqrt{23}$ B $\frac{4}{3}\sqrt{2}$ C $\frac{128}{25}\sqrt{6}$ D $\frac{13}{2}\sqrt{23}$ E $\frac{8}{3}\sqrt{6}$

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| 1. Cicely had he | er 21 st birthday in 1939. | _ | | |
|--|---|---|---|----------------------------|
| When did she A 2020 | e have her 100 th birthda <u>y</u> B 2019 | y? C 2018 | D 2010 | E 2008 |
| 2. The sequence 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, | e, formed from the sequ 0, 20, 20, 20, 30, | ence of primes by r | ounding each to the | nearest ten, begins 0, 0, |
| When contin | ued, how many terms in | this sequence are e | qual to 40? | |
| A 1 | B 2 | C 3 | D 4 | E 5 |
| 3. The diagram The angles m | shows two congruent re arked x° are equal. | gular pentagons and | d a triangle. | x° |
| What is the v | alue of <i>x</i> ? | | | |
| A 24 | B 30 C 36 | D 40 | E 45 | x° |
| 4. The positive What is the s | integer k is a solution of um of the digits of k? | f the equation $(k \div$ | $(12) \div (15 \div k) = 20.$ | |
| A 15 | B 12 | C 9 | D 6 | E 3 |
| 5. The sum of f What is the la | our consecutive primes argest of the four primes | is itself prime. | D 12 | E 7 |
| A 3/ | В 29 | C 19 | D 13 | E / |
| 6. Three points, O. The arc le In what ratio A 1:1: | P, Q and R are placed engths PQ, QR and RP are the areas of the sect B = 1 : 2 : 3 | on the circumferen are in the ratio 1 : 2 ors <i>POQ</i> , <i>QOR</i> an C 1 : π : π^2 | ce of a circle with ce 2 : 3. d <i>ROP</i> ? D 1 : 4 : 9 | ntre O• |
| E 1:8:2 | 27 | | | |
| 7. Which of the A 2 ⁵⁰⁰⁰ | se numbers is the larges B 3 ⁴⁰⁰⁰ | t? C 4 ³⁰⁰⁰ | D 5 ²⁰⁰⁰ | E 6 ¹⁰⁰⁰ |
| 8. What is the a | rea of the region inside | the quadrilateral P | QRS? | |
| A 18 E more in | B 24 C | C 36 D 4 | 8 3 4 | S 13 |
| | | | <i>R</i> ² | 12 Q |
| 9. Alison has a | set of ten fridge magnet | s showing the integ | ers from 0 to 9 inclu | sive. |
| In how many of 5? | different ways can she sp | plit the set into five p | pairs so that the sum c | of each pair is a multiple |
| A 1 | B 2 | C 3 | D 4 | E 5 |
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| 10. | In a survey, per The pie chart s with no rounding | apple | | | | |
|-----|--|--|---|---|--|---|
| | What is the sn surveyed? | nallest number | of people who | could have been | 109 | 140° |
| | A 45 | B 60 C | 2 80 D 90 | E 180 | cherry ¹⁰⁸ | 72° ^{40°} plum rhubarb |
| 11. | Alitta claims th | at if p is an odd | d prime then p^2 – | - 2 is also an odd p | orime. | |
| | Which of the fo | ollowing values | of p is a countered | example to this cla | im? | |
| | A 3 | B 5 | C 7 | D | 9 | E 11 |
| 12. | For how many | positive integer | s N is the remain | der 6 when 111 is | divided by N? | |
| | A 5 | B 4 | C 3 | D | 2 | E 1 |
| 13. | Which of these | is the mean of | the other four? | | | |
| | A $\sqrt{2}$ | B $\sqrt{18}$ | Cγ | 200 D | $\sqrt{32}$ | $E\sqrt{8}$ |
| 14. | What is the sm together withou | allest number of allest number of a second s | of rectangles, eac m a rectangle wh | h measuring 2 cm ose sides are in the | 1 by 3 cm, which e ratio 5 : 4 ? | are needed to fit |
| | A 10 | B 15 | C 2 | 0 D | 30 | E 60 |
| 15. | Three dice, eac dice is rolled or | h showing num nce. The total of | bers 1 to 6, are co the numbers rollo | loured red, blue ar ed is 10. In how ma | nd yellow respects any different ways | ively. Each of the s can this happen? |
| | A 36 | B 30 | C 2 | 7 D | 24 | E 21 |
| 16. | An array of 25 of in the bottom le which passes the How many such | equally spaced of ft corner. Linda arough <i>O</i> and ex h lines can Lind | dots is drawn in a a wants to draw a s kactly one other p la draw? | square grid as show straight line throug oint. | wn. Point <i>O</i> is th the diagram | • • • • • • • • • • • • • • • • • • |
| | A 4 | B 6 | C 8 | D 12 | E 24 | 0 |
| 17. | A circle of radii that one of the What is the sha A $\sqrt{2}r$ | us r and a righ shorter sides of ded area? B r^2 | t-angled isosceles the triangle is a c C $2\pi r$ | Is triangle are draw diameter of the circ D $\frac{\pi r^2}{4}$ | vn such cle. | |
| | E $(\sqrt{2} - 1)$ | πr^2 | | | | 回該 |

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D 12

G

Η

E 15

E

18. The number 840 can be written as $\frac{p!}{q!}$, where p and q are positive integers less than 10.

Note that, $n! = 1 \times 2 \times 3 \times \cdots \times (n-1) \times n$.

What is the value of p + q?

A 8

19. The diagram shows two overlapping triangles: triangle FGH with interior angles 60°, 30° and 90° and triangle EGH which is a right-angled isosceles triangle.

What is the ratio of the area of triangle *IFG* to the area of triangle *IEH*?

- A 1:1 B 1: $\sqrt{2}$ C 1: $\sqrt{3}$ D 1:2 E 1:3
- **20.** Laura and Dina have a running race. Laura runs at constant speed and Dina runs *n* times as fast where n > 1. Laura starts *s* m in front of Dina.

What distance, in metres, does Dina run before she overtakes Laura?

- A $\frac{ns}{n-1}$ B ns C $\frac{s}{n-1}$ D $\frac{ns}{n+1}$ E $\frac{s}{n}$
- **21.** The numbers *m* and *k* satisfy the equations $2^m + 2^k = p$ and $2^m 2^k = q$. What is the value of 2^{m+k} in terms of *p* and *q*?

A
$$\frac{p^2 - q^2}{4}$$
 B $\frac{pq}{2}$ C $p + q$ D $\frac{(p - q)^2}{4}$ E $\frac{p + q}{p - q}$

22. A triangle with interior angles 60° , 45° and 75° is inscribed in a circle of radius 2.

What is the area of the triangle?

- A $2\sqrt{3}$ B 4 C $6 + \sqrt{3}$ D $6\sqrt{3}$ E $3 + \sqrt{3}$
- **23.** Let x be a real number. What is the minimum value of $(x^2 4x + 3)(x^2 + 4x + 3)$?
 - A -16 B -9 C 0 D 9 E 16

24. Saba, Rayan and Derin are cooperating to complete a task. They each work at a constant rate independent of whoever else is working on the task. When all three work together, it takes 5 minutes to complete the task. When Saba is working with Derin, the task takes 7 minutes to complete. When Rayan is working with Derin, the task takes 15 minutes to complete.

How many minutes does it take for Derin to complete the task on his own?

A 21 B 28 C 35 D 48 E 105



What is the shaded area?

A 8 B 9 C 10 D 11 E 12



| 1. | What is the value | of $\frac{2020}{20 \times 20}$? | | | | | |
|-----|--|---|---|--------------------------------|--|---|--------|
| | A 10.1 | B 5.5 | C | 5.1 | D 5.05 | E 0.55 | |
| 2. | What is the remain | inder when 12 | 34 × 5678 is | divided by 5 | ? | | |
| | A 0 | B 1 | C | 2 | D 3 | E 4 | |
| 3. | A shape is made t | from five unit | cubes, as sho | wn. | | | |
| | What is the surface | ce area of the s | hape? | | | | 7 |
| | A 22 | B 24 | C 26 | D 28 | E 30 | | |
| 4. | The numbers p , $p \times q \times r \times s = 2$ | q, r and s sa 020. | tisfy the equ | utions $p =$ | $2, p \times q = 20,$ | $p \times q \times r = 202$ and | t |
| | What is the value | of $p + q + r +$ | <i>s</i> ? | | | | |
| | A 32 | B 32.1 | C | 33 | D 33.1 | E 34 | |
| 5. | What is $\sqrt{123454}$ | 321? | | | | | |
| | A 1111111 | B 11111 | 1 C | 11111 | D 1111 | E 111 | |
| 6. | There are fewer the one quarter play here are the play here. | han 30 student nockey and one | s in the A-le e seventh are | vel mathemat in the school | tics class. One hal l play. | f of them play the piano |), |
| | How many of the | students play | hockey? | | | | |
| | A 3 | B 4 | C | 5 | D 6 | E 7 | |
| 7. | Official UK accid However, in the fo | dent statistics sollowing year t | showed that here were 47 | there were 22 7 such accide | 25 accidents invol ¹ nts. | ving teapots in one yea | r. |
| | What was the app year to the second | roximate perce 1? | entage reduct | ion in recorde | ed accidents involv | ving teapots from the firs | t |
| | A 50% | B 60% | C | 70% | D 80% | E 90% | |
| 8. | What is the larges | st prime factor | of 106 ² – 15 | $5^2?$ | | | |
| | A 3 | B 7 | C | 11 | D 13 | E 17 | |
| 9. | In 2018, a racing within 1 second c km/h (in the same | driver was al of the car aheae e direction!). | lowed to use 1. Suppose t | the Drag Re | eduction System provident of the second aparts and the second apar | rovided that the car wa rt, each travelling at 180 | s D |
| | How many metre | s apart were th | ey? | | | | |
| | A 100 | B 50 | С | 10 | D 5 | E 1 | |
| 10. | Six friends Pat, Q people standing b Sam. Sam is not | asim, Roman, etween Pat and at either end o | Sam, Tara ar Qasim, two f the line. | nd Uma, stand between Qasi | l in a line for a pho im and Roman and | otograph. There are thre one between Roman and | e d |
| | How many people | e are standing | between Tara | a and Uma? | | | |
| | A 4 | B 3 | С | 2 | D 1 | E 0 | ţ |

E 0



11. Two congruent pentagons are each formed by removing a right-angled isosceles triangle from a square of side-length 1. The two pentagons are then fitted together as shown.

What is the length of the perimeter of the octagon formed?

A 4

B 4 + $2\sqrt{2}$ C 5 D 6 - $2\sqrt{2}$ E 6

12. A three-piece suit consists of a jacket, a pair of trousers and a waistcoat. Two jackets and three pairs of trousers cost £380. A pair of trousers costs the same as two waistcoats.

What is the cost of a three-piece suit?

A £150B £190C £200D £228E more information needed

13. The number $16! \div 2^k$ is an odd integer. Note that $n! = 1 \times 2 \times 3 \times \cdots \times (n-1) \times n$.

What is the value of *k*?

A 9 B 11 C 13 D 15 E 17

14. Diane has five identical blue disks, two identical red disks and one yellow disk. She wants to place them on the grid opposite so that each cell contains exactly one disk. The two red disks are not to be placed in cells that share a common edge.

How many different-looking completed grids can she produce?

A 96 B 108 C 144 D 180 E 216

15. The shaded area shown in the diagram consists of the interior of a circle of radius 3 together with the area between the circle and two tangents to the circle. The angle between the tangents at the point where they meet is 60°.

What is the shaded area?

A $6\pi + 9\sqrt{3}$ B $15\sqrt{3}$ C 9π D $9\pi + 4\sqrt{3}$ E $6\pi + \frac{9\sqrt{3}}{4}$

16. Which diagram represents the set of all points (x, y) satisfying $y^2 - 2y = x^2 + 2x$?



17. The positive integers *m*, *n* and *p* satisfy the equation $3m + \frac{3}{n + \frac{1}{n}} = 17$.

B 3

What is the value of *p*?

A 2

A

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C 4



60



D 6



E 9



p-gon

18. Two circles C_1 and C_2 have their centres at the point (3,4) and touch a third circle, C_3 . The centre of C_3 is at the point (0,0) and its radius is 2. What is the sum of the radii of the two circles C_1 and C_2 ? A 6 B 7 C 8 D 9 E 10 **19.** The letters p, q, r, s and t represent different positive single-digit numbers such that p - q = r and r-s=t. How many different values could *t* have? B 5 C 4 D 3 A 6 E 2 **20.** The real numbers x and y satisfy the equations $4^y = \frac{1}{8(\sqrt{2})^{x+2}}$ and $9^x \times 3^y = 3\sqrt{3}$. What is the value of 5^{x+y} ? $C \sqrt{5}$ $E \frac{1}{\sqrt{5}}$ A $5\sqrt{5}$ B 5 $D_{\frac{1}{5}}$ **21.** When written out in full, the number $(10^{2020} + 2020)^2$ has 4041 digits. What is the sum of the digits of this 4041-digit number? A 9 C 25 B 17 D 2048 E 4041 **22.** A square with perimeter 4 cm can be cut into two congruent right-angled triangles and two congruent trapezia as shown in the first diagram in such a way that the four pieces can be rearranged to form the rectangle shown in the second diagram. What is the perimeter, in centimetres, of this rectangle? D $4\sqrt{3}$ B $4\sqrt{2}$ A $2\sqrt{5}$ E $3\sqrt{7}$ C 5 **23.** A function f satisfies $y^3 f(x) = x^3 f(y)$ and $f(3) \neq 0$. What is the value of $\frac{f(20) - f(2)}{f(3)}$? B 20 C 216 A 6 D 296 E 2023 24. In the diagram shown, M is the mid-point of PQ. The line PS R bisects $\angle RPQ$ and intersects RQ at S. The line ST is parallel to *PR* and intersects *PQ* at T. The length of *PQ* is 12 and the length of MT is 1. The angle SQT is 120° . -120° What is the length of *SQ*? M7A 2 **B** 3 C 3.5 D 4 E 5 25. A regular *m*-gon, a regular *n*-gon and a regular *p*-gon share a vertex and pairwise share edges, as shown in the diagram. *m*-gon n-gon

What is the largest possible value of *p*?

A 6 B 20 C 42 D 50 E 100

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| 1. | What is the value | of $123^2 - 23^2$? | | | |
|----|-------------------|---------------------|----------------------|----------------------|---------|
| | A 10000 | B 10409 | C 12323 | D 14600 | E 15658 |
| 2. | What is the value | of (2019 – (2000 – | (10 - 9))) - (2000 - | -(10 - (9 - 2019)))? | |
| | A 4040 | B 40 | C -400 | D -4002 | E -4020 |

3. Used in measuring the width of a wire, one mil is equal to one thousandth of an inch. An inch is about 2.5 cm.

Which of these is approximately equal to one mil?

- A $\frac{1}{40}$ mm B $\frac{1}{25}$ mm C $\frac{1}{4}$ mm D 25 mm E 40 mm
- **4.** For how many positive integer values of *n* is $n^2 + 2n$ prime?
 - A 0 B 1 C 2 D 3
- **5.** Olive Green wishes to colour all the circles in the diagram so that, for each circle, there is exactly one circle of the same colour joined to it.

What is the smallest number of colours that Olive needs to complete this task?

- A 1 B 2 C 3 D 4 E 5
- 6. Each of the factors of 100 is to be placed in a 3 by 3 grid, one per cell, in such a way that the products of the three numbers in each row, column and diagonal are all equal. The positions of the numbers 1, 2, 50 and *x* are shown in the diagram.

C 10

What is the value of *x*?

A 4 B 5

7. Lucy is asked to choose p, q, r and s to be the numbers 1, 2, 3 and 4, in some order, so as to make the value of $\frac{p}{q} + \frac{r}{s}$ as small as possible.

D 20

E 25

What is the smallest value Lucy can achieve in this way?

A
$$\frac{7}{12}$$
 B $\frac{2}{3}$ C $\frac{3}{4}$ D $\frac{5}{6}$ E $\frac{11}{12}$

8. The number x is the solution to the equation $3^{(3^x)} = 333$.

Which of the following is true?

A 0 < x < 1 B 1 < x < 2 C 2 < x < 3 D 3 < x < 4 E 4 < x < 5

9. A square of paper is folded in half four times to obtain a smaller square. Then a corner is removed as shown.

Which of the following could be the paper after it is unfolded?













10. Which of the following five values of *n* is a counterexample to the statement in the box below?

| | | For a positiv | ve integer <i>n</i> , a | at least one of 6 <i>n</i> | -1 and $6n + 1$ is | prime. | | |
|-----|---|---|---------------------------------|---|---|---|--|--|
| | A 10 | B 19 | | C 20 | D 21 | E 30 | | |
| 11. | For how many i | integer values | of k is $\sqrt{200}$ | $\overline{) - \sqrt{k}}$ also an in | teger? | | | |
| | A 11 | B 13 | | C 15 | D 17 | E 20 | | |
| 12. | A circle with rather the smaller ang | dius 1 touches les between tl | s the sides of ne sides of th | a rhombus, as she e rhombus is 60° | own. Each of | | | |
| | What is the area | a of the rhom | bus? | | | | | |
| | A 6 | B 4 | C $2\sqrt{3}$ | D $3\sqrt{3}$ | $E \frac{8\sqrt{3}}{3}$ | 60° | | |
| 13. | Anish has a nur square of side <i>n</i> has 25 too few. | nber of small , he has 64 til | congruent sq es left over. V | uare tiles to use i Vhen he tries to f | n a mosaic. When form the tiles into a | he forms the tiles into a square of side $n + 1$, he | | |
| | How many tiles | does Anish l | nave? | | | | | |
| | A 89 | B 193 | 5 | C 1980 | D 2000 | E 2019 | | |
| 14. | One of the follo | wing is the la | argest square | that is a factor of | 10!. Which one? | | | |
| | Note that, $n! =$ | $1 \times 2 \times 3 \times \cdot$ | $\cdots \times (n-1) \times$ | < n. | | | | |
| | A $(4!)^2$ | B (5!) | 2 | C $(6!)^2$ | D $(7!)^2$ | $E (8!)^2$ | | |
| 15. | The highest con different primes | mmon factors | of all the pa | irs chosen from | the positive intege | ers Q , R and S are three | | |
| | What is the sma | allest possible | value of Q + | -R+S? | | | | |
| | A 41 | B 31 | | C 30 | D 21 | E 10 | | |
| 16. | The numbers x | , y and z satis | fy the equation | ons $9x + 3y - 5z$ | x = -4 and $5x + 2y$ | y - 2z = 13. | | |
| | What is the mea | an of x , y and | z? | - | | | | |
| | A 10 | B 11 | | C 12 | D 13 | E 14 | | |
| 17. | Jeroen writes a | list of 2019 c | onsecutive in | itegers. The sum | of his integers is 2 | 2019. | | |
| | What is the product of all the integers in Jeroen's list? | | | | | | | |
| | A 2019 ² | B $\frac{201}{2}$ | $\frac{19 \times 2020}{2}$ | C 2 ²⁰¹⁹ | D 2019 | E 0 | | |
| 18. | Alison folds a s diagram. After onto the dashed | quare piece o opening the p l line. | f paper in hal paper out aga | If along the dash in, she then fold | ed line shown in th s one of the corne | $\frac{1}{rs}$ | | |
| | What is the value | ue of α ? | | | | (C12-A2) (5 | | |
| | A 45 | B 60 | C 65 | D 70 | E 75 | (二) | | |

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С

19. Which of the following could be the graph of $y^2 = \sin(x^2)$?



20. The "heart" shown in the diagram is formed from an equilateral triangle ABC and two congruent semicircles on AB. The two semicircles meet at the point P. The point O is the centre of one of the semicircles. On the semicircle with centre O, lies a point X. The lines XO and XP are extended to meet AC at Y and Z respectively. The lines XY and XZ are of equal length.

What is $\angle ZXY$?

A 20° B 25° C 30° D 40° E 45°

21. In a square garden *PQRT* of side 10 m, a ladybird sets off from *Q* and moves along edge *QR* at 30 cm per minute. At the same time, a spider sets off from *R* and moves along edge *RT* at 40 cm per minute. What will be the shortest distance between them, in metres?
A 5 B 6 C 5√2 D 8 E 10

22. A function f satisfies the equation (n - 2019)f(n) - f(2019 - n) = 2019 for every integer n. What is the value of f(2019)?

A 0 B 1 C 2018×2019 D 2019^2 E 2019×2020

23. The edge-length of the solid cube shown is 2. A single plane cut goes through the points Y, T, V and W which are midpoints of the edges of the cube, as shown.

What is the area of the cross-section?

B -6

A
$$\sqrt{3}$$
 B $3\sqrt{3}$ C 6 D $6\sqrt{2}$ E 8

24. The numbers x, y and z are given by $x = \sqrt{12 - 3\sqrt{7}} - \sqrt{12 + 3\sqrt{7}}$, $y = \sqrt{7 - 4\sqrt{3}} - \sqrt{7 + 4\sqrt{3}}$ and $z = \sqrt{2 + \sqrt{3}} - \sqrt{2 - \sqrt{3}}$.

D 18

C -8

What is the value of *xyz* ?

A 1

25. Two circles of radius 1 are such that the centre of each circle lies on the other circle. A square is inscribed in the space between the circles.

What is the area of the square?

A $2 - \frac{\sqrt{7}}{2}$ B $2 + \frac{\sqrt{7}}{2}$ C $4 - \sqrt{5}$ D 1 E $\frac{\sqrt{5}}{5}$

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| 1. | . When the following are evaluated, how many of the answers are odd numbers? | | | | | | |
|-----|--|--|--|---------------------------|------------------------|--|--|
| | $1^2, 2^3, 3^4, 4^5, 5^6$ | | | | | | |
| | A 1 | B 2 | C 3 | D 4 | E 5 | | |
| 2. | The positive integer | 2018 is the pro | oduct of two primes. | | | | |
| | What is the sum of t | hese two prime | es? | | | | |
| | A 1001 | B 1010 | C 1011 | D 1100 | E 1101 | | |
| 3. | Which of the follow | ing shows the o | digit 6 after it has been | rotated clockwise thro | ough 135°? | | |
| | A (20 | BQ | ک ₂ | $_{\rm D}$ \mathfrak{S} | Е 6 | | |
| 4. | Which of the follow | ing is not a mu | ltiple of 5? | | | | |
| | A $2019^2 - 2014^2$ E $2015^2 \div 5^2$ | ² B 20 | $19^2 \times 10^2$ C 2 | $2020^2 \div 101^2$ | D $2010^2 - 2005^2$ | | |
| 5. | Which of the follow | ing numbers is | the largest? | | | | |
| | A $\frac{397}{101}$ | B $\frac{487}{121}$ | C $\frac{596}{153}$ | D $\frac{678}{173}$ | $E \frac{796}{203}$ | | |
| 6. | Which of the follow | ing is equal to | $25 \times 15 \times 9 \times 5.4 \times 3.24$ | 4? | | | |
| | A 3 ⁹ | B 3 ¹⁰ | C 3 ¹¹ | D 3 ¹⁴ | E 3 ¹⁷ | | |
| 7. | The circles P , Q ard diameter of P , as sh | nd <i>R</i> are all tan own in the figu | ngent to each other. Thure. | eir centres all lie on | a $P Q R$ | | |
| | What is the value of | circumference | e of Q + circumference c | $\frac{\text{of }R}{R}$? | | | |
| | Δ 1 | Cir D ¹ | cumference of P | D ¹ | | | |
| | E more informat | ion needed | $C_{\overline{3}}$ | $D_{\frac{1}{4}}$ | | | |
| 8. | What are the last tw | o digits of 7 ²⁰¹ | ⁸ ? | | | | |
| | A 07 | B 49 | C 43 | D 01 | E 18 | | |
| 9. | The diagram shows | a rectangle AE | <i>FJ</i> inside a regular deca | agon ABCDEFGHIJ | C B | | |
| | What is the ratio of | the area of the | rectangle to the area of t | the decagon? | $D \longrightarrow A$ | | |
| | A 2:5 | B 1:4 | C 3:5 D 3: | 10 E 3 : 20 | E J | | |
| | | | | | $F \overbrace{G H}{I}$ | | |
| 10. | On a training ride, I finally 18 km/h for | Laura averages 15 minutes. | speeds of 12 km/h for | 5 minutes, then 15 kr | n/h for 10 minutes and | | |
| | What was her average | ge speed over tl | he whole ride? | | | | |
| | A 13 km/h | B 14 km/ | h C 15 km/h | D 16 km/h | E 17 km/h | | |
| | | | | | | | |

11. How many of the following four equations has a graph that does *not* pass through the origin?

 $y = x^4 + 1$ $y = x^4 + x$ $y = x^4 + x^2$ $y = x^4 + x^3$

- B 1
- C 2

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A 0

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D 3

E 4



| 12. | A regular tetrahedron is a polyhedron with four faces, each of which is an equilateral triangle, as shown. A solid regular tetrahedron is cut into two pieces by a single plane cut. | | | | | | | |
|-----|--|---|--|---------------------------|-----------------------|--|--|--|
| | Which of the follo | | | | | | | |
| | A a pentagonC a rectangle tE a triangle th | hat is not a squa at is not equilate | B a square D a trapezium eral | n | | | | |
| 13. | The lines $y = x$ as | nd $y = mx - 4$ i | ntersect at the point <i>P</i> . | | | | | |
| | What is the sum of the positive integer values of <i>m</i> for which the coordinates of <i>P</i> are also positive integers? | | | | | | | |
| | A 3 | B 5 | C 7 | D 8 | E 10 | | | |
| 14. | The following two | elve integers are | written in ascending or | ler: | | | | |
| | | | 1, <i>x</i> , <i>x</i> , <i>x</i> , <i>y</i> , <i>y</i> , <i>y</i> , <i>y</i> , <i>y</i> , | , 8, 9, 11. | | | | |
| | The mean of these | e twelve integers | s is 7. What is the media | n? | | | | |
| | A 6 | В 7 | C 7.5 | D 8 | E 9 | | | |
| 15. | A square is inscrib | oed in a circle of | radius 1. An isosceles tr | iangle is inscribed in th | e | | | |
| | What is the ratio of | of the area of thi | is triangle to the area of | the shaded region? | | | | |
| | A $\pi:\sqrt{2}$ | B π :1 | C 1:4 D 1 | $\pi - 2$ E 2 : π | | | | |
| | | | | | | | | |
| 16. | 16. The numbers p , q , r and s satisfy the following equations: | | | | | | | |
| | | <i>p</i> + | $2q + 3r + 4s = k \qquad 4p$ | p = 3q = 2r = s. | | | | |
| | What is the smalle | est value of k fo | r which p, q, r and s are | all positive integers? | | | | |
| | A 20 | B 24 | C 25 | D 77 | E 154 | | | |
| 17. | Bethany has 11 pc coins is 52 pence. | ound coins and s | some 20p coins and some | e 50p coins in her purse. | The mean value of the | | | |
| | Which could not b | be the number o | f coins in the purse? | | | | | |
| | A 35 | B 40 | C 50 | D 65 | E 95 | | | |
| 18. | P, Q and R are the | e three angles o | f a triangle, when each h | as been rounded to the | nearest degree. | | | |
| | Which of the follo | wing is the con | plete list of possible val | ues of $P + Q + R$? | | | | |
| | A 179°, 180° o E 178°, 179°, | or 181° B 1 180°, 181° or 18 | 80°, 181° or 182° C 82° | 178°, 179° or 180° | D 180° | | | |
| 19. | How many pairs of | of numbers (<i>m</i> , <i>r</i> | i) are there such that the | following statement is | true? | | | |
| | 'A regular <i>m</i> -sided polygon has an exterior angle of size n° and a regular <i>m</i> -sided polygon has an exterior angle of size m° ' | | | | | | | |
| | A 24 | B 22 | C 20 | D 18 | E 16 | | | |
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20. The diagram shows a semicircle of radius 1 inside an isosceles triangle. The diameter of the semicircle lies along the 'base' of the triangle, and the angle of the triangle opposite the 'base' is equal to 2θ . Each of the two equal sides of the triangle is tangent to the semicircle.

What is the area of the triangle?

A $\frac{1}{2} \tan 2\theta$ B $\sin \theta \cos \theta$ C $\sin \theta + \cos \theta$ D $\frac{1}{2} \cos 2\theta$ E $\frac{1}{\sin \theta \cos \theta}$

21. The graph of $y = \frac{1}{x}$ is reflected in the line y = 1. The resulting image is reflected in the line y = -x. What is the equation of the final graph?

A
$$y = \frac{-1}{(x+2)}$$
 B $y = \frac{1}{(x-1)}$ C $y = \frac{1}{(x-2)}$ D $y = \frac{-1}{(x-1)}$ E $y = \frac{-1}{(x-2)}$

22. The diagram shows two overlapping triangles; an isosceles triangle with an angle of 120° and an equilateral triangle with area 36. Two of the vertices of the equilateral triangle are midpoints of the equal sides of the isosceles triangle.

What is the total area of the shaded regions (inside the isosceles triangle but outside the equilateral triangle)?



E 32

- A 24 B 26 C 28 D 30
- **23.** For particular real numbers a and b, the function f is defined by f(x) = ax + b, and is such that f(f(f(x))) = 27x 52.

Which of the following formulas defines the function g such that, for all values of x, g(f(x)) = x?

A $\frac{1}{3}x - 4$ B $\frac{1}{3}x + \frac{4}{3}$ C 4x - 3 D $\frac{1}{3}x - \frac{4}{3}$ E 3x - 4

24. The diagram shows a circle with centre *O* which lies in a horizontal plane. The diameter *AB* has length 4. Point *P* lies vertically above *O* and $PO = 2\sqrt{2}$. Point *C* lies on the semicircular arc *AB* such that the ratio of the lengths of the arcs *AC* and *CB* is 2 : 1.

What is the shortest distance from A to PC?

A $\sqrt{2}$ B $\sqrt{3}$ C 2 D $2\sqrt{2}$ E 3





25. A semicircle is inscribed in a quarter circle as shown.

What fraction of the quarter circle is shaded?

A $\frac{1}{3}$ B $\frac{1}{\sqrt{3}}$ C $\frac{2}{3}$ D $\frac{\sqrt{3}}{2}$ E $\frac{1}{\sqrt{2}}$

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