United Kingdom Mathematics Trust

Senior Kangaroo Past Paper

2011 – 2019 Collection

Last updated: August 18, 2020

Instructions

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **60 minutes**.
   No answers, or personal details, may be entered after the allowed time is over.
3. The use of blank or lined paper for rough working is allowed; **squared paper**, **calculators and measuring instruments are forbidden**.
4. **Use a B or an HB non-propelling pencil** to record your answer to each problem as a three-digit number from 000 to 999.
   Pay close attention to the example on the Answer Sheet that shows how to code your answers.
5. **Do not expect to finish the whole paper in the time allowed.** The questions in this paper have been arranged in approximate order of difficulty with the harder questions towards the end. You are not expected to complete all the questions during the time. You should bear this in mind when deciding which questions to tackle.
6. **Scoring rules:**
   5 marks are awarded for each correct answer;
   There is no penalty for giving an incorrect answer.
7. **The questions on this paper are designed to challenge you to think, not to guess.** You will gain more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. This paper is about solving interesting problems, not about lucky guessing.

Comments and suggestions to 89272376@QQ.com .

Contents

Answers:

	11	12	13	14	15	16	17	18	19	20	
1	071	001	009	005	180	121	132	240	403		1
2	390	206	110	001	028	950	231	110	256		2
3	016	016	025	011	011	027	221	135	210		3
4	100	002	012	108	007	125	216	150	087		4
5	036	009	025	015	050	009	625	343	130		5
6	084	004	002	343	007	160	373	150	338		6
7	030	002	013	220	156	315	300	216	012		7
8	013	962	007	011	372	007	276	180	004		8
9	005	005	060	153	028	256	013	672	100		9
10	004	015	003	035	288	103	004	100	140		10
11	060	099	128	048	052	183	714	270	094		11
12	040	004	011	123	035	144	121	128	105		12
13	065	400	008	008	013	143	002	112	198		13
14	010	010	992	004	030	035	027	320	216		14
15	891	186	008	921	014	110	829	038	129		15
16	021	343	030	225	064	087	074	227	317		16
17	021	223	150	012	016	649	447	006	246		17
18	004	096	804	015	002	441	603	493	014		18
19	003	116	741	016	100	421	343	122	735		19
20	017	341	050	024	247	116	077	147	295		20

1. What is the sum of all the factors of 144?

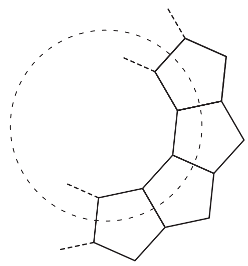
2. When I noticed that $2^4 = 4^2$, I tried to find other pairs of numbers with this property. Trying 2 and 16, I realised that 2^{16} is larger than 16^2. How many times larger is 2^{16}?

3. The two diagonals of a quadrilateral are perpendicular. The lengths of the diagonals are 14 and 30. What is the area of the quadrilateral?

4. The integer n satisfies the inequality $n + (n + 1) + (n + 2) + \cdots + (n + 20) > 2019$.

What is the minimum possible value of n?

5. Identical regular pentagons are arranged in a ring. The partially completed ring is shown in the diagram. Each of the regular pentagons has a perimeter of 65. The regular polygon formed as the inner boundary of the ring has a perimeter of P. What is the value of P?

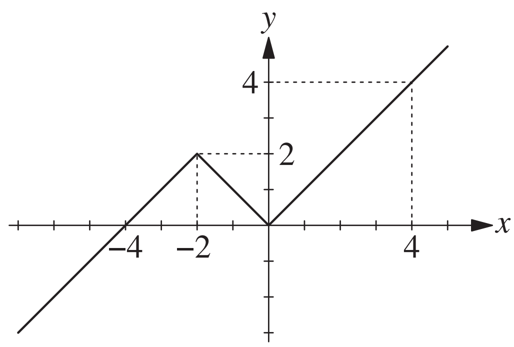


6. For natural numbers a and b we are given that $2019 = a^2 - b^2$. It is known that $a < 1000$. What is the value of a?

7. How many positive? integers n exist such that both $\frac{n+1}{3}$ and $3n + 1$ are three-digit integers?

8. The function $J(x)$ is defined by:

$$J(x) = \begin{cases} 4 + x & \text{for } x \leq -2, \\ -x & \text{for } -2 < x \leq 0, \\ x & \text{for } x > 0. \end{cases}$$



How many distinct real solutions has the equation $J(J(J(x))) = 0$?

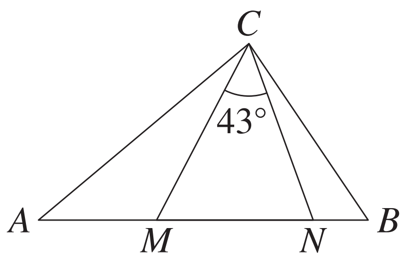
9. What is the smallest three-digit number K which can be written as $K = a^b + b^a$, where both a and b are one-digit positive integers?

10. What is the value of $\sqrt{13 + \sqrt{28 + \sqrt{281}}} \times \sqrt{13 - \sqrt{28 + \sqrt{281}}} \times \sqrt{141 + \sqrt{281}}$?

11. In the triangle ABC the points M and N lie on the side AB such that $AN = AC$ and $BM = BC$.

We know that $\angle MCN = 43°$.

Find the size in degrees of $\angle ACB$.



12. What is the value of $A^2 + B^3 + C^5$, given that:

$A = \sqrt[3]{16\sqrt{2}}$

$B = \sqrt{9\sqrt[3]{9}}$

$C = [(\sqrt[5]{2})^2]^2$

13. The real numbers a and b, where $a > b$, are solutions to the equation $3^{2x} - 10 \times 3^{x+1} + 81 = 0$. What is the value of $20a^2 + 18b^2$?

14. A number N is the product of three distinct primes. How many distinct factors does N^5 have?

15. Five Bunchkins sit in a horizontal field. No three of the Bunchkins are sitting in a straight line. Each Bunchkin knows the four distances between her and each of the others. Each Bunchkin calculates and then announces the total of these distances. These totals are 17, 43, 56, 66 and 76. A straight line is painted joining each pair of Bunchkins. What is the total length of paint required?

16. The real numbers x and y satisfy the equations:

$$xy - x = 180 \quad \text{and} \quad y + xy = 208.$$

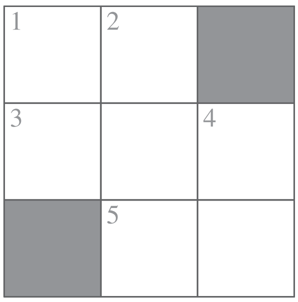
Let the two solutions be (x_1, y_1) and (x_2, y_2).
What is the value of $x_1 + 10y_1 + x_2 + 10y_2$?

17. In triangle ABC, $\angle BAC$ is 120°. The length of AB is 123. The point M is the midpoint of side BC. The line segments AB and AM are perpendicular.
What is the length of side AC?

18. An integer is said to be *chunky* if it consists only of non-zero digits by which it is divisible when written in base 10.
For example, the number 936 is Chunky since it is divisible by 9, 3 and 6.
How many chunky integers are there between 13 and 113?

19. The square $ABCD$ has sides of length 105. The point M is the midpoint of side BC. The point N is the midpoint of BM. The lines BD and AM meet at the point P. The lines BD and AN meet at the point Q.
What is the area of triangle APQ?

20. Each square in this cross-number can be filled with a non-zero digit such that all of the conditions in the clues are fulfilled. The digits used are not necessarily distinct.
What is the answer to 3 ACROSS?



ACROSS
1. A composite factor of 1001
3. Not a palindrome
5. pq^3 where p, q prime and $p \neq q$

DOWN
1. One more than a prime, one less than a prime
2. A multiple of 9
4. p^3q using the same p, q as 5 ACROSS

1. My age is a two-digit number that is a power of 5. My cousin's age is a two-digit number that is a power of 2. The sum of the digits of our ages is an odd number.

What is the product of the digits of our ages?

2. Let K be the largest integer for which $n^{200} < 5^{300}$. What is the value of $10K$?

3. In triangle ABC, we are given that $AC = 5\sqrt{2}$, $BC = 5$ and $\angle BAC = 30°$.

What is the largest possible size in degrees of $\angle ABC$?

4. In a list of five numbers, the first number is 60 and the last number is 300. The product of the first three numbers is 810 000, the product of the three in the middle is 2 430 000 and the product of the last three numbers is 8 100 000.

Which number is third in the list?

5. Rachel and Steven play games of chess. If either wins two consecutive games s/he is declared the champion.

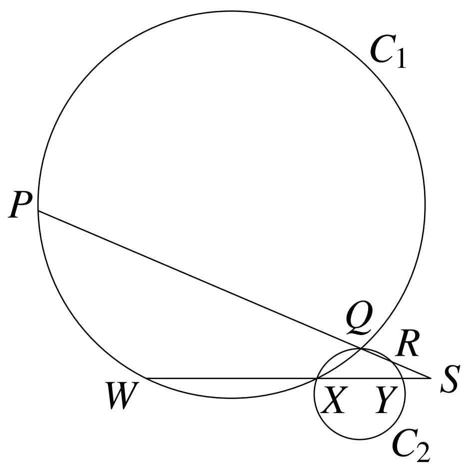
The probability that Rachel will win any given game is 0.6.

The probability that Steven will win any given game is 0.3.

There is a 0.1 probability that any given game is drawn.

The probability that neither is the champion after at most three games is P. Find the value of $1000P$.

6. The line segments $PQRS$ and $WXYS$ intersect circle C_1 at points P, Q, W and X.



The line segments intersect circle C_2 at points Q, R, X and Y. The lengths QR, RS and XY are 7, 9 and 18 respectively. The length WX is six times the length YS.

What is the sum of the lengths of PS and WS?

7. The volume of a cube in cubic metres and its surface area in square metres is numerically equal to four-thirds of the sum of the lengths of its edges in metres.

What is the total volume in cubic metres of twenty-seven such cubes?

8. An integer x satisfies the inequality $x^2 \leq 729 \leq -x^3$. P and Q are possible values of x. What is the maximum possible value of $10(P - Q)$?

9. The two science classes 7A and 7B each consist of a number of boys and a number of girls. Each class has exactly 30 students.

The girls in 7A have a mean score of 48. The overall mean across both classes is 60.

The mean score across all the girls of both classes is also 60.

The 5 girls in 7B have a mean score that is double that of the 15 boys in 7A.

The mean score of the boys in 7B is μ. What is the value of 10μ?

10. The function SPF(n) denotes the sum of the prime factors of n, where the prime factors are not necessarily distinct. For example, $120 = 2^3 \times 3 \times 5$, so SPF$(120) = 2 + 2 + 2 + 3 + 5 = 14$.

Find the value of SPF$(2^{22} - 4)$.

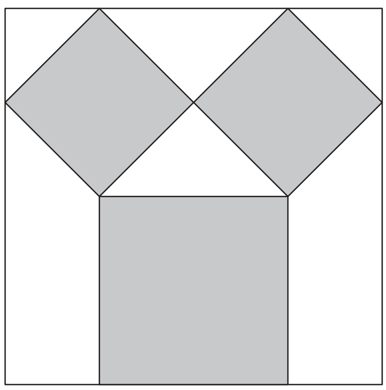
11. A sequence $U_1, U_2, U_3, \ldots$ is defined as follows:

- $U_1 = 2$;
- if U_n is prime then U_{n+1} is the smallest positive integer not yet in the sequence;
- if U_n is not prime then U_{n+1} is the smallest prime not yet in the sequence.

The integer k is the smallest such that $U_{k+1} - U_k > 10$.

What is the value of $k \times U_k$?

12. The diagram shows a 16 metre by 16 metre wall. Three grey squares are painted on the wall as shown.



The two smaller grey squares are equal in size and each makes an angle of 45° with the edge of the wall. The grey squares cover a total area of B metres squared.

What is the value of B?

13. A nine-digit number is odd. The sum of its digits is 10. The product of the digits of the number is non-zero. The number is divisible by seven.

When rounded to three significant figures, how many millions is the number equal to?

14. A square $ABCD$ has side 40 units. Point F is the midpoint of side AD. Point G lies on CF such that $3CG = 2GF$.

What is the area of triangle BCG?

15. In the sequence $20, 18, 2, 20, -18, \ldots$ the first two terms a_1 and a_2 are 20 and 18 respectively. The third term is found by subtracting the second from the first, $a_3 = a_1 - a_2$. The fourth is the sum of the two preceding elements, $a_4 = a_2 + a_3$. Then $a_5 = a_3 - a_4$, $a_6 = a_4 + a_5$, and so on.

What is the sum of the first 2018 terms of this sequence?

16. A right-angled triangle has sides of integer length. One of its sides has length 20. Toni writes down a list of all the different possible hypotenuses of such triangles.

What is the sum of all the numbers in Toni's list?

17. Sarah chooses two numbers a and b from the set $\{1, 2, 3, \ldots, 26\}$. The product ab is equal to the sum of the remaining 24 numbers.

What is the difference between a and b?

18. How many zeros are there at the end of $\dfrac{2018!}{30! \times 11!}$?

19. Shan solves the simultaneous equations

$$xy = 15 \text{ and } (2x - y)^4 = 1$$

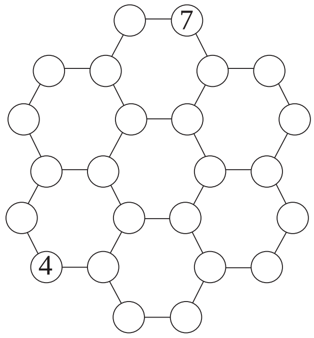
where x and y are real numbers. She calculates z, the sum of the squares of all the y-values in her solutions.

What is the value of z?

20. Determine the value of the integer y given that $y = 3x^2$ and

$$\frac{2x}{5} = \cfrac{1}{1 - \cfrac{2}{3 + \cfrac{1}{4 - \cfrac{5}{6 - x}}}}$$

1. An integer is to be written in each circle of the network shown. The integers must be written so that the sum of the numbers at the end of each line segment is the same. Two of the integers have already been written. What is the total of all the integers in the completed diagram?



2. Three sportsmen called Primus, Secundus and Tertius take part in a race every day. Primus wears the number '1' on his shirt, Secundus wears '2' and Tertius wears '3'.

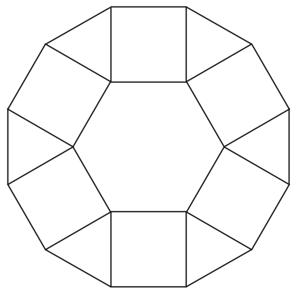
   On Saturday Primus wins, Secundus is second and Tertius is third. Using their shirt numbers this result is recorded as '123'.

   On Sunday Primus starts the race in the lead with Secundus in second. During Sunday's race Primus and Secundus change places exactly 9 times, Secundus and Tertius change places exactly 10 times while Primus and Tertius change places exactly 11 times.

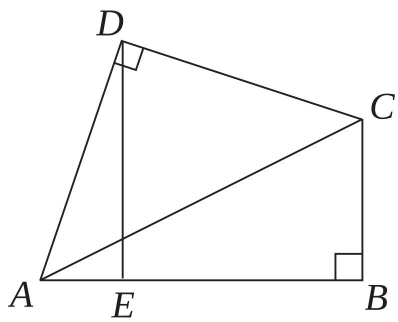
   How will Sunday's result be recorded?

3. All three-digit positive integers whose digit sum is 5 are listed in ascending order. What is the median of this list?

4. The figure shows a shape consisting of a regular hexagon of side 18 cm, six triangles and six squares. The outer perimeter of the shape is P cm. What is the value of P?



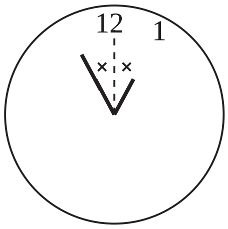
5. The figure shows a quadrilateral $ABCD$ in which $AD = DC$ and $\angle ADC = \angle ABC = 90°$. The point E is the foot of the perpendicular from D to AB. The length DE is 25. What is the area of quadrilateral $ABCD$?



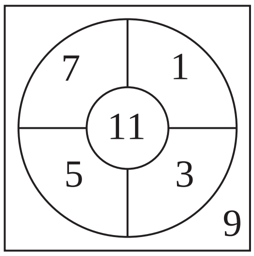
6. Winnie wrote all the integers from 1 to 2017 inclusive on a board. She then erased all the integers that are a multiple of 3. Next she reinstated all those integers that are a multiple of 6. Finally she erased all integers then on the board which are a multiple of 27. Of the 2017 integers that began in the list, how many are now missing?

7. Three rectangles are placed mutually adjacent and without gaps or overlaps to form a larger rectangle. One of the three rectangles has dimensions 70 by 110. Another of the rectangles has dimensions 40 by 80. What is the maximum perimeter of the third rectangle?

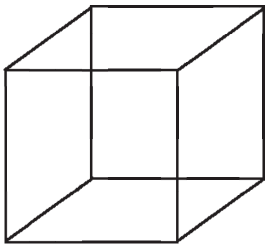
8. Priti is learning a new language called Tedio. During her one hour lesson, which started at midday, she looks at the clock and notices that the hour hand and the minute hand make exactly the same angle with the vertical, as shown in the diagram. How many whole seconds remain until the end of the lesson?



9. Robin shoots three arrows at a target. He earns points for each shot as shown in the figure. However, if any of his arrows miss the target or if any two of his arrows hit adjacent regions of the target, he scores a total of zero. How many different scores can he obtain?

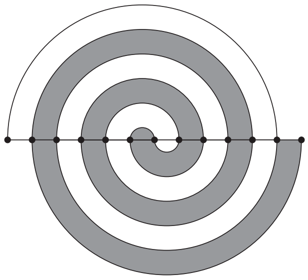


10. At each of the vertices of a cube sits a Bunchkin. Two Bunchkins are said to be adjacent if and only if they sit at either end of one of the cube's edges. Each Bunchkin is either a 'truther', who always tells the truth, or a 'liar', who always lies. All eight Bunchkins say 'I am adjacent to exactly two liars'. What is the maximum number of Bunchkins who are telling the truth?



11. An infinite arithmetic progression of positive integers contains the terms 7, 11, 15, 71, 75 and 79. The first term in the progression is 7. Kim writes down all the possible values of the one-hundredth term in the progression. What is the sum of the numbers Kim writes down?

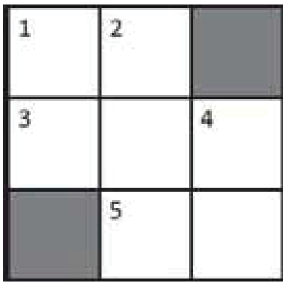
12. The pattern shown in the diagram is constructed using semicircles. Each semicircle has a diameter that lies on the horizontal axis shown and has one of the black dots at either end. The distance between each pair of adjacent black dots is 1 cm. The area, in cm^2, of the pattern that is shaded in grey is $\frac{1}{8}k\pi$. What is the value of k?



13. In the expression $\frac{k.a.n.g.a.r.o.o}{g.a.m.e}$, different letters stand for different non-zero digits but the same letter always stands for the same digit. What is the smallest possible integer value of the expression?

14. The set S is given by $S = \{1, 2, 3, 4, 5, 6\}$. A non-empty subset T of S has the property that it contains no pair of integers that share a common factor other than 1. How many distinct possibilities are there for T?

15. Each square in this cross-number can be filled with a non-zero digit such that all of the conditions in the clues are fulfilled. The digits used are not necessarily distinct.



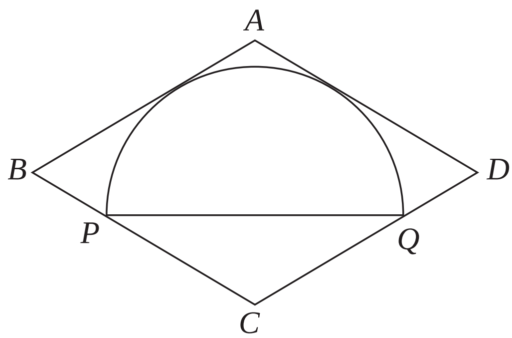
ACROSS
1. A square
3. The answer to this Kangaroo question
5. A square

DOWN
1. 4 down minus eleven
2. One less than a cube
4. The highest common factor of 1 down and 4 down is greater than one

16. The curve $x^2 + y^2 = 25$ is drawn. Points on the curve whose x-coordinate and y-coordinate are both integers are marked with crosses. All of those crosses are joined in turn to create a convex polygon P. What is the area of P?

17. Matthew writes a list of all three-digit squares backwards. For example, in his list Matthew writes the three-digit square '625' as '526'. Norma looks at Matthew's list and notices that some of the numbers are prime numbers. What is the mean of those prime numbers in Matthew's list?

18. The diagram shows a semicircle with diameter PQ inscribed in a rhombus $ABCD$. The rhombus is tangent to the arc of the semicircle in two places. Points P and Q lie on sides BC and CD of the rhombus respectively. The line of symmetry of the semicircle is coincident with the diagonal AC of the rhombus. It is given that $\angle CBA = 60°$. The semicircle has radius 10. The area of the rhombus can be written in the form $a\sqrt{b}$ where a and b are integers and b is prime. What is the value of $ab + a + b$?



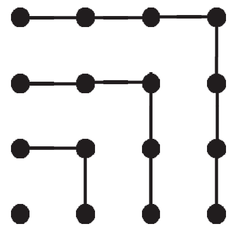
19. The sequence of functions $F_1(x)$, $F_2(x)$, … satisfies the following conditions:

$$F_1(x) = x, \qquad F_{n+1}(x) = \frac{1}{1 - F_n(x)}.$$

The integer C is a three-digit cube such that $F_C(C) = C$.
What is the largest possible value of C?

20. Let a, b and c be positive integers such that $a^2 = 2b^3 = 3c^5$. What is the minimum possible number of factors of abc (including 1 and abc)?

1. Using this picture we can observe that
$1 + 3 + 5 + 7 = 4 \times 4$.
What is the value of
$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21$?



2. Both rows of the following grid have the same sum. What is the value of * ?

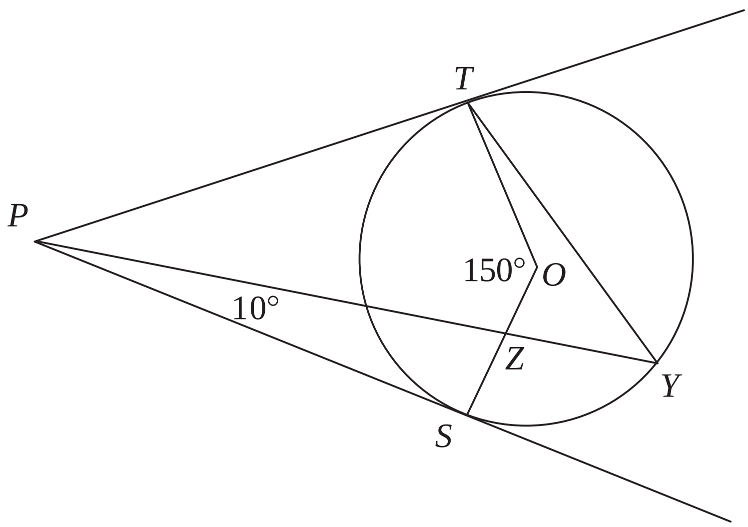
1	2	3	4	5	6	7	8	9	10	1050
11	12	13	14	15	16	17	18	19	20	*

3. Andrew has two containers for carrying water. The containers are cubes without tops and have base areas of 4 dm^2 and 36 dm^2 respectively. Andrew has to completely fill the larger cube with pond water, which must be carried from the pond using the smaller cube. What is the smallest number of visits Andrew has to make to the pond with the smaller cube?

4. How many four-digit numbers formed only of odd digits are divisible by five?

5. The notation $|x|$ is used to denote the absolute value of a number, regardless of sign. For example, $|7| = |-7| = 7$.
The graphs $y = |2x| - 3$ and $y = |x|$ are drawn on the same set of axes. What is the area enclosed by them?

6.



In the diagram, PT and PS are tangents to a circle with centre O. The point Y lies on the circumference of the circle; and the point Z is where the line PY meets the radius OS.

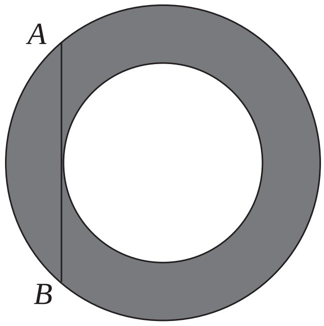
Also, $\angle SPZ = 10°$ and $\angle TOS = 150°$.

How many degrees are there in the sum of $\angle PTY$ and $\angle PYT$?

7. Bav is counting the edges on a particular prism. The prism has more than 310 edges, it has fewer than 320 edges and its number of edges is odd. How many edges does the prism have?

8. The real numbers x, y and z are a solution (x, y, z) of the equation $(x^2 - 9)^2 + (y^2 - 4)^2 + (z^2 - 1)^2 = 0$. How many different possible values are there for $x + y + z$?

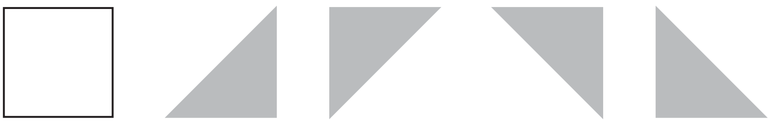
9. The diagram shows two concentric circles. Chord AB of the larger circle is tangential to the smaller circle.
The length of AB is 32 cm and the area of the shaded region is $k\pi$ cm^2.
What is the value of k?



10. Consider the expression $1 * 2 * 3 * 4 * 5 * 6$.
Each star in the expression is to be replaced with either '+' or '×'.
N is the largest possible value of the expression. What is the largest prime factor of N?

11. Stephanie enjoys swimming. She goes for a swim on a particular date if, and only if, the day, month (where January is replaced by '01' through to December by '12') and year are all of the same parity (that is they are all odd, or all are even). On how many days will she go for a swim in the two-year period between January 1st of one year and December 31st of the following year inclusive?

12. Delia is joining three vertices of a square to make four right-angled triangles.
She can create four triangles doing this, as shown.

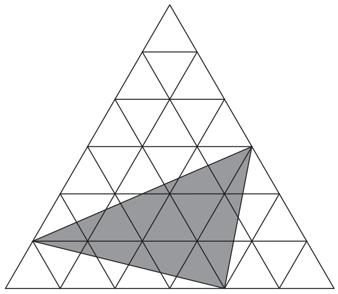


How many right-angled triangles can Delia make by joining three vertices of a regular polygon with 18 sides?

13. This year, 2016, can be written as the sum of two positive integers p and q where $2p = 5q$ (as $2016 = 1440 + 576$). How many years between 2000 and 3000 inclusive have this property?

14. The lengths of the sides of a triangle are the integers 13, x, y. It is given that $xy = 105$. What is the length of the perimeter of the triangle?

15. The large equilateral triangle shown consists of 36 smaller equilateral triangles. Each of the smaller equilateral triangles has area 10 cm^2.
The area of the shaded triangle is K cm^2. Find K.



16. A function $f(x)$ has the property that, for all positive x, $3f(x) + 7f\left(\frac{2016}{x}\right) = 2x$.
What is the value of $f(8)$?

17. Students in a class take turns to practise their arithmetic skills. Initially a board contains the integers from 1 to 10 inclusive, each written ten times. On each turn a student first deletes two of the integers and then writes on the board the number that is one more than the sum of those two deleted integers. Turns are taken until there is only one number remaining on the board. Assuming no student makes a mistake, what is the remaining number?

18. The sum of the squares of four consecutive positive integers is equal to the sum of the squares of the next three consecutive integers. What is the square of the smallest of these integers?

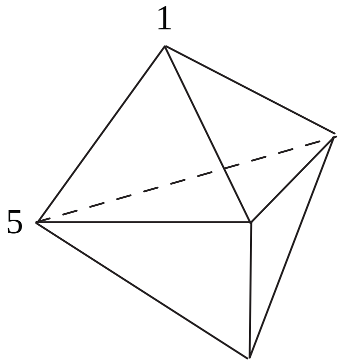
19. Erin lists all three-digit primes that are 21 less than a square. What is the mean of the numbers in Erin's list?

20. A barcode of the type shown in the two examples is composed of alternate strips of black and white, where the leftmost and rightmost strips are always black. Each strip (of either colour) has a width of 1 or 2. The total width of the barcode is 12. The barcodes are always read from left to right. How many distinct barcodes are possible?

1. In a pile of 200 coins, 2% are gold coins and the rest are silver. Simple Simon removes one silver coin every day until the pile contains 20% gold coins. How many silver coins does Simon remove?

2. The value of the expression $1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{5}}}$ is $\frac{a}{b}$, where a and b are integers whose only common factor is 1. What is the value of $a + b$?

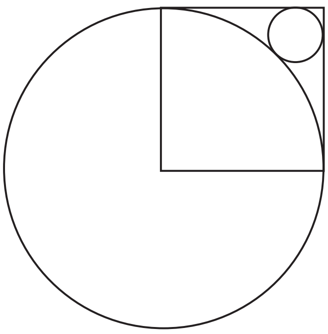
3. The diagram shows a solid with six triangular faces and five vertices. Andrew wants to write an integer at each of the vertices so that the sum of the numbers at the three vertices of each face is the same. He has already written the numbers 1 and 5 as shown.



What is the sum of the other three numbers he will write?

4. A box contains two white socks, three blue socks and four grey socks. Rachel knows that three of the socks have holes in, but does not know what colour these socks are. She takes one sock at a time from the box without looking. How many socks must she take for her to be certain she has a pair of socks of the same colour without holes?

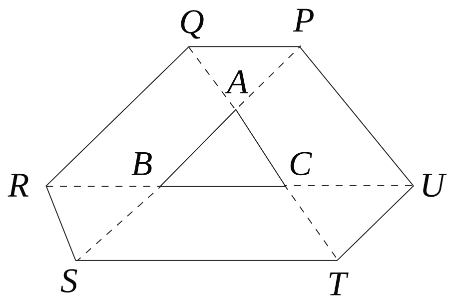
5. The diagram shows two circles and a square with sides of length 10 cm. One vertex of the square is at the centre of the large circle and two sides of the square are tangents to both circles. The small circle touches the large circle. The radius of the small circle is $\left(a - b\sqrt{2}\right)$ cm.



What is the value of $a + b$?

6. The median of a set of five positive integers is one more than the mode and one less than the mean. What is the largest possible value of the range of the five integers?

7. The diagram shows a triangle ABC with area 12 cm^2. The sides of the triangle are extended to points P, Q, R, S, T and U as shown so that $PA = AB = BS$, $QA = AC = CT$ and $RB = BC = CU$.



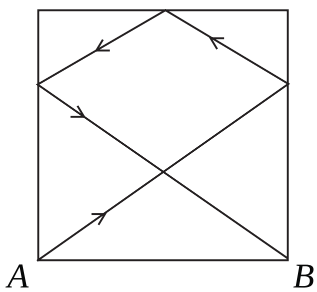
What is the area (in cm^2) of hexagon $PQRSTU$?

8. A mob of 2015 kangaroos contains only red and grey kangaroos. One grey kangaroo is taller than exactly one red kangaroo, one grey kangaroo is taller than exactly three red kangaroos, one grey kangaroo is taller than exactly five red kangaroos and so on with each successive grey kangaroo being taller than exactly two more red kangaroos than the previous grey kangaroo. The final grey kangaroo is taller than all the red kangaroos. How many grey kangaroos are in the mob?

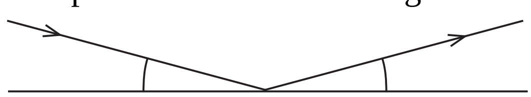
9. A large rectangle is divided into four identical smaller rectangles by slicing parallel to one of its sides. The perimeter of the large rectangle is 18 metres more than the perimeter of each of the smaller rectangles. The area of the large rectangle is 18 m^2 more than the area of each of the smaller rectangles. What is the perimeter in metres of the large rectangle?

10. Katherine and James are jogging in the same direction around a pond. They start at the same time and from the same place and each jogs at a constant speed. Katherine, the faster jogger, takes 3 minutes to complete one lap and first overtakes James 8 minutes after starting. How many seconds does it take James to complete one lap?

11. A ball is propelled from corner A of a *square* snooker table of side 2 metres. After bouncing off three cushions as shown, the ball goes into a pocket at B. The total distance travelled by the ball is $\sqrt{k}$ metres. What is the value of k?



(Note that when the ball bounces off a cushion, the angle its path makes with the cushion as it approaches the point of impact is equal to the angle its path makes with the cushion as it moves away from the point of impact as shown in the diagram below.)



12. Chris planned a 210 km bike ride. However, he rode 5 km/h faster than he planned and finished his ride 1 hour earlier than he planned. His average speed for the ride was x km/h. What is the value of x?

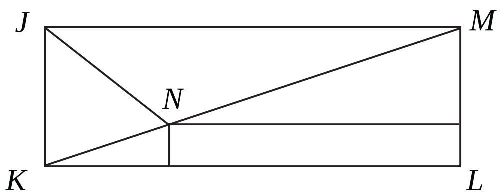
13. Twenty-five people who always tell the truth or always lie are standing in a queue. The man at the front of the queue says that everyone behind him always lies. Everyone else says that the person immediately in front of them always lies. How many people in the queue always lie?

14. Four problems were attempted by 100 contestants in a Mathematics competition. The first problem was solved by 90 contestants, the second by 85 contestants, the third by 80 contestants and the fourth by 75 contestants. What is the smallest possible number of contestants who solved all four problems?

15. The 5-digit number '$XX4XY$' is exactly divisible by 165. What is the value of $X + Y$?

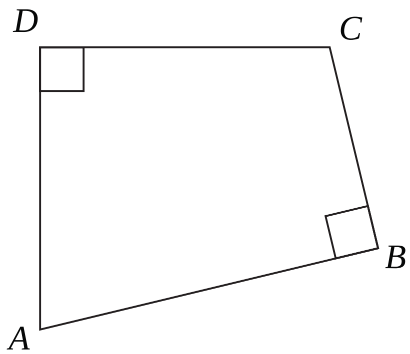
16. How many 10-digit numbers are there whose digits are all 1, 2 or 3 and in which adjacent digits differ by 1?

17. In rectangle $JKLM$, the bisector of angle KJM cuts the diagonal KM at point N as shown. The distances between N and sides LM and KL are 8 cm and 1 cm respectively. The length of KL is $(a + \sqrt{b})$ cm. What is the value of $a + b$?



18. Numbers a, b and c are such that $\frac{a}{b + c} = \frac{b}{c + a} = \frac{c}{a + b} = k$.
How many possible values of k are there?

19. In quadrilateral $ABCD$, $\angle ABC = \angle ADC = 90°$, $AD = DC$ and $AB + BC = 20$ cm.



What is the area in cm^2 of quadrilateral $ABCD$?

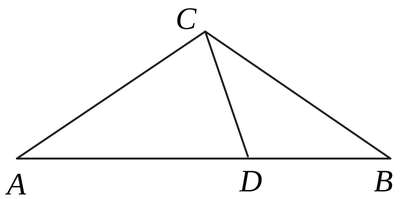
20. The number $N = 3^{16} - 1$ has a divisor of 193. It also has some divisors between 75 and 85 inclusive. What is the sum of these divisors?

1. Three standard dice are stacked in a tower so that the numbers on each pair of touching faces add to 5. The number on the top of the tower is even. What is the number on the base of the tower?

2. How many prime numbers p have the property that $p^4 + 1$ is also prime?

3. Neil has a combination lock. He knows that the combination is a four-digit number with first digit 2 and fourth digit 8 and that the number is divisible by 9. How many different numbers with that property are there?

4. In the diagram, triangle ABC is isosceles with $CA = CB$ and point D lies on AB with $AD = AC$ and $DB = DC$. What is the size in degrees of angle BCA?

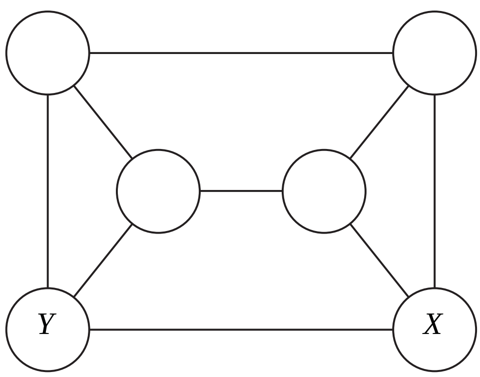


5. Six of the seven numbers 11, 20, 15, 25, 16, 19 and 17 are divided into three groups of two numbers so that the sum of the two numbers in each group is the same. Which number is not used?

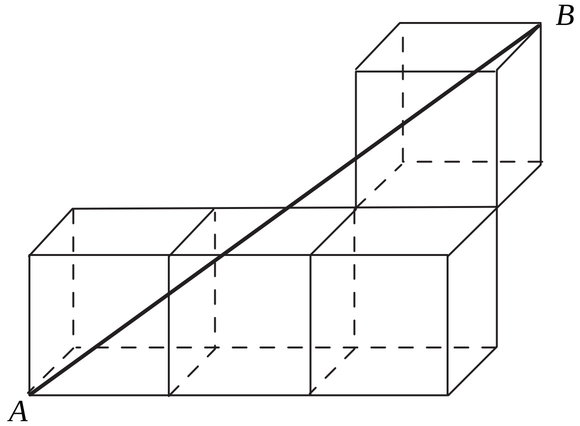
6. The numbers x, y and z satisfy the equations $x^2yz^3 = 7^3$ and $xy^2 = 7^9$. What is the value of $\frac{xyz}{7}$?

7. A table of numbers has 21 columns labelled 1, 2, 3, ..., 21 and 33 rows labelled 1, 2, 3, ..., 33. Every element of the table is equal to 2. All the rows whose label is not a multiple of 3 are erased. All the columns whose label is not an even number are erased. What is the sum of the numbers that remain in the table?

8. Andrew wishes to place a number in each circle in the diagram. The sum of the numbers in the circles of any closed loop of length three must be 30. The sum of the numbers in the circles of any closed loop of length four must be 40. He places the number 9 in the circle marked X. What number should he put in the circle marked Y?



9. Each of the cubes in the diagram has side length 3 cm. The length of AB is $\sqrt{k}$ cm. What is the value of k?

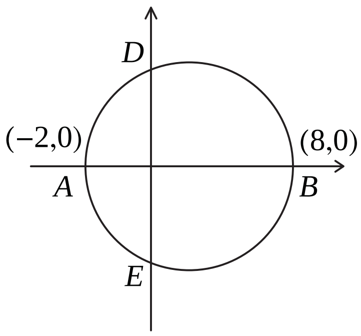


10. A Mathematical Challenge consists of five problems, each of which is worth a different whole number of marks. Carl solved all five problems correctly. He scored 10 marks for the two problems with the lowest numbers of marks and 18 marks for the two problems with the highest numbers of marks. How many marks did he score for all five problems?

11. The mean weight of five children is 45 kg. The mean weight of the lightest three children is 42 kg and the mean weight of the heaviest three children is 49 kg. What is the median weight of the children in kg?

12. On Old MacDonald's farm, the numbers of horses and cows are in the ratio 6:5, the numbers of pigs and sheep are in the ratio 4:3 and the numbers of cows and pigs are in the ratio 2:1. What is the smallest number of animals that can be on the farm?

13. The diagram shows a circle with diameter AB. The coordinates of A are $(-2, 0)$ and the coordinates of B are $(8, 0)$. The circle cuts the y-axis at points D and E. What is the length of DE?

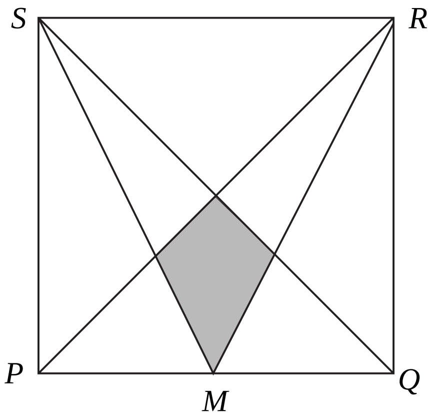


14. Rachel draws 36 kangaroos using three different colours. 25 of the kangaroos are drawn using some grey, 28 are drawn using some pink and 20 are drawn using some brown. Five of the kangaroos are drawn using all three colours. How many kangaroos did she draw that use only one colour?

15. A box contains seven cards numbered from 301 to 307. Graham picks three cards from the box and then Zoe picks two cards from the remainder. Graham looks at his cards and then says "I know that the sum of the numbers on your cards is even". What is the sum of the numbers on Graham's cards?

16. The numbers x, y and z satisfy the equations $x + y + z = 15$ and $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$. What is the value of $x^2 + y^2 + z^2$?

17. In the diagram, $PQRS$ is a square. M is the midpoint of PQ. The area of the square is k times the area of the shaded region. What is the value of k?



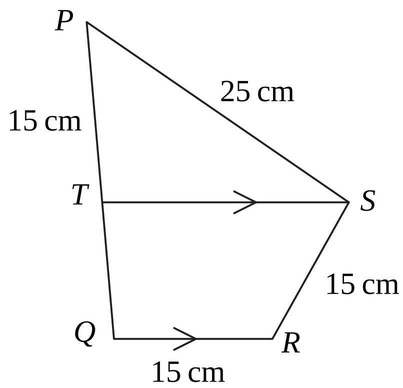
18. Twenty-five workmen have completed a fifth of a project in eight days. Their foreman then decides that the project must be completed in the next 20 days. What is the smallest number of additional workmen required to complete the project on time?

19. In the long multiplication sum shown, each asterisk stands for one digit.

$$\begin{array}{r} *** \\ \times\ \ *** \\ \hline 22** \\ 90{*}0 \\ **2** \\ \hline 56*** \end{array}$$

What is the sum of the digits of the answer?

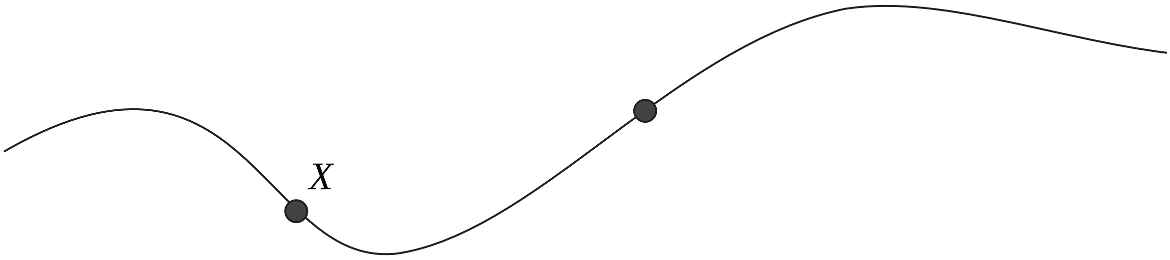
20. In the quadrilateral $PQRS$ with $PQ = PS = 25$ cm and $QR = RS = 15$ cm, point T lies on PQ so that $PT = 15$ cm and so that TS is parallel to QR. What is the length in centimetres of TS?



1. Adam, Bill and Carl have 30 sweets between them. Bill gives 5 sweets to Carl, Carl gives 4 sweets to Adam and Adam gives 2 sweets to Bill. Now each of them has the same number of sweets. How many sweets did Carl have initially?

2. An i-rectangle is defined to be a rectangle all of whose sides have integer length. Two i-rectangles are considered to be the same if they have the same side-lengths. The sum of the areas of all the different i-rectangles with perimeter 22 cm is A cm^2. What is the value of A?

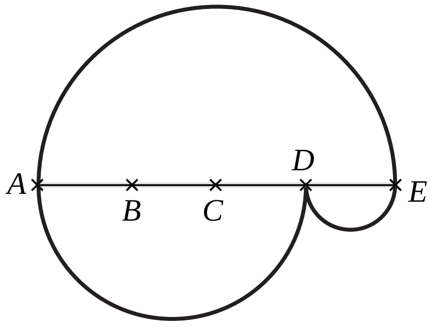
3. Some historians claim that the ancient Egyptians used a rope with two knots tied in it to construct a right-angled triangle by joining the two ends of the rope and taking the vertices of the triangle to be at the two knots and at the join. The length of the rope shown is 60 m and one of the knots is at X, which is 15 m from one end of the rope. How many metres from the other end of the rope should the second knot be placed to be able to create a right-angled triangle with the right angle at X?



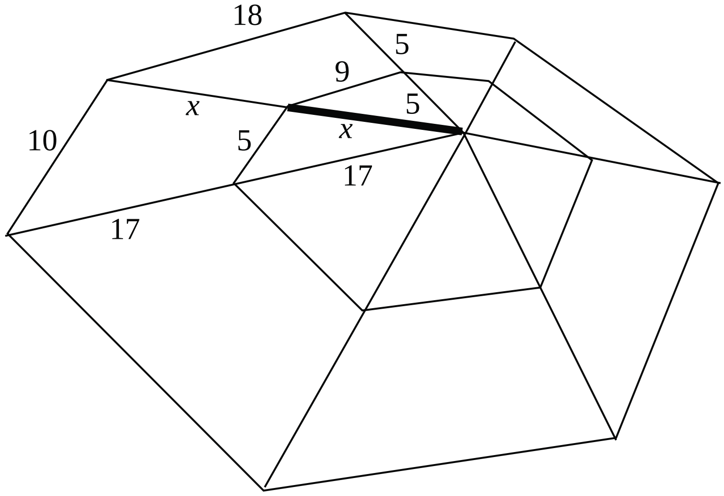
4. The height, width and length of a cube are multiplied by 2, 3 and 6 respectively to create a cuboid. The surface area of the cuboid is N times the surface area of the original cube. What is the value of N?

5. In a university admissions test, Dean gets exactly 10 of the first 15 questions correct. He then answers all the remaining questions correctly. Dean finds out he has answered 80% of all the questions correctly. How many questions are there on the test?

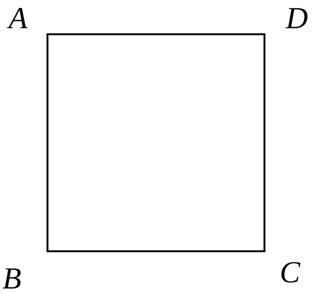
6. In the diagram, AE is divided into four equal parts and semicircles have been drawn with AE, AD and DE as diameters. This has created two new paths, an upper path and a lower path, from A to E. The ratio of the length of the upper path to the length of the lower path can be written as $a : b$ in its lowest terms. What is the value of $a + b$?



7. A mathematically skilful spider has spun a web and the lengths of some of the strands (which are all straight lines) are as shown in the diagram. It is known that x is an integer. What is the value of x?



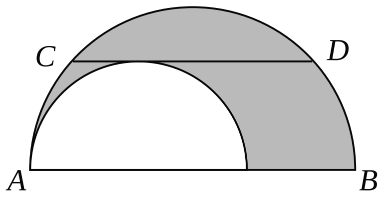
8. The square $ABCD$ has sides of length 1. All possible squares that share two vertices with $ABCD$ are drawn. The boundary of the region formed by the union of these squares is an irregular polygon. What is the area of this polygon?



9. In triangle ABC, angle B is 25% smaller than angle C and 50% larger than angle A. What is the size in degrees of angle B?

10. In the equation $2^{m+1} + 2^m = 3^{n+2} - 3^n$, m and n are integers. What is the value of m?

11. The diagram shows two semicircles. The chord CD of the larger semicircle is parallel to AB, and touches the smaller semicircle. The length of CD is 32 m. The area of the shaded region is $k\pi$ m^2. What is the value of k?



12. The sum of five consecutive integers is equal to the sum of the next three consecutive integers. What is the largest of these eight integers?

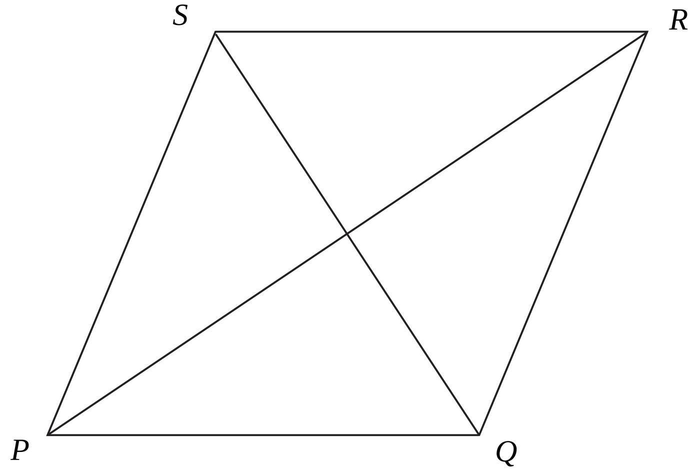
13. Zoe was born on her mother's 24th birthday so they share birthdays. Assuming they both live long lives, on how many birthdays will Zoe's age be a factor of her mother's age?

14. What is the largest three-digit integer that can be written in the form $n + \sqrt{n}$ where n is an integer?

15. How many integers a are there for which the roots of the quadratic equation $x^2 + ax + 2013 = 0$ are integers?

16. A sphere of radius 3 has its centre at the origin. How many points on the surface of the sphere have coordinates that are all integers?

17. The length of each side of the rhombus $PQRS$ is equal to the geometric mean of the lengths of its diagonals. What is the size in degrees of the obtuse angle PQR?



[The geometric mean of 2 values x_1 and x_2 is given by $\sqrt{x_1 x_2}$.]

18. How many of the first 2013 triangular numbers are multiples of 5?

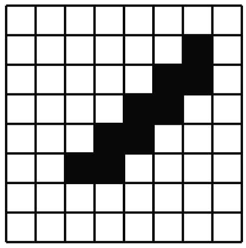
19. The increasing sequence 1, 3, 4, 9, 10, 12, 13, … contains all the powers of 3 and all the numbers that can be written as the sum of two or more distinct powers of 3. What is the 70th number in the sequence?

20. Rachel and Nicky stand at either end of a straight track. They then run at constant (but different) speeds to the other end of the track, turn and run back to their original end at the same speed they ran before. On their first leg, they pass each other 20 m from one end of the track. When they are both on their return leg, they pass each other for a second time 10 m from the other end of the track. How many metres long is the track?

1. How many zeroes are there at the end of the number which is the product of the first 2012 prime numbers?

2. The size of the increase from each term to the next in the list a, $225\frac{1}{2}$, c, d, 284 is always the same. What is the value of a?

3. On the grid shown in the diagram, the shaded squares form a region, A.



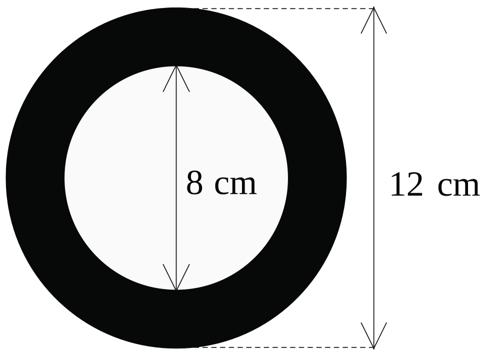
What is the maximum number of additional grid squares which can be shaded to form a region B such that B contains A and that the lengths of the perimeters of A and B are the same?

4. Five cards are laid on a table, as shown. Every card has a letter on one side and a number on the other side.

Peter says: "For every card on the table, if there is a vowel on one side of the card, then there is an even number on the other side."

What is the smallest number of cards Sylvia must turn over in order to be certain that Peter is telling the truth?

5. Susan has two pendants made of the same material. They are equally thick and weigh the same. The first pendant is in the shape of an annulus created from two concentric circles, with diameters 8 cm and 12 cm, as shown. The shape of the second pendant is a disc. The diameter of the second pendant is written in the form $a\sqrt{b}$, where a is an integer and b is a prime integer.



What is the value of $a + b$?

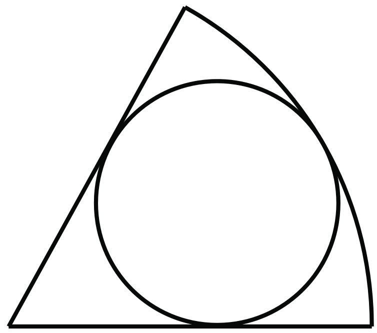
6. Given that $4^x = 9$ and $9^y = 256$, what is the value of xy?

7. When 1001 is divided by a single-digit number, the remainder is 5.
What is the remainder when 2012 is divided by the same single-digit number?

8. The three prime numbers a, b and c are such that $a > b > c$, $a + b + c = 52$ and $a - b - c = 22$.
What is the value of abc?

9. The diagram shows a circle touching a sector of another circle in three places. The ratio of the radius of the sector to the radius of the small circle is 3:1. The ratio of the area of the sector to the area of the small circle, written in its simplest form, is $p : q$.

What is the value of $p + q$?



10. Sixteen teams play in a volleyball league. Each team plays one game against every other team. For each game, the winning team is awarded 1 point, and the losing team 0 points. There are no draws. After all the games have been played and the teams have been ranked according to their total scores, the total scores form a sequence where the difference between consecutive terms is constant.

How many points did the team in first place receive?

11. Last year there were 30 more boys than girls in the school choir. This year the number of choir members has increased by 10%, the number of girls has increased by 20% and the number of boys by 5%.

How many members does the choir have this year?

12. The cells of a 4×4 grid are coloured black and white as shown in Figure 1. One move allows us to exchange the colourings of any two cells positioned in the same row or in the same column.

What is the minimum number of moves needed to obtain Figure 2?

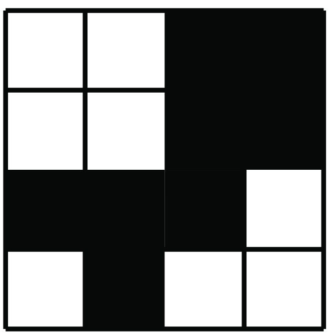


Figure 1

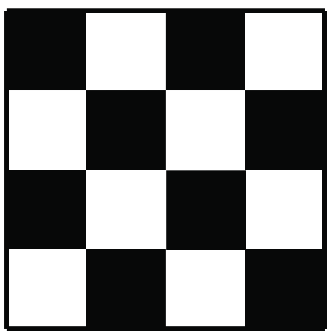
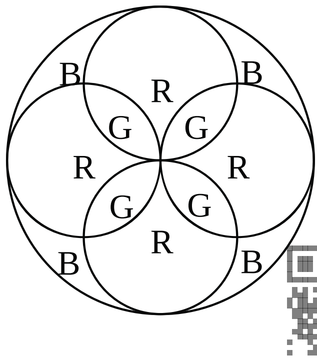


Figure 2

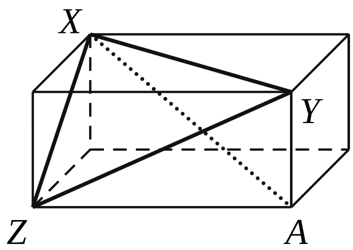
13. A circular stained-glass window is shown in the diagram. The four smaller circles are the same size and are positioned at equal intervals around the centre of the large circle. The letters R, G and B have been placed in regions of red, green and blue glass respectively. The total area of the green glass is 400.

What is the area of the blue glass?



14. The diagram shows a cuboid. In triangle XYZ, the lengths of XY, XZ and YZ are 9, 8 and $\sqrt{55}$ respectively.

What is the length of the diagonal XA shown?



15. The equation $x^2 - bx + 80 = 0$, where $b > 0$, has two integer-valued solutions. What is the sum of the possible values of b?

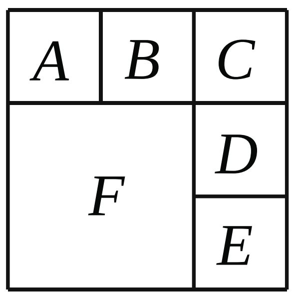
16. Given that $a + b = 5$ and $ab = 3$, what is the value of $a^4 + b^4$?

17. David removed one number from ten consecutive natural numbers. The sum of the remaining numbers was 2012.

Which number did he remove?

18. The diagram shows a square divided into six smaller squares labelled A, B, C, D, E and F. Two squares are considered to be adjacent if they have more than one point in common. The numbers 1, 2, 3, 4, 5 and 6 are to be placed in the smaller squares, one in each, so that no two adjacent squares contain numbers differing by 3.

How many different arrangements are possible?



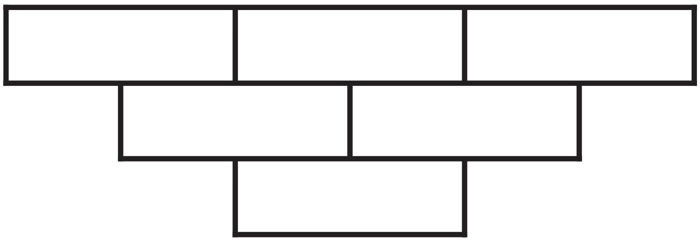
19. A rectangle which has integer-length sides and area 36 is cut from a square with sides of length 20 so that one of the sides of the rectangle forms part of one of the sides of the square.

What is the largest possible perimeter of the remaining shape?

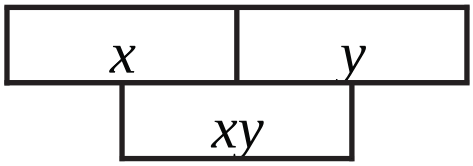
20. How many subsets of the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ exist in which the sum of the largest element and the smallest element is 11?

1. The diagram below is to be completed so that:
   a) each cell contains a positive integer;
   b) apart from the top row, the number in each cell is the product of the numbers in the two cells immediately above;
   c) the six numbers are all different.

   What is the smallest possible total of the six numbers?



RULE:



2. The mean number of students accepted by a school in the four years 2007 to 2010 was 325. The mean number of students accepted by the school in the five years 2007 to 2011 was 4% higher.

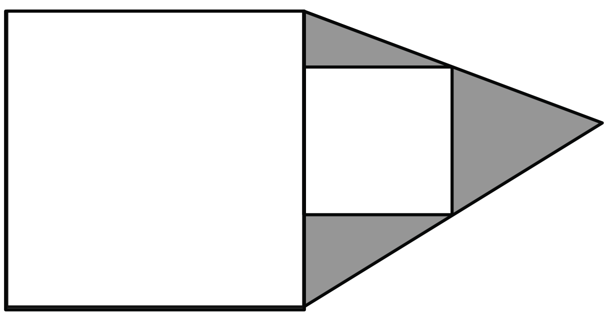
   How many students did this school accept in 2011?

3. 200 people stand in a line. The prize-giver walks along the line 200 times, always starting at the same end. On the first pass, the prize-giver gives each person a pound coin. On the second pass along the line, the prize-giver gives every second person another pound. On the third pass, every third person is given another pound, and so on.

   After 200 passes, how many pounds has the 120th person been given?

4. The diagram below includes two squares: one has sides of length 20 and the other has sides of length 10.

   What is the area of the shaded region?

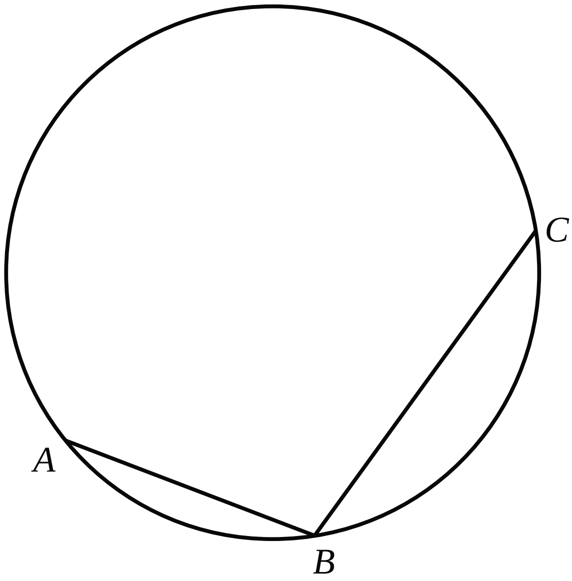


5. How many positive two-digit numbers are there whose square and cube both end in the same digit?

6. The lengths of two sides of an acute-angled triangle and the perpendicular height from the third side of the triangle are 12, 13 and 15 (possibly not in that order).

   What is the area of the triangle?

7. In the diagram, the radius of the circle is equal to the length *AB*.
What is the size of angle *ACB*, in degrees?

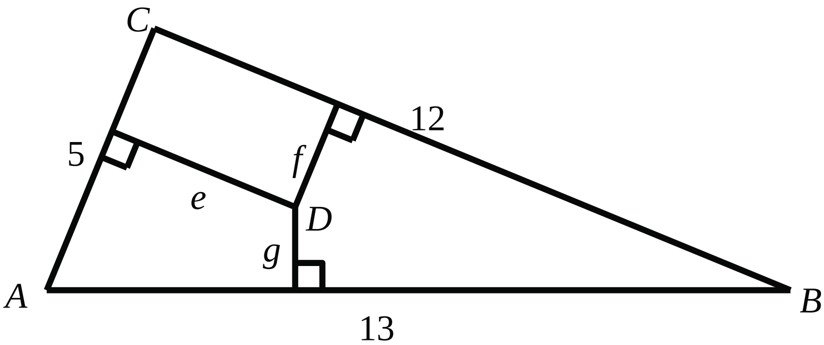


8. The price of an item in pounds and pence is increased by 4%. The new price is exactly n pounds where n is a whole number.
What is the smallest possible value of n?

9. How many squares have $(-1, -1)$ as a vertex and at least one of the coordinate axes as an axis of symmetry?

10. What is the value of $\left(\sqrt{8 + 2\sqrt{7}} - \sqrt{8 - 2\sqrt{7}}\right)^2$?

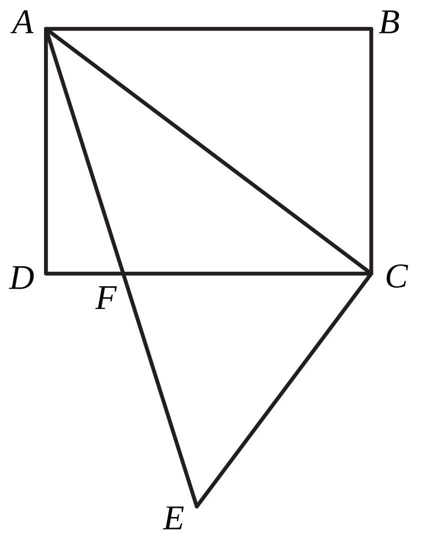
11. In the diagram, *ABC* is a triangle with sides $AB = 13$, $BC = 12$ and $AC = 5$. The point *D* is any point inside the triangle with $CD = 4$ and the perpendicular distances from *D* to the sides of the triangle are e, f and g, as shown.
What is the value of $5e + 12f + 13g$?



12. Elections in Herbville were held recently. Everyone who voted for the Broccoli Party had already eaten broccoli. Of those who voted for other parties, 90% had never eaten broccoli. Of those who voted, 46% had eaten broccoli.
What percentage voted for the Broccoli Party?

13. A manager in a store has to determine the price of a sweater. Market research gives him the following data: If the price is €75, then 100 teenagers will buy the sweater. Each time the price is increased by €5, 20 fewer teenagers will buy the sweater. However, each time the price is decreased by €5, 20 sweaters more will be sold. The sweaters cost the company €30 apiece.
What is the sale price that maximizes profits?

14. The diagram shows a rectangle $ABCD$ with $AB = 16$ and $BC = 12$. Angle ACE is a right angle and $CE = 15$. The line segments AE and CD meet at F.
What is the area of triangle ACF?



15. For each real number x, let $f(x)$ be the minimum of the numbers $3x + 1$, $2x + 3$ and $-4x + 24$.
What is the maximum value of $f(x)$?

16. The integer m has ninety-nine digits, all of them nines.
What is the sum of the digits of m^2 ?

17. In rectangle $ABCD$, the midpoints of sides BC, CD and DA are P, Q and R respectively. The point M is the midpoint of QR. The area of triangle APM is a fraction m/n of the area of rectangle $ABCD$, where m and n are integers and m/n is in its simplest form.
What is the value of $m + n$?

18. The integers a, b and c are such that $0 < a < b < c < 10$. The sum of all three-digit numbers that can be formed by a permutation of these three integers is 1554.
What is the value of c?

19. Given that $\left(a + \frac{1}{a}\right)^2 = 6$ and $a^3 + \frac{1}{a^3} = N\sqrt{6}$ and $a > 0$, what is the value of N?

20. The polynomial $f(x)$ is such that $f(x^2 + 1) \equiv x^4 + 4x^2$ and $f(x^2 - 1) \equiv ax^4 + 4bx^2 + c$.
What is the value of $a^2 + b^2 + c^2$?

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