United Kingdom Mathematics Trust

# Senior Mathematical Challenge

## Questions by Topic

2007 – 2016 Collection

Last updated: August 22, 2020

## Contents






Comments and suggestions to 89272376@QQ.com .

# Answers

| | 07 | 08 | 09 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | C | C | A | C | D | E | A | C | D | B | C | C | | | | 1 |
| 2 | A | B | D | B | D | B | C | B | A | D | B | C | | | | 2 |
| 3 | B | D | E | C | B | D | D | D | B | B | E | D | | | | 3 |
| 4 | C | C | E | B | D | B | C | A | B | C | E | E | | | | 4 |
| 5 | E | E | A | D | B | C | E | C | A | A | C | B | | | | 5 |
| 6 | A | E | C | B | C | C | E | B | E | D | B | B | | | | 6 |
| 7 | D | D | B | C | D | D | B | B | B | A | C | A | | | | 7 |
| 8 | E | A | C | D | C | C | B | C | D | B | B | B | | | | 8 |
| 9 | E | D | D | D | B | C | A | D | B | C | B | A | | | | 9 |
| 10 | B | D | B | E | C | E | B | C | D | B | D | D | | | | 10 |
| 11 | D | B | C | D | C | D | A | E | C | E | D | B | | | | 11 |
| 12 | E | C | E | A | D | E | B | D | A | C | B | A | | | | 12 |
| 13 | B | C | C | A | D | B | D | E | E | A | C | E | | | | 13 |
| 14 | B | D | D | E | C | D | D | E | E | D | A | D | | | | 14 |
| 15 | D | A | D | E | B | A | B | C | C | B | B | D | | | | 15 |
| 16 | A | A | B | E | E | A | D | A | A | D | D | D | | | | 16 |
| 17 | C | E | C | E | A | B | A | C | C | D | D | B | | | | 17 |
| 18 | A | C | A | A | E | A | C | D | D | E | E | A | | | | 18 |
| 19 | D | B | C | B | B | E | C | E | A | D | B | C | | | | 19 |
| 20 | C | B | E | C | B | E | E | A | E | E | B | E | | | | 20 |
| 21 | B | B | B | A | C | D | B | B | C | C | C | A | | | | 21 |
| 22 | D | A | C | B | A | B | C | A | B | C | D | C | | | | 22 |
| 23 | E | B | D | B | B | C | A | E | D | E | C | B | | | | 23 |
| 24 | D | E | B | E | B | B | E | A | C | B | D | E | | | | 24 |
| 25 | D | D | A | D | C | B | D | B | C | D | A | C | | | | 25 |

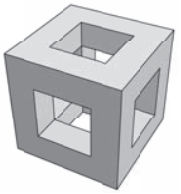


Comments and suggestions to 89272376@QQ.com .

# 1 3D shapes

**Q1 :** 2007_Q11

A $4 \times 4 \times 4$ cube has three $2 \times 2 \times 4$ holes drilled symmetrically all the way through, as shown.

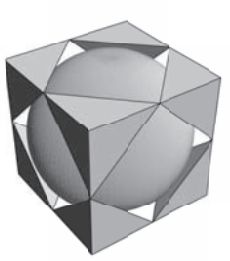


What is the surface area of the resulting solid?

A 192 B 144 C 136 D 120 E 96

**Q2 :** 2007_Q24

A paperweight is made from a glass cube of side 2 units by first shearing off the eight tetrahedral corners which touch at the midpoints of the edges of the cube. The remaining inner core of the cube is discarded and replaced by a sphere. The eight corner pieces are now stuck onto the sphere so that they have the same positions relative to each other as they did originally. What is the diameter of the sphere?



A $\sqrt{8} - 1$ B $\sqrt{8} + 1$ C $\frac{1}{3}(6 + \sqrt{3})$ D $\frac{4}{3}\sqrt{3}$ E $2\sqrt{3}$

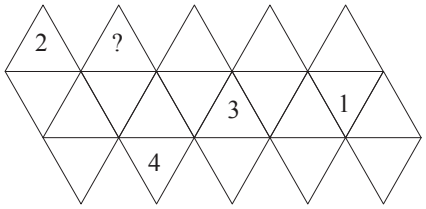
**Q3 :** 2009_Q17

A solid cube is divided into two pieces by a single rectangular cut. As a result, the total surface area increases by a fraction $f$ of the surface area of the original cube. What is the greatest possible value of $f$?

A $\frac{1}{3}$ B $\frac{\sqrt{3}}{4}$ C $\frac{\sqrt{2}}{3}$ D $\frac{1}{2}$ E $\frac{1}{\sqrt{3}}$

**Q4 :** 2009_Q23

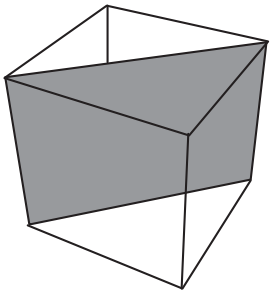
The net shown is folded into an icosahedron and the remaining faces are numbered such that at each vertex the numbers 1 to 5 all appear. What number must go on the face with a question mark?




A 1 B 2 C 3 D 4 E 5

**Q5 :** 2010_Q18

A solid cube of side 2 cm is cut into two triangular prisms by a plane passing through four vertices, as shown. What is the total surface area of these two prisms?



A $8(3 + \sqrt{2})$ B $2(8 + \sqrt{2})$ C $8(3 + 2\sqrt{2})$
D $16(3 + \sqrt{2})$ E $8\sqrt{2}$

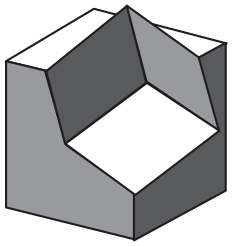
**Q6 :** 2010_Q24

Three spheres of radius 1 are placed on a horizontal table and inside a vertical hollow cylinder of height 2 units which is just large enough to surround them. What fraction of the internal volume of the cylinder is occupied by the spheres?

A $\frac{2}{7+4\sqrt{3}}$ B $\frac{2}{2+\sqrt{3}}$ C $\frac{1}{3}$ D $\frac{3}{2+\sqrt{3}}$ E $\frac{6}{7+4\sqrt{3}}$

**Q7 :** 2011_Q25

A solid sculpture consists of a $4 \times 4 \times 4$ cube with a $3 \times 3 \times 3$ cube sticking out, as shown. Three vertices of the smaller cube lie on edges of the larger cube, the same distance along each.



What is the total volume of the sculpture?

A 79 B 81 C 82 D 84 E 85

**Q8 :** 2011_Q9

Sam has a large collection of $1 \times 1 \times 1$ cubes, each of which is either red or yellow. Sam makes a $3 \times 3 \times 3$ block from twenty-seven cubes, so that no cubes of the same colour meet face-to-face.

What is the difference between the largest number of red cubes that Sam can use and the smallest number?

A 0 B 1 C 2 D 3 E 4

**Q9 :** 2012_Q13

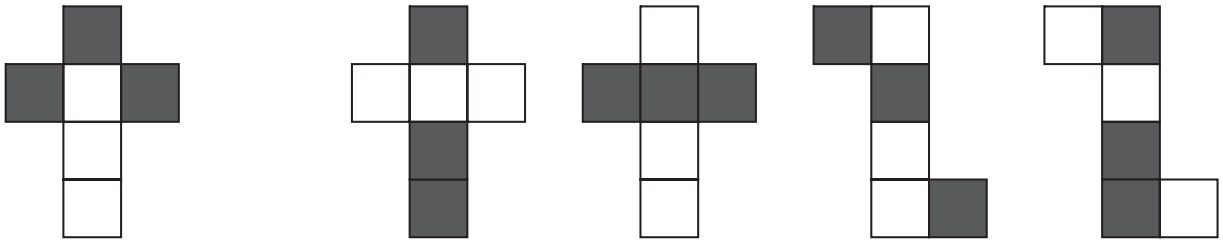
A cube is placed with one face on square 1 in the maze shown, so that it completely covers the square with no overlap. The upper face of the cube is covered in wet paint. The cube is then 'rolled' around the maze, rotating about an edge each time, until it reaches square 25. It leaves paint on all of the squares on which the painted face lands, but on no others. The cube is removed on reaching the square 25. What is the sum of the numbers on the squares which are now marked with paint?

| | | | | |
|---|---|---|---|---|
| 5 | 6 | 7 | 8 | 9 |
| 4 | 19 | 20 | 21 | 10 |
| 3 | 18 | 25 | 22 | 11 |
| 2 | 17 | 24 | 23 | 12 |
| 1 | 16 | 15 | 14 | 13 |

A 78 B 80 C 82 D 169 E 625

**Q10 :** 2014_Q13

Each of the five nets P, Q, R, S and T is made from six squares. Both sides of each square have the same colour. Net P is folded to form a cube.



P Q R S T

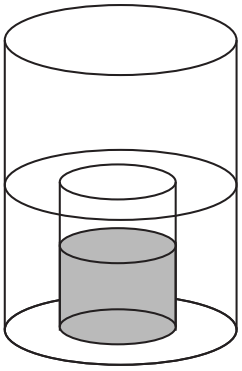
How many of the nets Q, R, S and T can be folded to produce a cube that looks the same as that produced by P?

A 0 B 1 C 2 D 3 E 4

**Q11 :** 2015_Q15

Two vases are cylindrical in shape. The larger vase has diameter 20 cm. The smaller vase has diameter 10 cm and height 16 cm. The larger vase is partially filled with water. Then the empty smaller vase, with the open end at the top, is slowly pushed down into the water, which flows over its rim. When the smaller vase is pushed right down, it is half full of water.

What was the original depth of the water in the larger vase?



A 10 cm B 12 cm C 14 cm D 16 cm E 18 cm

**Q12 :** 2016_Q23

A cuboid has sides of lengths 22, 2 and 10. It is contained within a sphere of the smallest possible radius. What is the side-length of the largest cube that will fit inside the same sphere?

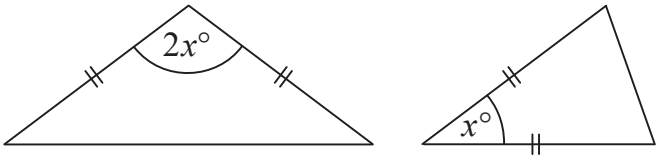
A 10 B 11 C 12 D 13 E 14

## 2 Angles

**Q13 :** 2007_Q17

The two triangles have equal areas and the four marked lengths are equal.

What is the value of $x$?

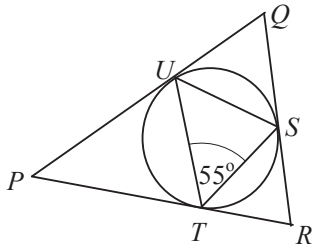



A 30 B 45 C 60 D 75 E more information needed

**Q14 :** 2007_Q19

The largest circle which it is possible to draw inside triangle $PQR$ touches the triangle at $S$, $T$ and $U$, as shown in the diagram.
The size of $\angle STU = 55°$. What is the size of $\angle PQR$?

A $55°$ B $60°$ C $65°$ D $70°$ E $75°$

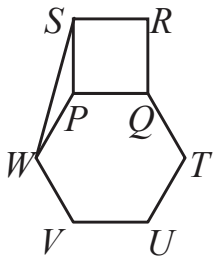



**Q15 :** 2007_Q4

The diagram shows square $PQRS$ and regular hexagon $PQTUVW$.

What is the size of $\angle PSW$?

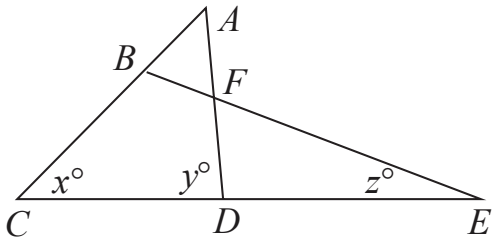
A $10°$ B $12°$ C $15°$ D $24°$ E $30°$




**Q16 :** 2008_Q8

In the figure shown, $AB = AF$ and $ABC$, $AFD$, $BFE$ and $CDE$ are all straight lines.
Which of the following expressions gives $z$ in terms of $x$ and $y$?

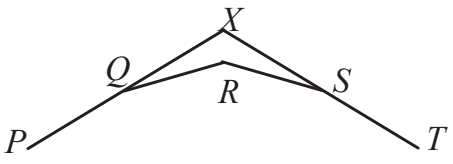
A $\frac{y-x}{2}$ B $y-\frac{x}{2}$ C $\frac{y-x}{3}$ D $y-\frac{x}{3}$ E $y-x$




**Q17 :** 2009_Q14

$P$, $Q$, $R$, $S$, $T$ are vertices of a regular polygon. The sides $PQ$ and $TS$ are produced to meet at $X$, as shown in the diagram, and $\angle QXS = 140°$. How many sides does the polygon have?

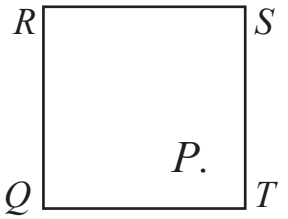
A 9 B 18 C 24 D 27 E 40




**Q18 :** 2009_Q20

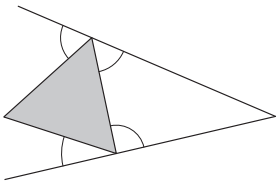
A point $P$ is chosen at random inside a square $QRST$. What is the probability that $\angle RPQ$ is acute?

A $\frac{3}{4}$ B $\sqrt{2}-1$ C $\frac{1}{2}$ D $\frac{\pi}{4}$ E $1-\frac{\pi}{8}$



**Q19 :** 2010_Q3

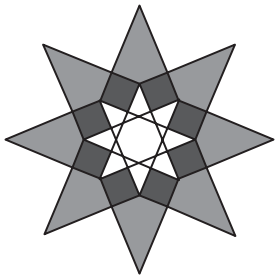
The diagram shows an equilateral triangle touching two straight lines.
What is the sum of the four marked angles?



A 120° B 180° C 240° D 300° E 360°

**Q20 :** 2011_Q19

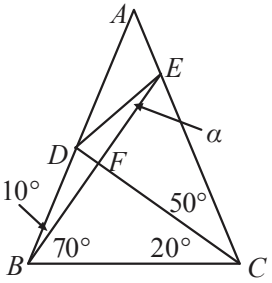
The diagram shows a small regular octagram (an eight-sided star) surrounded by eight squares (dark grey) and eight kites (light grey) to make a large regular octagram. Each square has area 1.
What is the area of one of the light grey kites?



A 2 B $\sqrt{2}+1$ C $\frac{21}{8}$ D $4\sqrt{2}-3$ E $\frac{11}{4}$

**Q21 :** 2011_Q22

In the diagram, $\angle ABE = 10°$; $\angle EBC = 70°$; $\angle ACD = 50°$; $\angle DCB = 20°$; $\angle DEF = \alpha$.
Which of the following is equal to $\tan\alpha$?

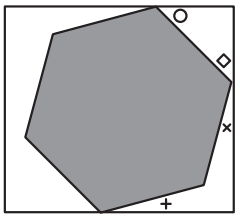



A $\frac{\tan 10° \tan 20°}{\tan 50°}$ B $\frac{\tan 10° \tan 20°}{\tan 70°}$ C $\frac{\tan 10° \tan 50°}{\tan 70°}$

D $\frac{\tan 20° \tan 50°}{\tan 70°}$ E $\frac{\tan 10° \tan 70°}{\tan 50°}$

**Q22 :** 2011_Q5

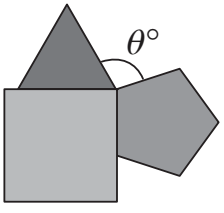
The diagram shows a regular hexagon inside a rectangle.
What is the sum of the four marked angles?



A 90° B 120° C 150° D 180° E 210°

**Q23 :** 2012_Q2

The diagram shows an equilateral triangle, a square and a regular pentagon which all share a common vertex. What is the value of $\theta$?

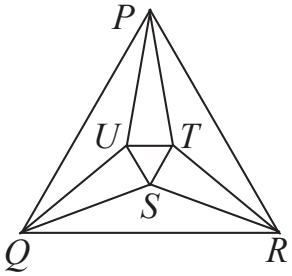



A 98 B 102 C 106 D 110 E 112

**Q24 :** 2013_Q17

The equilateral triangle $PQR$ has side-length 1. The lines $PT$ and $PU$ trisect the angle $RPQ$, the lines $RS$ and $RT$ trisect the angle $QRP$ and the lines $QS$ and $QU$ trisect the angle $PQR$.

What is the side-length of the equilateral triangle $STU$?



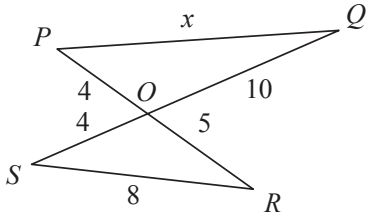

A $\dfrac{\cos 80^\circ}{\cos 20^\circ}$ B $\frac{1}{3}\cos 20^\circ$ C $\cos^2 20^\circ$

D $\frac{1}{6}$ E $\cos 20^\circ \cos 80^\circ$

**Q25 :** 2013_Q24

The diagram shows two straight lines $PR$ and $QS$ crossing at $O$.

What is the value of $x$?




A $7\sqrt{2}$ B $2\sqrt{29}$ C $14\sqrt{2}$ D $7(1+\sqrt{13})$ E $9\sqrt{2}$

**Q26 :** 2014_Q25

Figure 1 shows a tile in the form of a trapezium, where $\alpha = 83\frac{1}{3}^\circ$. Several copies of the tile are placed together to form a symmetrical pattern, part of which is shown in Figure 2. The outer border of the complete pattern is a regular 'star polygon'. Figure 3 shows an example of a regular 'star polygon'.

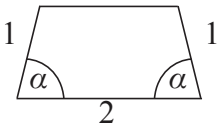
Figure 1 

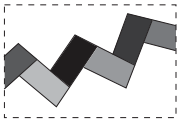
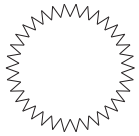

Figure 2 

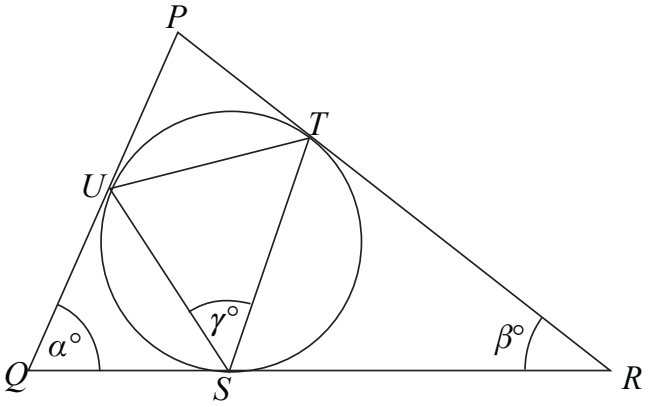
Figure 3 

How many tiles are there in the complete pattern?

A 48 B 54 C 60 D 66 E 72

**Q27 :** 2015_Q12

A circle touches the sides of triangle $PQR$ at the points $S$, $T$ and $U$ as shown. Also $\angle PQR = \alpha^\circ$, $\angle PRQ = \beta^\circ$ and $\angle TSU = \gamma^\circ$. Which of the following gives $\gamma$ in terms of $\alpha$ and $\beta$?




A $\frac{1}{2}(\alpha + \beta)$ B $180 - \frac{1}{2}(\alpha + \beta)$

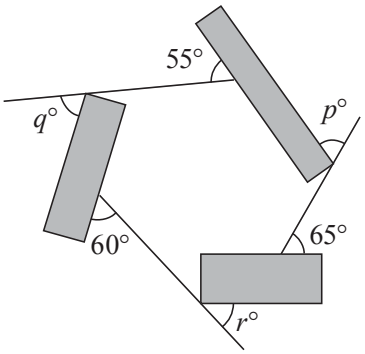
C $180 - (\alpha + \beta)$ D $\alpha + \beta$

E $\frac{1}{3}(\alpha + \beta)$

**Q28 :** 2016_Q15

The diagram shows three rectangles and three straight lines.

What is the value of $p + q + r$?




A 135 B 180 C 210 D 225 E 270

**Q29 :** 2016_Q4

Alex draws a scalene triangle. One of the angles is 80°.

Which of the following could be the difference between the other two angles in Alex's triangle?

A 0° B 60° C 80° D 100° E 120°

## 3 Averages

**Q30 :** 2007_Q10

In 1954, a total of 6 527 mm of rain fell at Sprinkling Tarn and this set a UK record for annual rainfall. The tarn has a surface area of 23 450 $m^2$. Roughly how many million litres of water fell on Sprinkling Tarn in 1954?

A 15 B 150 C 1 500 D 15 000 E 150 000

**Q31 :** 2007_Q8

Travelling at an average speed of 100 km/hr, a train took 3 hours to travel to Birmingham. Unfortunately the train then waited just outside the station, which reduced the average speed for the whole journey to 90 km/hr. For how many minutes was the train waiting?

A 1 B 5 C 10 D 15 E 20

**Q32 :** 2013_Q10

Frank's teacher asks him to write down five integers such that the median is one more than the mean, and the mode is one greater than the median. Frank is also told that the median is 10. What is the smallest possible integer that he could include in his list?

A 3 B 4 C 5 D 6 E 7

**Q33 :** 2015_Q5

The integer $n$ is the mean of the three numbers 17, 23 and $2n$. What is the sum of the digits of $n$?

A 4 B 5 C 6 D 7 E 8

**Q34 :** 2016_Q11

In the grid below each of the blank squares and the square marked $X$ are to be filled by the mean of the two numbers in its adjacent squares. Which number should go in the square marked $X$?

| | | | | | |
|---|---|---|---|---|---|
| 10 | | | $X$ | | 25 |

A 15 B 16 C 17 D 18 E 19

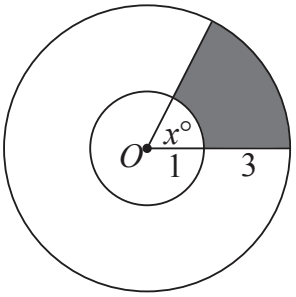
## 4 Circles

**Q35 :** 2007_Q14

The point $O$ is the centre of both circles and the shaded area is one-sixth of the area of the outer circle.

What is the value of $x$?

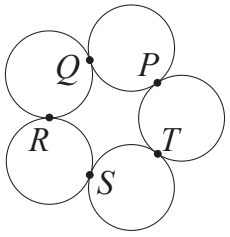
A 60 B 64 C 72 D 80 E 84




**Q36 :** 2008_Q14

Five touching circles each have radius 1 and their centres are at the vertices of a regular pentagon. What is the radius of the circle through the points of contact $P$, $Q$, $R$, $S$ and $T$?

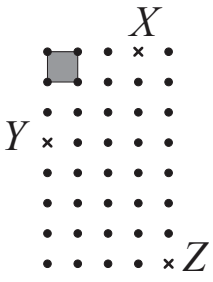
A $\tan 18°$ B $\tan 36°$ C $\tan 45°$ D $\tan 54°$ E $\tan 72°$




**Q37 :** 2008_Q18

The shaded square of the lattice shown has area 1. What is the area of the circle through the points $X$, $Y$ and $Z$?

A $\frac{9\pi}{2}$ B $8\pi$ C $\frac{25\pi}{2}$ D $25\pi$ E $50\pi$



[This page is intentionally left blank.]