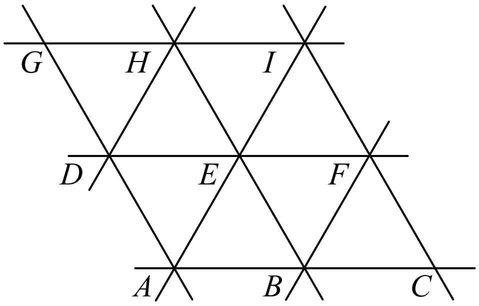
1



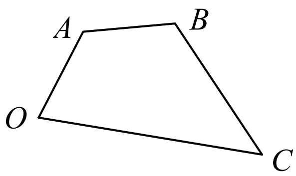
The diagram shows three sets of equally-spaced parallel lines.

Given that $\overrightarrow{AC} = \mathbf{p}$ and that $\overrightarrow{AD} = \mathbf{q}$, express the following vectors in terms of **p** and **q**.

a $\overrightarrow{CA}$ **b** $\overrightarrow{AG}$ **c** $\overrightarrow{AB}$ **d** $\overrightarrow{DF}$ **e** $\overrightarrow{HE}$ **f** $\overrightarrow{AF}$

g $\overrightarrow{AH}$ **h** $\overrightarrow{DC}$ **i** $\overrightarrow{CG}$ **j** $\overrightarrow{IA}$ **k** $\overrightarrow{EC}$ **l** $\overrightarrow{IB}$

2

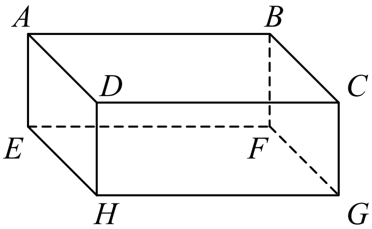


In the quadrilateral shown, $\overrightarrow{OA} = \mathbf{u}$, $\overrightarrow{AB} = \mathbf{v}$ and $\overrightarrow{OC} = \mathbf{w}$.

Find expressions in terms of **u**, **v** and **w** for

a $\overrightarrow{OB}$ **b** $\overrightarrow{AC}$ **c** $\overrightarrow{CB}$

3

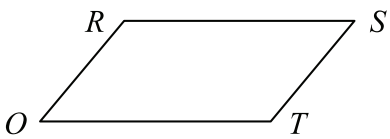


The diagram shows a cuboid.

Given that $\overrightarrow{AB} = \mathbf{p}$, $\overrightarrow{AD} = \mathbf{q}$ and $\overrightarrow{AE} = \mathbf{r}$, find expressions in terms of **p**, **q** and **r** for

a $\overrightarrow{BC}$ **b** $\overrightarrow{AF}$ **c** $\overrightarrow{DE}$ **d** $\overrightarrow{AG}$ **e** $\overrightarrow{GB}$ **f** $\overrightarrow{BH}$

4



The diagram shows parallelogram *ORST*.

Given that $\overrightarrow{OR} = \mathbf{a} + 2\mathbf{b}$ and that $\overrightarrow{OT} = \mathbf{a} - 2\mathbf{b}$,

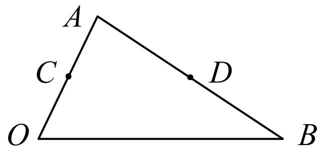
a find expressions in terms of **a** and **b** for

i $\overrightarrow{OS}$ **ii** $\overrightarrow{TR}$

Given also that $\overrightarrow{OA} = \mathbf{a}$ and that $\overrightarrow{OB} = \mathbf{b}$,

b copy the diagram and show the positions of the points *A* and *B*.

5



The diagram shows triangle OAB in which $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

The points C and D are the mid-points of OA and AB respectively.

a Find and simplify expressions in terms of **a** and **b** for

i $\overrightarrow{OC}$ **ii** $\overrightarrow{AB}$ **iii** $\overrightarrow{AD}$ **iv** $\overrightarrow{OD}$ **v** $\overrightarrow{CD}$

b Explain what your expression for $\overrightarrow{CD}$ tells you about $\overrightarrow{OB}$ and $\overrightarrow{CD}$.

6 Given that vectors **p** and **q** are not parallel, state whether or not each of the following pairs of vectors are parallel.

a $2\mathbf{p}$ and $3\mathbf{p}$ **b** $(\mathbf{p} + 2\mathbf{q})$ and $(2\mathbf{p} - 4\mathbf{q})$ **c** $(3\mathbf{p} - \mathbf{q})$ and $(\mathbf{p} - \frac{1}{3}\mathbf{q})$

d $(\mathbf{p} - 2\mathbf{q})$ and $(4\mathbf{q} - 2\mathbf{p})$ **e** $(\frac{3}{4}\mathbf{p} + \mathbf{q})$ and $(6\mathbf{p} + 8\mathbf{q})$ **f** $(2\mathbf{q} - 3\mathbf{p})$ and $(\frac{3}{2}\mathbf{q} - \mathbf{p})$

7 The points O, A, B and C are such that $\overrightarrow{OA} = 4\mathbf{m}$, $\overrightarrow{OB} = 4\mathbf{m} + 2\mathbf{n}$ and $\overrightarrow{OC} = 2\mathbf{m} + 3\mathbf{n}$, where **m** and **n** are non-parallel vectors.

a Find an expression for $\overrightarrow{BC}$ in terms of **m** and **n**.

The point M is the mid-point of OC.

b Show that AM is parallel to BC.

8 The points O, A, B and C are such that $\overrightarrow{OA} = 6\mathbf{u} - 4\mathbf{v}$, $\overrightarrow{OB} = 3\mathbf{u} - \mathbf{v}$ and $\overrightarrow{OC} = \mathbf{v} - 3\mathbf{u}$, where **u** and **v** are non-parallel vectors.

The point M is the mid-point of OA and the point N is the point on AB such that $AN : NB = 1 : 2$

a Find $\overrightarrow{OM}$ and $\overrightarrow{ON}$.

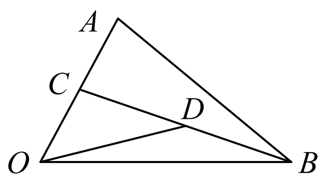
b Prove that C, M and N are collinear.

9 Given that vectors **p** and **q** are not parallel, find the values of the constants a and b such that

a $a\mathbf{p} + 3\mathbf{q} = 5\mathbf{p} + b\mathbf{q}$ **b** $(2\mathbf{p} + a\mathbf{q}) + (b\mathbf{p} - 4\mathbf{q}) = \mathbf{0}$

c $4a\mathbf{q} - \mathbf{p} = b\mathbf{p} - 2\mathbf{q}$ **d** $(2a\mathbf{p} + b\mathbf{q}) - (a\mathbf{q} - 6\mathbf{p}) = \mathbf{0}$

10



The diagram shows triangle OAB in which $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.
The point C is the mid-point of OA and the point D is the mid-point of BC.

a Find an expression for $\overrightarrow{OD}$ in terms of **a** and **b**.

b Show that if the point E lies on AB then $\overrightarrow{OE}$ can be written in the form $\mathbf{a} + k(\mathbf{b} - \mathbf{a})$, where k is a constant.

Given also that OD produced meets AB at E,

c find $\overrightarrow{OE}$,

d show that $AE : EB = 2 : 1$

1 The points A, B and C have coordinates (6, 1), (2, 3) and (–4, 3) respectively and O is the origin. Find, in terms of **i** and **j**, the vectors

a $\overrightarrow{OA}$ **b** $\overrightarrow{AB}$ **c** $\overrightarrow{BC}$ **d** $\overrightarrow{CA}$

2 Given that $\mathbf{p} = \mathbf{i} - 3\mathbf{j}$ and $\mathbf{q} = 4\mathbf{i} + 2\mathbf{j}$, find expressions in terms of **i** and **j** for

a $4\mathbf{p}$ **b** $\mathbf{q} - \mathbf{p}$ **c** $2\mathbf{p} + 3\mathbf{q}$ **d** $4\mathbf{p} - 2\mathbf{q}$

3 Given that $\mathbf{p} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, find

a $|\,\mathbf{p}\,|$ **b** $|\,2\mathbf{q}\,|$ **c** $|\,\mathbf{p} + 2\mathbf{q}\,|$ **d** $|\,3\mathbf{q} - 2\mathbf{p}\,|$

4 Given that $\mathbf{p} = 2\mathbf{i} + \mathbf{j}$ and $\mathbf{q} = \mathbf{i} - 3\mathbf{j}$, find, in degrees to 1 decimal place, the angle made with the vector **i** by the vector

a $\mathbf{p}$ **b** $\mathbf{q}$ **c** $5\mathbf{p} + \mathbf{q}$ **d** $\mathbf{p} - 3\mathbf{q}$

5 Find a unit vector in the direction

a $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ **b** $\begin{pmatrix} 7 \\ -24 \end{pmatrix}$ **c** $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$ **d** $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$

6 Find a vector

a of magnitude 26 in the direction $5\mathbf{i} + 12\mathbf{j}$,

b of magnitude 15 in the direction $-6\mathbf{i} - 8\mathbf{j}$,

c of magnitude 5 in the direction $2\mathbf{i} - 4\mathbf{j}$.

7 Given that $\mathbf{m} = 2\mathbf{i} + \lambda\mathbf{j}$ and $\mathbf{n} = \mu\mathbf{i} - 5\mathbf{j}$, find the values of λ and μ such that

a $\mathbf{m} + \mathbf{n} = 3\mathbf{i} - \mathbf{j}$ **b** $2\mathbf{m} - \mathbf{n} = -3\mathbf{i} + 8\mathbf{j}$

8 Given that $\mathbf{r} = 6\mathbf{i} + c\mathbf{j}$, where c is a positive constant, find the value of c such that

a **r** is parallel to the vector $2\mathbf{i} + \mathbf{j}$ **b** **r** is parallel to the vector $-9\mathbf{i} - 6\mathbf{j}$

c $|\,\mathbf{r}\,| = 10$ **d** $|\,\mathbf{r}\,| = 3\sqrt{5}$

9 Given that $\mathbf{p} = \mathbf{i} + 3\mathbf{j}$ and $\mathbf{q} = 4\mathbf{i} - 2\mathbf{j}$,

a find the values of a and b such that $a\mathbf{p} + b\mathbf{q} = -5\mathbf{i} + 13\mathbf{j}$,

b find the value of c such that $c\mathbf{p} + \mathbf{q}$ is parallel to the vector **j**,

c find the value of d such that $\mathbf{p} + d\mathbf{q}$ is parallel to the vector $3\mathbf{i} - \mathbf{j}$.

10 Relative to a fixed origin O, the points A and B have position vectors $\begin{pmatrix} 3 \\ 6 \end{pmatrix}$ and $\begin{pmatrix} -5 \\ 2 \end{pmatrix}$ respectively.

Find

a the vector $\overrightarrow{AB}$,

b $|\,\overrightarrow{AB}\,|$,

c the position vector of the mid-point of AB,

d the position vector of the point C such that $OABC$ is a parallelogram.

11 Given the coordinates of the points A and B, find the length AB in each case.

a A (4, 0, 9), B (2, –3, 3) **b** A (11, –3, 5), B (7, –1, 3)

12 Find the magnitude of each vector.

a $4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$ **b** $\mathbf{i} + \mathbf{j} + \mathbf{k}$ **c** $-8\mathbf{i} - \mathbf{j} + 4\mathbf{k}$ **d** $3\mathbf{i} - 5\mathbf{j} + \mathbf{k}$

13 Find

a a unit vector in the direction $5\mathbf{i} - 2\mathbf{j} + 14\mathbf{k}$,

b a vector of magnitude 10 in the direction $2\mathbf{i} + 11\mathbf{j} - 10\mathbf{k}$,

c a vector of magnitude 20 in the direction $-5\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$.

14 Given that $\mathbf{r} = \lambda\mathbf{i} + 12\mathbf{j} - 4\mathbf{k}$, find the two possible values of λ such that $|\,\mathbf{r}\,| = 14$.

15 Given that $\mathbf{p} = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}$, $\mathbf{q} = \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} -2 \\ 5 \\ -3 \end{pmatrix}$, find as column vectors,

a $\mathbf{p} + 2\mathbf{q}$ **b** $\mathbf{p} - \mathbf{r}$ **c** $\mathbf{p} + \mathbf{q} + \mathbf{r}$ **d** $2\mathbf{p} - 3\mathbf{q} + \mathbf{r}$

16 Given that $\mathbf{r} = -2\mathbf{i} + \lambda\mathbf{j} + \mu\mathbf{k}$, find the values of λ and μ such that

a $\mathbf{r}$ is parallel to $4\mathbf{i} + 2\mathbf{j} - 8\mathbf{k}$ **b** $\mathbf{r}$ is parallel to $-5\mathbf{i} + 20\mathbf{j} - 10\mathbf{k}$

17 Given that $\mathbf{p} = \mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$, $\mathbf{q} = -\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ and $\mathbf{r} = 2\mathbf{i} - 4\mathbf{j} - 7\mathbf{k}$,

a find $|\,2\mathbf{p} - \mathbf{q}\,|$,

b find the value of k such that $\mathbf{p} + k\mathbf{q}$ is parallel to $\mathbf{r}$.

18 Relative to a fixed origin O, the points A, B and C have position vectors $(-2\mathbf{i} + 7\mathbf{j} + 4\mathbf{k})$, $(-4\mathbf{i} + \mathbf{j} + 8\mathbf{k})$ and $(6\mathbf{i} - 5\mathbf{j})$ respectively.

a Find the position vector of the mid-point of AB.

b Find the position vector of the point D on AC such that $AD : DC = 3 : 1$

19 Given that $\mathbf{r} = \lambda\mathbf{i} - 2\lambda\mathbf{j} + \mu\mathbf{k}$, and that $\mathbf{r}$ is parallel to the vector $(2\mathbf{i} - 4\mathbf{j} - 3\mathbf{k})$,

a show that $3\lambda + 2\mu = 0$.

Given also that $|\,\mathbf{r}\,| = 2\sqrt{29}$ and that $\mu > 0$,

b find the values of λ and μ.

20 Relative to a fixed origin O, the points A, B and C have position vectors $\begin{pmatrix} 6 \\ -2 \\ -4 \end{pmatrix}$, $\begin{pmatrix} 12 \\ -7 \\ -4 \end{pmatrix}$ and $\begin{pmatrix} 6 \\ 1 \\ -8 \end{pmatrix}$ respectively.

a Find the position vector of the point M, the mid-point of BC.

b Show that O, A and M are collinear.

21 The position vector of a model aircraft at time t seconds is $(9 - t)\mathbf{i} + (1 + 2t)\mathbf{j} + (5 - t)\mathbf{k}$, relative to a fixed origin O. One unit on each coordinate axis represents 1 metre.

a Find an expression for d^2 in terms of t, where d metres is the distance of the aircraft from O.

b Find the value of t when the aircraft is closest to O and hence, the least distance of the aircraft from O.

1 Sketch each line on a separate diagram given its vector equation.

a $\mathbf{r} = 2\mathbf{i} + s\mathbf{j}$ **b** $\mathbf{r} = s(\mathbf{i} + \mathbf{j})$ **c** $\mathbf{r} = \mathbf{i} + 4\mathbf{j} + s(\mathbf{i} + 2\mathbf{j})$

d $\mathbf{r} = 3\mathbf{j} + s(3\mathbf{i} - \mathbf{j})$ **e** $\mathbf{r} = -4\mathbf{i} + 2\mathbf{j} + s(2\mathbf{i} - \mathbf{j})$ **f** $\mathbf{r} = (2s + 1)\mathbf{i} + (3s - 2)\mathbf{j}$

2 Write down a vector equation of the straight line

a parallel to the vector $(3\mathbf{i} - 2\mathbf{j})$ which passes through the point with position vector $(-\mathbf{i} + \mathbf{j})$,

b parallel to the x-axis which passes through the point with coordinates (0, 4),

c parallel to the line $\mathbf{r} = 2\mathbf{i} + t(\mathbf{i} + 5\mathbf{j})$ which passes through the point with coordinates (3, −1).

3 Find a vector equation of the straight line which passes through the points with position vectors

a $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}3\\1\end{pmatrix}$ **b** $\begin{pmatrix}-3\\4\end{pmatrix}$ and $\begin{pmatrix}-1\\1\end{pmatrix}$ **c** $\begin{pmatrix}2\\-2\end{pmatrix}$ and $\begin{pmatrix}-2\\3\end{pmatrix}$

4 Find the value of the constant c such that line with vector equation $\mathbf{r} = 3\mathbf{i} - \mathbf{j} + \lambda(c\mathbf{i} + 2\mathbf{j})$

a passes through the point (0, 5),

b is parallel to the line $\mathbf{r} = -2\mathbf{i} + 4\mathbf{j} + \mu(6\mathbf{i} + 3\mathbf{j})$.

5 Find a vector equation for each line given its cartesian equation.

a $x = -1$ **b** $y = 2x$ **c** $y = 3x + 1$

d $y = \frac{3}{4}x - 2$ **e** $y = 5 - \frac{1}{2}x$ **f** $x - 4y + 8 = 0$

6 A line has the vector equation $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + \lambda(3\mathbf{i} + 2\mathbf{j})$.

a Write down parametric equations for the line.

b Hence find the cartesian equation of the line in the form $ax + by + c = 0$, where a, b and c are integers.

7 Find a cartesian equation for each line in the form $ax + by + c = 0$, where a, b and c are integers.

a $\mathbf{r} = 3\mathbf{i} + \lambda(\mathbf{i} + 2\mathbf{j})$ **b** $\mathbf{r} = \mathbf{i} + 4\mathbf{j} + \lambda(3\mathbf{i} + \mathbf{j})$ **c** $\mathbf{r} = 2\mathbf{j} + \lambda(4\mathbf{i} - \mathbf{j})$

d $\mathbf{r} = -2\mathbf{i} + \mathbf{j} + \lambda(5\mathbf{i} + 2\mathbf{j})$ **e** $\mathbf{r} = 2\mathbf{i} - 3\mathbf{j} + \lambda(-3\mathbf{i} + 4\mathbf{j})$ **f** $\mathbf{r} = (\lambda + 3)\mathbf{i} + (-2\lambda - 1)\mathbf{j}$

8 For each pair of lines, determine with reasons whether they are identical, parallel but not identical or not parallel.

a $\mathbf{r} = \begin{pmatrix}1\\2\end{pmatrix} + s\begin{pmatrix}3\\-1\end{pmatrix}$, $\mathbf{r} = \begin{pmatrix}-2\\3\end{pmatrix} + t\begin{pmatrix}-6\\2\end{pmatrix}$

b $\mathbf{r} = \begin{pmatrix}-1\\2\end{pmatrix} + s\begin{pmatrix}1\\4\end{pmatrix}$, $\mathbf{r} = \begin{pmatrix}-2\\4\end{pmatrix} + t\begin{pmatrix}4\\1\end{pmatrix}$

c $\mathbf{r} = \begin{pmatrix}2\\-5\end{pmatrix} + s\begin{pmatrix}2\\4\end{pmatrix}$, $\mathbf{r} = \begin{pmatrix}-1\\1\end{pmatrix} + t\begin{pmatrix}3\\6\end{pmatrix}$

9 Find the position vector of the point of intersection of each pair of lines.

a $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + \lambda\mathbf{i}$, $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + \mu(3\mathbf{i} + \mathbf{j})$

b $\mathbf{r} = 4\mathbf{i} + \mathbf{j} + \lambda(-\mathbf{i} + \mathbf{j})$, $\mathbf{r} = 5\mathbf{i} - 2\mathbf{j} + \mu(2\mathbf{i} - 3\mathbf{j})$

c $\mathbf{r} = \mathbf{j} + \lambda(2\mathbf{i} - \mathbf{j})$, $\mathbf{r} = 2\mathbf{i} + 10\mathbf{j} + \mu(-\mathbf{i} + 3\mathbf{j})$

d $\mathbf{r} = -\mathbf{i} + 5\mathbf{j} + \lambda(-4\mathbf{i} + 6\mathbf{j})$, $\mathbf{r} = 2\mathbf{i} - 2\mathbf{j} + \mu(-\mathbf{i} + 2\mathbf{j})$

e $\mathbf{r} = -2\mathbf{i} + 11\mathbf{j} + \lambda(-3\mathbf{i} + 4\mathbf{j})$, $\mathbf{r} = -3\mathbf{i} - 7\mathbf{j} + \mu(5\mathbf{i} + 3\mathbf{j})$

f $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + \lambda(3\mathbf{i} + 2\mathbf{j})$, $\mathbf{r} = 3\mathbf{i} + 5\mathbf{j} + \mu(\mathbf{i} + 4\mathbf{j})$

10 Write down a vector equation of the straight line

a parallel to the vector $(\mathbf{i} + 3\mathbf{j} - 2\mathbf{k})$ which passes through the point with position vector $(4\mathbf{i} + \mathbf{k})$,

b perpendicular to the xy-plane which passes through the point with coordinates $(2, 1, 0)$,

c parallel to the line $\mathbf{r} = 3\mathbf{i} - \mathbf{j} + t(2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k})$ which passes through the point with coordinates $(-1, 4, 2)$.

11 The points A and B have position vectors $(5\mathbf{i} + \mathbf{j} - 2\mathbf{k})$ and $(6\mathbf{i} - 3\mathbf{j} + \mathbf{k})$ respectively.

a Find $\overrightarrow{AB}$ in terms of **i**, **j** and **k**.

b Write down a vector equation of the straight line l which passes through A and B.

c Show that l passes through the point with coordinates $(3, 9, -8)$.

12 Find a vector equation of the straight line which passes through the points with position vectors

a $(\mathbf{i} + 3\mathbf{j} + 4\mathbf{k})$ and $(5\mathbf{i} + 4\mathbf{j} + 6\mathbf{k})$

b $(3\mathbf{i} - 2\mathbf{k})$ and $(\mathbf{i} + 5\mathbf{j} + 2\mathbf{k})$

c $\mathbf{0}$ and $(6\mathbf{i} - \mathbf{j} + 2\mathbf{k})$

d $(-\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$ and $(4\mathbf{i} - 7\mathbf{j} + \mathbf{k})$

13 Find the value of the constants a and b such that line $\mathbf{r} = 3\mathbf{i} - 5\mathbf{j} + \mathbf{k} + \lambda(2\mathbf{i} + a\mathbf{j} + b\mathbf{k})$

a passes through the point $(9, -2, -8)$,

b is parallel to the line $\mathbf{r} = 4\mathbf{j} - 2\mathbf{k} + \mu(8\mathbf{i} - 4\mathbf{j} + 2\mathbf{k})$.

14 Find cartesian equations for each of the following lines.

a $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 5 \\ 2 \end{pmatrix}$

b $\mathbf{r} = \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 6 \\ 3 \end{pmatrix}$

c $\mathbf{r} = \begin{pmatrix} -1 \\ 5 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -2 \\ -1 \end{pmatrix}$

15 Find a vector equation for each line given its cartesian equations.

a $\dfrac{x-1}{3} = \dfrac{y+4}{2} = z - 5$

b $\dfrac{x}{4} = \dfrac{y-1}{-2} = \dfrac{z+7}{3}$

c $\dfrac{x+5}{-4} = y + 3 = z$

16 Show that the lines with vector equations $\mathbf{r} = 4\mathbf{i} + 3\mathbf{k} + s(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$ and $\mathbf{r} = 7\mathbf{i} + 2\mathbf{j} - 5\mathbf{k} + t(-3\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ intersect, and find the coordinates of their point of intersection.

17 Show that the lines with vector equations $\mathbf{r} = 2\mathbf{i} - \mathbf{j} + 4\mathbf{k} + \lambda(\mathbf{i} + \mathbf{j} + 3\mathbf{k})$ and $\mathbf{r} = \mathbf{i} + 4\mathbf{j} + 3\mathbf{k} + \mu(\mathbf{i} - 2\mathbf{j} + \mathbf{k})$ are skew.

18 For each pair of lines, find the position vector of their point of intersection or, if they do not intersect, state whether they are parallel or skew.

a $\mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 1 \\ -1 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$

b $\mathbf{r} = \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 6 \\ -2 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} -4 \\ 2 \\ 6 \end{pmatrix}$

c $\mathbf{r} = \begin{pmatrix} 8 \\ 2 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} -2 \\ 2 \\ 8 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -3 \\ -4 \end{pmatrix}$

d $\mathbf{r} = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 7 \\ -6 \\ -5 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$

e $\mathbf{r} = \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 5 \\ -3 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ -3 \\ -4 \end{pmatrix}$

f $\mathbf{r} = \begin{pmatrix} 0 \\ 7 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ -4 \\ 8 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} -12 \\ -1 \\ 11 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ 2 \\ -3 \end{pmatrix}$

1 Calculate

a $(\mathbf{i} + 2\mathbf{j}).(3\mathbf{i} + \mathbf{j})$ **b** $(4\mathbf{i} - \mathbf{j}).(3\mathbf{i} + 5\mathbf{j})$ **c** $(\mathbf{i} - 2\mathbf{j}).(-5\mathbf{i} - 2\mathbf{j})$

2 Show that the vectors $(\mathbf{i} + 4\mathbf{j})$ and $(8\mathbf{i} - 2\mathbf{j})$ are perpendicular.

3 Find in each case the value of the constant c for which the vectors **u** and **v** are perpendicular.

a $\mathbf{u} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$, $\mathbf{v} = \begin{pmatrix} c \\ 3 \end{pmatrix}$ **b** $\mathbf{u} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, $\mathbf{v} = \begin{pmatrix} 3 \\ c \end{pmatrix}$ **c** $\mathbf{u} = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$, $\mathbf{v} = \begin{pmatrix} c \\ -4 \end{pmatrix}$

4 Find, in degrees to 1 decimal place, the angle between the vectors

a $(4\mathbf{i} - 3\mathbf{j})$ and $(8\mathbf{i} + 6\mathbf{j})$ **b** $(7\mathbf{i} + \mathbf{j})$ and $(2\mathbf{i} + 6\mathbf{j})$ **c** $(4\mathbf{i} + 2\mathbf{j})$ and $(-5\mathbf{i} + 2\mathbf{j})$

5 Relative to a fixed origin O, the points A, B and C have position vectors $(9\mathbf{i} + \mathbf{j})$, $(3\mathbf{i} - \mathbf{j})$ and $(5\mathbf{i} - 2\mathbf{j})$ respectively. Show that $\angle ABC = 45°$.

6 Calculate

a $(\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}).(3\mathbf{i} + \mathbf{j} + 2\mathbf{k})$ **b** $(6\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}).(\mathbf{i} - 3\mathbf{j} - \mathbf{k})$

c $(-5\mathbf{i} + 2\mathbf{k}).(\mathbf{i} + 4\mathbf{j} - 3\mathbf{k})$ **d** $(3\mathbf{i} + 2\mathbf{j} - 8\mathbf{k}).(-\mathbf{i} + 11\mathbf{j} - 4\mathbf{k})$

e $(3\mathbf{i} - 7\mathbf{j} + \mathbf{k}).(9\mathbf{i} + 4\mathbf{j} - \mathbf{k})$ **f** $(7\mathbf{i} - 3\mathbf{j}).(-3\mathbf{j} + 6\mathbf{k})$

7 Given that $\mathbf{p} = 2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$, $\mathbf{q} = \mathbf{i} + 5\mathbf{j} - \mathbf{k}$ and $\mathbf{r} = 6\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$,

a find the value of $\mathbf{p}.\mathbf{q}$,

b find the value of $\mathbf{p}.\mathbf{r}$,

c verify that $\mathbf{p}.(\mathbf{q} + \mathbf{r}) = \mathbf{p}.\mathbf{q} + \mathbf{p}.\mathbf{r}$

8 Simplify

a $\mathbf{p}.(\mathbf{q} + \mathbf{r}) + \mathbf{p}.(\mathbf{q} - \mathbf{r})$ **b** $\mathbf{p}.(\mathbf{q} + \mathbf{r}) + \mathbf{q}.(\mathbf{r} - \mathbf{p})$

9 Show that the vectors $(5\mathbf{i} - 3\mathbf{j} + 2\mathbf{k})$ and $(3\mathbf{i} + \mathbf{j} - 6\mathbf{k})$ are perpendicular.

10 Relative to a fixed origin O, the points A, B and C have position vectors $(3\mathbf{i} + 4\mathbf{j} - 6\mathbf{k})$, $(\mathbf{i} + 5\mathbf{j} - 2\mathbf{k})$ and $(8\mathbf{i} + 3\mathbf{j} + 2\mathbf{k})$ respectively. Show that $\angle ABC = 90°$.

11 Find in each case the value or values of the constant c for which the vectors **u** and **v** are perpendicular.

a $\mathbf{u} = (2\mathbf{i} + 3\mathbf{j} + \mathbf{k})$, $\mathbf{v} = (c\mathbf{i} - 3\mathbf{j} + \mathbf{k})$ **b** $\mathbf{u} = (-5\mathbf{i} + 3\mathbf{j} + 2\mathbf{k})$, $\mathbf{v} = (c\mathbf{i} - \mathbf{j} + 3c\mathbf{k})$

c $\mathbf{u} = (c\mathbf{i} - 2\mathbf{j} + 8\mathbf{k})$, $\mathbf{v} = (c\mathbf{i} + c\mathbf{j} - 3\mathbf{k})$ **d** $\mathbf{u} = (3c\mathbf{i} + 2\mathbf{j} + c\mathbf{k})$, $\mathbf{v} = (5\mathbf{i} - 4\mathbf{j} + 2c\mathbf{k})$

12 Find the exact value of the cosine of the angle between the vectors

a $\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} 8 \\ 1 \\ -4 \end{pmatrix}$ **b** $\begin{pmatrix} 4 \\ 1 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ 3 \\ -6 \end{pmatrix}$ **c** $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -7 \\ 2 \end{pmatrix}$ **d** $\begin{pmatrix} 5 \\ -3 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ -4 \\ -1 \end{pmatrix}$

13 Find, in degrees to 1 decimal place, the angle between the vectors

a $(3\mathbf{i} - 4\mathbf{k})$ and $(7\mathbf{i} - 4\mathbf{j} + 4\mathbf{k})$ **b** $(2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k})$ and $(\mathbf{i} - 3\mathbf{j} - \mathbf{k})$

c $(6\mathbf{i} - 2\mathbf{j} - 9\mathbf{k})$ and $(3\mathbf{i} + \mathbf{j} + 4\mathbf{k})$ **d** $(\mathbf{i} + 5\mathbf{j} - 3\mathbf{k})$ and $(-3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k})$

14 The points A (7, 2, –2), B (–1, 6, –3) and C (–3, 1, 2) are the vertices of a triangle.

a Find $\overrightarrow{BA}$ and $\overrightarrow{BC}$ in terms of **i**, **j** and **k**.

b Show that $\angle ABC = 82.2°$ to 1 decimal place.

c Find the area of triangle ABC to 3 significant figures.

15 Relative to a fixed origin, the points A, B and C have position vectors $(3\mathbf{i} - 2\mathbf{j} - \mathbf{k})$, $(4\mathbf{i} + 3\mathbf{j} - 2\mathbf{k})$ and $(2\mathbf{i} - \mathbf{j})$ respectively.

a Find the exact value of the cosine of angle BAC.

b Hence show that the area of triangle ABC is $3\sqrt{2}$.

16 Find, in degrees to 1 decimal place, the acute angle between each pair of lines.

a $\mathbf{r} = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix} + \lambda\begin{pmatrix} 4 \\ -4 \\ 2 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 5 \\ -2 \\ 1 \end{pmatrix} + \mu\begin{pmatrix} 8 \\ 0 \\ -6 \end{pmatrix}$

b $\mathbf{r} = \begin{pmatrix} 0 \\ -3 \\ 7 \end{pmatrix} + \lambda\begin{pmatrix} 6 \\ -1 \\ -18 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 4 \\ 6 \\ -3 \end{pmatrix} + \mu\begin{pmatrix} 4 \\ -12 \\ 3 \end{pmatrix}$

c $\mathbf{r} = \begin{pmatrix} 7 \\ 1 \\ 5 \end{pmatrix} + \lambda\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} -2 \\ 6 \\ -3 \end{pmatrix} + \mu\begin{pmatrix} 2 \\ -5 \\ 3 \end{pmatrix}$

d $\mathbf{r} = \begin{pmatrix} 2 \\ -3 \\ -9 \end{pmatrix} + \lambda\begin{pmatrix} -4 \\ -6 \\ 7 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 11 \\ 1 \\ -2 \end{pmatrix} + \mu\begin{pmatrix} 5 \\ -1 \\ -8 \end{pmatrix}$

17 Relative to a fixed origin, the points A and B have position vectors $(5\mathbf{i} + 8\mathbf{j} - \mathbf{k})$ and $(6\mathbf{i} + 5\mathbf{j} + \mathbf{k})$ respectively.

a Find a vector equation of the straight line l_1 which passes through A and B.

The line l_2 has the equation $\mathbf{r} = 4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k} + \mu(-5\mathbf{i} + \mathbf{j} - 2\mathbf{k})$.

b Show that lines l_1 and l_2 intersect and find the position vector of their point of intersection.

c Find, in degrees, the acute angle between lines l_1 and l_2.

18 Find, in degrees to 1 decimal place, the acute angle between the lines with cartesian equations

$$\frac{x-2}{3} = \frac{y}{2} = \frac{z+5}{-6} \quad \text{and} \quad \frac{x-4}{-4} = \frac{y+1}{7} = \frac{z-3}{-4}.$$

19 The line l has the equation $\mathbf{r} = 7\mathbf{i} - 2\mathbf{k} + \lambda(2\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ and the line m has the equation $\mathbf{r} = -4\mathbf{i} + 7\mathbf{j} - 6\mathbf{k} + \mu(5\mathbf{i} - 4\mathbf{j} - 2\mathbf{k})$.

a Find the coordinates of the point A where lines l and m intersect.

b Find, in degrees, the acute angle between lines l and m.

The point B has coordinates (5, 1, –4).

c Show that B lies on the line l.

d Find the distance of B from m.

20 Relative to a fixed origin O, the points A and B have position vectors $(9\mathbf{i} + 6\mathbf{j})$ and $(11\mathbf{i} + 5\mathbf{j} + \mathbf{k})$ respectively.

a Show that for all values of λ, the point C with position vector $(9 + 2\lambda)\mathbf{i} + (6 - \lambda)\mathbf{j} + \lambda\mathbf{k}$ lies on the straight line l which passes through A and B.

b Find the value of λ for which OC is perpendicular to l.

c Hence, find the position vector of the foot of the perpendicular from O to l.

21 Find the coordinates of the point on each line which is closest to the origin.

a $\mathbf{r} = -4\mathbf{i} + 2\mathbf{j} + 7\mathbf{k} + \lambda(\mathbf{i} + 3\mathbf{j} - 4\mathbf{k})$

b $\mathbf{r} = 7\mathbf{i} + 11\mathbf{j} - 9\mathbf{k} + \lambda(6\mathbf{i} - 9\mathbf{j} + 3\mathbf{k})$