1 $f : x \to 3x - 5,\ x \in \mathbb{R}$ $g : x \to \frac{4}{6-x},\ x \in \mathbb{R},\ x \neq 6$ $h : x \to x^2 + 4x - 1,\ x \in \mathbb{R}$

Find the value of

a f(3) **b** g(4) **c** h(2) **d** f(1) **e** h(–1) **f** g(8)

g g(–4) **h** $f(\frac{2}{3})$ **i** $h(\frac{1}{2})$ **j** f(–1) **k** h(–3) **l** $g(1\frac{2}{3})$

2 $f : x \to \ln(2 - 5x),\ x \in \mathbb{R},\ x < 0.4$ $g : x \to \sin(2x + \frac{\pi}{3}),\ x \in \mathbb{R}$ $h : x \to 3 + 2e^{1-x},\ x \in \mathbb{R}$

Find, correct to 3 significant figures where appropriate, the value of

a $g(\frac{\pi}{3})$ **b** f(0) **c** h(1) **d** $g(\frac{\pi}{6})$ **e** h(2) **f** $f(-\frac{1}{2})$

g h(–0.8) **h** f(0.2) **i** g(0.3) **j** $h(\frac{2}{3})$ **k** g(–1) **l** $f(-\frac{3}{4})$

3 Sketch each function and state its range.

a $f : x \to 2x + 1,\ x \in \mathbb{R},\ 0 \le x \le 7$ **b** $f : x \to 3x - 2,\ x \in \mathbb{R},\ x \ge 0$

c $f : x \to 5 - x,\ x \in \mathbb{R},\ -5 \le x \le 5$ **d** $f : x \to 4 - 7x,\ x \in \mathbb{R}$

e $f : x \to x^2,\ x \in \mathbb{R},\ -3 < x < 3$ **f** $f : x \to x^2 + 3,\ x \in \mathbb{R}$

g $f : x \to x^2 - 6,\ x \in \mathbb{R},\ x \ge 0$ **h** $f : x \to (x-1)^2,\ x \in \mathbb{R},\ -2 \le x \le 4$

i $f : x \to (x+2)^2,\ x \in \mathbb{R}$ **j** $f : x \to 4 - x^2,\ x \in \mathbb{R}$

k $f : x \to x^3,\ x \in \mathbb{R},\ -10 < x \le 10$ **l** $f : x \to -x^3,\ x \in \mathbb{R}$

4 Sketch each function and state its range.

a $f : x \to x^2 + 2x - 8,\ x \in \mathbb{R}$ **b** $f : x \to \frac{1}{x},\ x \in \mathbb{R},\ x \neq 0$

c $f : x \to \frac{1}{x^2},\ x \in \mathbb{R},\ x \neq 0$ **d** $f : x \to \cos x,\ x \in \mathbb{R},\ 0 \le x \le 2\pi$

e $f : x \to 5^x,\ x \in \mathbb{R}$ **f** $f : x \to \tan x,\ x \in \mathbb{R},\ -\frac{\pi}{4} \le x \le \frac{\pi}{4}$

5 Find the domain of each function given its range.

a $f : x \to x - 1,\ f(x) \in \mathbb{R},\ -1 \le f(x) < 6$ **b** $f : x \to 4 - 3x,\ f(x) \in \mathbb{R},\ f(x) \le 4$

c $f : x \to x^3,\ f(x) \in \mathbb{R},\ 0 \le f(x) \le 125$ **d** $f : x \to \frac{1}{x},\ f(x) \in \mathbb{R},\ 2 < f(x) < 10$

6 Given that for $x \in \mathbb{R}$, $f(x) \equiv 4x + 3$, $g(x) \equiv x^2 - 7$ and $h(x) \equiv \frac{9}{x+2},\ x \neq -2$, solve the equations

a $f(x) = 9$ **b** $g(x) = 18$ **c** $h(x) = 6$

d $f(x) = h(x)$ **e** $g(x) - \frac{1}{h(x)} = -6\frac{1}{3}$ **f** $f(x) + g(x) = 0$

7 Express each function in the form indicated and hence, state its range.

a $f : x \to x^2 + 4x + 11,\ x \in \mathbb{R}$ in the form $(x + a)^2 + b$

b $f : x \to x^2 - 2x - 6,\ x \in \mathbb{R}$ in the form $(x + a)^2 + b$

c $f : x \to 4x^2 + 12x + 3,\ x \in \mathbb{R}$ in the form $(ax + b)^2 + c$

d $f : x \to 9x^2 - 6x + 16,\ x \in \mathbb{R}$ in the form $(ax + b)^2 + c$

e $f : x \to 15 - 4x - x^2,\ x \in \mathbb{R}$ in the form $a - (x + b)^2$

1 $f : x \to 4x - 3,\ x \in \mathbb{R}$ $g : x \to 2 - x,\ x \in \mathbb{R}$ $h : x \to x^2 + 5,\ x \in \mathbb{R}$

Evaluate

a gf(2) **b** gh(1) **c** fg(–3) **d** hf(3)

e gg(5) **f** ff($\frac{1}{2}$) **g** hg(8) **h** fh($1\frac{1}{2}$)

2 $f : x \to 5x + 2,\ x \in \mathbb{R}$ $g : x \to \cos x,\ x \in \mathbb{R}$ $h: x \to \ln x,\ x \in \mathbb{R},\ x > 0$

Evaluate, giving your answers to 3 significant figures

a fh(20) **b** gh(3) **c** fg(5) **d** gg(–4)

e gf($1\frac{3}{4}$) **f** hg(6.7) **g** hh(50) **h** hf(–0.3)

3 $f : x \to 2x + 1,\ x \in \mathbb{R}$ $g : x \to 1 - 3x,\ x \in \mathbb{R}$ $h : x \to x^2 + 4,\ x \in \mathbb{R}$

Given the functions f, g and h, express the following composite functions in a similar form.

a fg **b** ff **c** fh **d** hf

e gh **f** gg **g** hg **h** gf

4 $f : x \to 4 - x,\ x \in \mathbb{R}$ $g : x \to e^x,\ x \in \mathbb{R}$ $h : x \to 2x^2 + 7,\ x \in \mathbb{R}$

Given the functions f, g and h, express the following composite functions in a similar form.

a gf **b** hg **c** fh **d** gg

e gh **f** ff **g** fg **h** hf

5 $f : x \to 5x - 3,\ x \in \mathbb{R}$ $g : x \to 3x^2 + 1,\ x \in \mathbb{R}$ $h : x \to \frac{1}{x-2},\ x \in \mathbb{R},\ x \neq 2$

Solve

a $ff(x) = -8$ **b** $hf(x) = 2$ **c** $gf(x) = 28$ **d** $hg(x) = \frac{1}{2}$

e $fh(x) = 7$ **f** $fg(x) = 32$ **g** $gh(x) = 4$ **h** $hh(x) = -2$

6 $f : x \to \ln x,\ x \in \mathbb{R},\ x > 0$ $g : x \to 3 + 2x,\ x \in \mathbb{R}$ $h : x \to e^x,\ x \in \mathbb{R}$

Solve, giving your answers to 2 decimal places,

a $gh(x) = 9$ **b** $fg(x) = 3.6$ **c** $hg(x) = 4$ **d** $gf(x) = 10.4$

7 The functions f and g are defined by

$$f : x \to \frac{x+1}{5},\ x \in \mathbb{R} \qquad g : x \to e^x,\ x \in \mathbb{R}$$

a State the range of g.

b Solve $fg(x) = 17$.

8 The functions f and g are defined by

$$f(x) \equiv 4x - 9,\ x \in \mathbb{R} \qquad g(x) \equiv x^2,\ x \in \mathbb{R}$$

a Evaluate ff($3\frac{1}{4}$).

b Solve $gf(x) = 25$.

c Sketch the graph of $y = fg(x)$, showing the coordinates of any points of intersection with the coordinate axes.

9 $\text{f} : x \to \tan x,\ x \in \mathbb{R}$ $\quad \text{g} : x \to 4 + \ln x,\ x \in \mathbb{R}^+$ $\quad \text{h} : x \to \text{e}^{2x-1},\ x \in \mathbb{R}$

Evaluate

a $\text{gf}(\frac{\pi}{4})$ **b** $\text{hg}(\text{e}^{-2})$ **c** $\text{gh}(-1)$ **d** $\text{ff}(1)$

e $\text{hf}(0.2)$ **f** $\text{fg}(7)$ **g** $\text{hh}(\frac{1}{4})$ **h** $\text{fg}(\text{e}^{\text{e}})$

10 $\text{f} : x \to 3\text{e}^x + 2,\ x \in \mathbb{R}$ $\quad \text{g} : x \to 4x + 1,\ x \in \mathbb{R}$ $\quad \text{h} : x \to \frac{1}{x+1},\ x \in \mathbb{R},\ x \neq -1$

Express the following composite functions in a similar form, stating the domain in each case.

a fg **b** gf **c** hf **d** gg

e hg **f** gh **g** hh **h** ggg

11 $\text{f} : x \to \sqrt{x+4},\ x \in \mathbb{R},\ x > -4$ $\quad \text{g} : x \to \text{e}^{1+2x},\ x \in \mathbb{R}$ $\quad \text{h} : x \to \frac{x+1}{3},\ x \in \mathbb{R}$

Solve

a $\text{fh}(x) = 3$ **b** $\text{fg}(x) = 7$ **c** $\text{gh}(x) = 11$ **d** $\text{hh}(x) = \frac{2}{3}$

e $\text{hg}(x) = 1.2$ **f** $\text{hf}(x) = \frac{1}{2}$ **g** $\text{ff}(x) = 3$ **h** $\text{ghh}(x) = \frac{1}{2}$

12 $\text{f}(x) \equiv x^3,\ x \in \mathbb{R}$ $\quad \text{g}(x) \equiv x + 2,\ x \in \mathbb{R}$

Find the composition of the functions f and g that corresponds to the function h, where

a $\text{h}(x) \equiv (x+2)^3,\ x \in \mathbb{R}$ **b** $\text{h}(x) \equiv x^3 + 2,\ x \in \mathbb{R}$ **c** $\text{h}(x) \equiv x + 4,\ x \in \mathbb{R}$

d $\text{h}(x) \equiv x^9,\ x \in \mathbb{R}$ **e** $\text{h}(x) \equiv x^9 + 2,\ x \in \mathbb{R}$ **f** $\text{h}(x) \equiv (x+2)^3 + 2,\ x \in \mathbb{R}$

13 $\text{f}(x) \equiv x - 4,\ x \in \mathbb{R}$ $\quad \text{g}(x) \equiv 3x^2,\ x \in \mathbb{R}$ $\quad \text{h}(x) \equiv \frac{1}{x},\ x \in \mathbb{R},\ x \neq 0$

Find the composition of the functions f, g and h that corresponds to the function j, where

a $\text{j}(x) \equiv 3x^2 - 4,\ x \in \mathbb{R}$ **b** $\text{j}(x) \equiv \frac{1}{x-4},\ x \in \mathbb{R},\ x \neq 4$

c $\text{j}(x) \equiv \frac{3}{x^2},\ x \in \mathbb{R},\ x \neq 0$ **d** $\text{j}(x) \equiv 27x^4,\ x \in \mathbb{R}$

e $\text{j}(x) \equiv \frac{1}{3x^2} - 4,\ x \in \mathbb{R},\ x \neq 0$ **f** $\text{j}(x) \equiv \frac{1}{3x^2 - 4},\ x \in \mathbb{R},\ x \neq \pm\frac{2}{\sqrt{3}}$

14 The functions f and g are defined by

$$\text{f} : x \to 5^x - 7,\ x \in \mathbb{R} \qquad \text{g} : x \to 2x + 3,\ x \in \mathbb{R}$$

a Find and simplify an expression for gf, stating its domain.

b Solve the equation $\text{gf}(x) = 10$.

15 The functions f and g are defined by

$$\text{f} : x \to 2(x+1),\ x \in \mathbb{R} \qquad \text{g} : x \to x^2 - 9,\ x \in \mathbb{R}$$

a Express gf in terms of x and state its domain and range.

b Sketch the graph of $y = \text{gf}(x)$, showing the coordinates of any points of intersection with the coordinate axes.

The equation $\text{gf}(x) - 2\text{f}(x) = a$, where a is a constant, has no real roots.

c Show that $a < -10$.

1 The domain of each of the following functions is $x \in \mathbb{R}$. For each function, find its inverse $f^{-1}(x)$.

a $f : x \to 10x + 3$ **b** $f : x \to 9 + 2x$ **c** $f : x \to 5 - 6x$

d $f : x \to \frac{x+3}{4}$ **e** $f : x \to \frac{1}{3}(2x - 5)$ **f** $f : x \to 8 - \frac{3}{5}x$

2 For each function, find $f^{-1}(x)$ and state its domain.

a $f(x) \equiv \ln x,\ x \in \mathbb{R},\ x > 0$ **b** $f(x) \equiv \frac{1}{x},\ x \in \mathbb{R},\ x \neq 0$

c $f(x) \equiv \sqrt[4]{x},\ x \in \mathbb{R},\ x > 0$ **d** $f(x) \equiv 3x - 4,\ x \in \mathbb{R},\ 0 \leq x < 3$

e $f(x) \equiv \frac{1}{x-5},\ x \in \mathbb{R},\ x \neq 5$ **f** $f(x) \equiv 2 + \frac{1}{x},\ x \in \mathbb{R},\ x \neq 0$

3 For each of the following functions,

i find, in the form $f^{-1} : x \to \ldots$, the inverse function of f and state its domain,

ii sketch $y = f(x)$ and $y = f^{-1}(x)$ on the same set of axes.

a $f : x \to 2x + 1,\ x \in \mathbb{R}$ **b** $f : x \to \frac{1-x}{5},\ x \in \mathbb{R}$ **c** $f : x \to \frac{10}{x},\ x \in \mathbb{R},\ x \neq 0$

d $f : x \to x^2,\ x \in \mathbb{R},\ x > 0$ **e** $f : x \to e^x,\ x \in \mathbb{R}$ **f** $f : x \to x^3,\ x \in \mathbb{R}$

4 For each of the following, solve the equation $f^{-1}(x) = g(x)$.

a $f : x \to 5x + 1,\ x \in \mathbb{R}$ $g : x \to 2,\ x \in \mathbb{R}$

b $f : x \to \frac{2x-4}{3},\ x \in \mathbb{R}$ $g : x \to 7 - x,\ x \in \mathbb{R}$

c $f : x \to e^x + 2,\ x \in \mathbb{R}$ $g : x \to \ln(3x - 8),\ x \in \mathbb{R},\ x > \frac{8}{3}$

d $f : x \to \sqrt{x+2},\ x \in \mathbb{R},\ x \geq -2$ $g : x \to 3x - 4,\ x \in \mathbb{R}$

e $f : x \to \frac{4}{x+3},\ x \in \mathbb{R},\ x \neq -3$ $g : x \to 5(x + 1),\ x \in \mathbb{R}$

5 The function f is defined by $f : x \to 4 - 2x,\ x \in \mathbb{R}$.

a Sketch $y = f(x)$ and $y = f^{-1}(x)$ on the same set of axes.

b Find the coordinates of the point where the lines $y = f(x)$ and $y = f^{-1}(x)$ intersect.

6 The functions f and g are defined by

$$f : x \to 3 - 2x,\ x \in \mathbb{R} \qquad g : x \to \frac{1}{2x+4},\ x \in \mathbb{R},\ x \neq -2$$

a Find $g^{-1}(x)$ and state its domain and range.

b Express gf in terms of x and state its domain.

c Solve the equation $gf(x) = f^{-1}(x)$.

7 The functions f and g are defined by

$$f : x \to 5x + 2,\ x \in \mathbb{R} \qquad g : x \to \frac{1}{x},\ x \in \mathbb{R},\ x \neq 0$$

a Find the following functions, stating the domain in each case.

i f^{-1} **ii** fg **iii** $(fg)^{-1}$

b Solve the equation $f^{-1}(x) = fg(x)$, giving your answers correct to 2 decimal places.

8 For each of the following functions, find the inverse function in the form $f^{-1}: x \to \dots$ and state its domain.

a $f: x \to \frac{1}{2}\ln(4x-9),\ x \in \mathbb{R},\ x > 2\frac{1}{4}$

b $f: x \to \frac{x-2}{x+5},\ x \in \mathbb{R},\ x \neq -5$

c $f: x \to e^{0.4x-2},\ x \in \mathbb{R}$

d $f: x \to \sqrt[3]{x^5-3},\ x \in \mathbb{R}$

e $f: x \to \log_{10}(2-7x),\ x \in \mathbb{R},\ x < \frac{2}{7}$

f $f: x \to \frac{4-x}{3x+2},\ x \in \mathbb{R},\ x \neq -\frac{2}{3}$

9 For each of the following functions,

i find, in the form $f^{-1}: x \to \dots$, the inverse function of f and state its domain,

ii sketch $y = f(x)$ and $y = f^{-1}(x)$ on the same set of axes.

a $f: x \to e^{2x},\ x \in \mathbb{R}$

b $f: x \to x^2+4,\ x \in \mathbb{R},\ x > 0$

c $f: x \to \ln(x-3),\ x \in \mathbb{R},\ x > 3$

d $f: x \to x^2+6x+9,\ x \in \mathbb{R},\ x > -3$

10 For each of the following functions,

i find the range of f,

ii find $f^{-1}(x)$, stating its domain.

a $f(x) \equiv x^2+6x+3,\ x \in \mathbb{R},\ x < -3$

b $f(x) \equiv x^2-4x+5,\ x \in \mathbb{R},\ x \geq 2$

c $f(x) \equiv x^2+5x-2,\ x \in \mathbb{R},\ x < -2\frac{1}{2}$

d $f(x) \equiv x^2-3x+5,\ x \in \mathbb{R},\ 2 < x < 4$

e $f(x) \equiv (2-x)(4+x),\ x \in \mathbb{R},\ x \geq -1$

f $f(x) \equiv 20x-5x^2,\ x \in \mathbb{R},\ x > 2$

11 For each of the following, solve the equation $f^{-1}(x) = g(x)$.

a $f: x \to \frac{1}{3}(2x-5),\ x \in \mathbb{R}$ $\quad g: x \to \frac{4}{2-x},\ x \in \mathbb{R},\ x \neq 2$

b $f: x \to \ln\frac{x+3}{5},\ x \in \mathbb{R},\ x > -3$ $\quad g: x \to 10-6e^{-x},\ x \in \mathbb{R}$

c $f: x \to x^2-4,\ x \in \mathbb{R},\ x > 0$ $\quad g: x \to \frac{x+6}{3},\ x \in \mathbb{R}$

12 The function f is defined by

$$f: x \to \frac{x+b}{x+a},\ x \in \mathbb{R},\ x \neq 2.$$

a State the value of the constant a.

Given that $f(6) = 4$,

b find the value of the constant b,

c find $f^{-1}(x)$ and state its domain.

13 The functions f and g are defined by

$$f: x \to x^2-3x,\ x \in \mathbb{R},\ x \geq 1\tfrac{1}{2},$$
$$g: x \to 2x+3,\ x \in \mathbb{R}.$$

a Find, in the form $f^{-1}: x \to \dots$, the inverse function of f and state its domain.

b On the same set of axes, sketch $y = f(x)$ and $y = f^{-1}(x)$.

Given that $f^{-1}g^{-1}(12) = a(1+\sqrt{3})$,

c show that $a = 1\frac{1}{2}$.

1 $\mathrm{f} : x \to |x - 4|,\ x \in \mathbb{R}$ $\quad \mathrm{g} : x \to |x| - 4,\ x \in \mathbb{R}$

Find the value of

a f(6) **b** f(3) **c** f(−2) **d** g(2) **e** g(−8) **f** g(−1)

2 $\mathrm{f} : x \to x^2 + 2x - 3,\ x \in \mathbb{R}$ $\quad \mathrm{g} : x \to |2x + 1|,\ x \in \mathbb{R}$

Find the value of

a gf(0) **b** fg(0) **c** fg(4) **d** gg(−3) **e** gf(−3) **f** fg(−1)

3 Sketch each of the following graphs, showing the coordinates of any points of intersection with the axes. Where it occurs, a is a positive constant.

a $y = |x + 4|$ **b** $y = |2x - 5|$ **c** $y = |2 - 3x|$

d $y = |x^2 - 9|$ **e** $y = |x^3|$ **f** $y = |\sin x|,\ 0 \le x \le 2\pi$

g $y = |x - a|$ **h** $y = |3x + a|$ **i** $y = |a - 2x|$

j $y = |16 - x^2|$ **k** $y = |(x + 3)(2x - 1)|$ **l** $y = \left|\frac{1}{x}\right|,\ x \ne 0$

m $y = |\ln x|,\ x > 0$ **n** $y = |10 - 3x - x^2|$ **o** $y = |3x^2 + 5ax - 2a^2|$

4 For each of the following,

i sketch $y = \mathrm{f}(x)$ and $y = \mathrm{g}(x)$ on the same diagram,

ii solve the equation $\mathrm{f}(x) = \mathrm{g}(x)$.

The domain of all the functions is $x \in \mathbb{R}$ and a is a positive constant where it occurs.

a $\mathrm{f}(x) \equiv |2x - 3|,\ \mathrm{g}(x) \equiv 2$ **b** $\mathrm{f}(x) \equiv |7 - 3x|,\ \mathrm{g}(x) \equiv 7$

c $\mathrm{f}(x) \equiv |4x + 3a|,\ \mathrm{g}(x) \equiv 5a$ **d** $\mathrm{f}(x) \equiv |x^2 - 4|,\ \mathrm{g}(x) \equiv 9$

e $\mathrm{f}(x) \equiv |x^2 - 4x - 12|,\ \mathrm{g}(x) \equiv 20$ **f** $\mathrm{f}(x) \equiv |2a - 5x|,\ \mathrm{g}(x) \equiv x$

5 Solve each equation.

a $|x - 5| = 3$ **b** $|x + 1| = 15$ **c** $|2x - 7| = 4$

d $|x - 2| = |x + 4|$ **e** $|x - 5| = |7 - x|$ **f** $|2x + 1| = |9 - 2x|$

g $|x + 3| = |2x|$ **h** $|4x - 1| = |2 - x|$ **i** $|3x - 4| = |2x + 3|$

6 Find the set of values of x for which

a $|x - 20| < 2$ **b** $|2x - 11| \le 5$ **c** $|x - 17| > 12$

d $|5x - 22| < 40$ **e** $|x + 4| \le |x + 1|$ **f** $|x + 2| > |2x - 5|$

7 For each of the following, sketch $y = |\mathrm{f}(x)|$ and $y = \mathrm{f}(|x|)$ on separate diagrams showing the coordinates of any points of intersection with the axes.

a $\mathrm{f} : x \to 3x - 1,\ x \in \mathbb{R}$ **b** $\mathrm{f} : x \to 3 - 4x,\ x \in \mathbb{R}$

c $\mathrm{f} : x \to 4x^2 - 25,\ x \in \mathbb{R}$ **d** $\mathrm{f} : x \to (1 + x)(5 - x),\ x \in \mathbb{R}$

e $\mathrm{f} : x \to \tan x,\ x \in \mathbb{R},\ -\frac{\pi}{2} < x < \frac{\pi}{2}$ **f** $\mathrm{f} : x \to \mathrm{e}^x,\ x \in \mathbb{R}$

1 Describe how the graph of $y = \mathrm{f}(x)$ is transformed to give the graph of

a $y = 2 + \mathrm{f}(x + 3)$ **b** $y = 2\mathrm{f}(-x)$ **c** $y = 3\mathrm{f}(x - 1)$ **d** $y = 4 - \mathrm{f}(x)$

2 **a** Express $x^2 + 6x + 2$ in the form $a(x + b)^2 + c$.

b Hence, describe two transformations that would map the graph of $y = x^2$ onto the graph of $y = x^2 + 6x + 2$.

3 Each of the following graphs is translated by 3 units in the positive x-direction and then stretched by a factor of 2 in the y-direction, about the x-axis.

Find and simplify an equation of the graph obtained in each case.

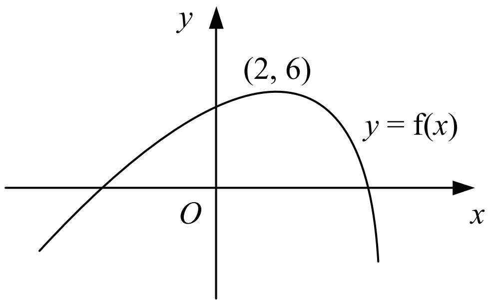
a $y = 2x + 7$ **b** $y = 3e^x$ **c** $y = x^2 - 3x + 1$ **d** $y = \frac{1}{x}$

4 Describe in order two transformations that would map the graph of

a $y = |x|$ onto the graph of $y = -|3x|$ **b** $y = e^x$ onto the graph of $y = 5 + e^{-x}$

c $y = \frac{1}{x}$ onto the graph of $y = \frac{3}{x+4}$ **d** $y = \ln x$ onto the graph of $y = 2 + 3 \ln x$

5



The diagram shows the curve with equation $y = \mathrm{f}(x)$ which is stationary at the point (2, 6).

Showing the coordinates of the stationary point in each case, sketch on separate diagrams the graphs of

a $y = 1 + \mathrm{f}(x - 4)$ **b** $y = 3 - \mathrm{f}(x)$ **c** $y = 2\mathrm{f}(x + 1)$ **d** $y = \frac{1}{2}\mathrm{f}(2x)$

6 The graph of $y = x^2 + 4x - 2$ undergoes the following three transformations:

first: translation by -2 units in the positive x-direction,
second: stretch by a factor of 3 in the y-direction, about the x-axis,
third: reflection in the y-axis.

Find and simplify an equation of the graph obtained.

7 **a** Express $2x^2 - 4x + 7$ in the form $a(x + b)^2 + c$.

b Hence, describe in order a sequence of transformations that would map the graph of $y = 2x^2 - 4x + 7$ onto the graph of $y = x^2$.

8 $$\mathrm{f}(x) \equiv x^3 - 3x^2 + 4, \quad x \in \mathbb{R}.$$

a Find the coordinates of the stationary points on the graph of $y = \mathrm{f}(x)$.

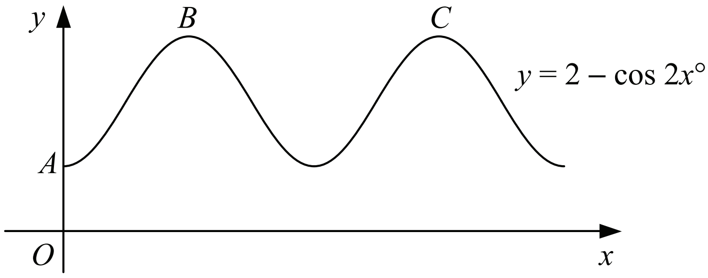
b Hence, find the coordinates of the stationary points on each of the following graphs.

i $y = -2\mathrm{f}(x)$ **ii** $y = 3 + \mathrm{f}(\frac{1}{2}x)$ **iii** $y = \frac{1}{4}\mathrm{f}(x - 2)$

9 **a** Describe clearly, in order, the sequence of transformations that would map the graph of $y = \sqrt{x}$ onto the graph of $y = 2 - 3\sqrt{x}$.

b Sketch the graph of $y = 2 - 3\sqrt{x}$ showing the coordinates of any points where the graph meets the coordinate axes.

10



The diagram shows part of the curve with equation $y = 2 - \cos 2x°$, $x > 0$.

a State the period of the curve.

b Write down the coordinates of the point A where the curve meets the y-axis.

c Write down the coordinates of B and C, the first two maximum points on the curve.

11 Sketch each of the following curves for x in the interval $0 \le x \le 360$. Show the coordinates of any turning points and the equations of any asymptotes.

a $y = 3\cos 2x°$ **b** $y = \tan(-2x°)$ **c** $y = 1 + 2\sin x°$

d $y = -\sin(x + 60)°$ **e** $y = 2\cos(x - 45)°$ **f** $y = 3 - \tan x°$

g $y = 2 + \cos \frac{1}{2}x°$ **h** $y = 4\sin \frac{3}{2}x°$ **i** $y = 1 - 2\cos x°$

12 State the period of the curves with the equations

a $y = 2\tan 3x°$,

b $y = 1 + \sin kx°$, giving your answer in terms of k.

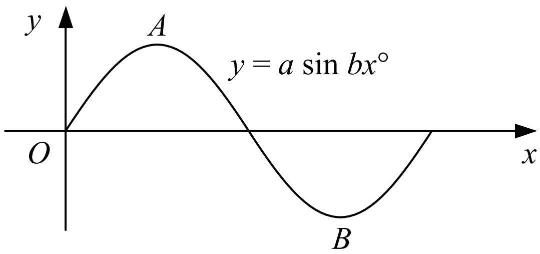
13 $\mathrm{f}(x) \equiv 2\sin \frac{1}{2}x, \quad 0 \le x \le 2\pi.$

a Sketch the graph $y = \mathrm{f}(x)$.

b State the coordinates of the maximum point of the curve.

c Solve the equation $\mathrm{f}(x) = \sqrt{2}$, giving your answers in terms of π.

14



The graph shows the curve $y = a\sin bx°$, $0 \le x \le 180$.

The curve has a maximum at the point A with coordinates (45, 4).

a Find the values of the constants a and b.

b Write down the coordinates of the minimum point of the curve, B.