Solomon Practice Paper

Pure Mathematics 6D

Time allowed: 90 **minutes**

Centre: www.CasperYC.club

Name:

Teacher:

Question	Points	Score
1	7	
2	9	
3	10	
4	11	
5	11	
6	13	
7	14	
Total:	75	

How I can achieve better:

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Last updated: May 5, 2023

1. Given that [7]

$$y = \frac{1}{1-x},$$

prove by induction that

$$\frac{\mathrm{d}^n y}{\mathrm{d}x^n} = \frac{n!}{(1-x)^{n+1}}$$

for all integers $n, n \geq 1$.

2. The variable y satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = x^2 + y + 2, \quad y = 0 \quad \text{at} \quad x = 0.$$

(a) Given that $y \approx 2h$ when $x = h$, use the approximation $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_0 \approx \frac{y_1 - y_{-1}}{2h}$ once to obtain an estimate for y as a function of h when $x = 2h$. [4]

(b) Use the same approximation to show that an estimate for y when $x = 3h$ is given by [3]

$$y \approx 2h(2h^3 + 8h^2 + 4h + 3).$$

(c) Hence find an estimate for y when $x = 0.3$. [2]

Total: 9

3. Given that

$$z^6 - z^3\sqrt{3} + 1 = 0,$$

(a) find the possible values of z^3, giving your answers in the form $x + \mathbf{i}y$ where $x, y \in \mathbb{R}$. [3]

(b) Hence find all possible values of z in the form $re^{\mathbf{i}\theta}$, where $r > 0$ and $-\pi \leq \theta < \pi$. [7]

Total: 10

4. (a) Write down the first three terms of the series of e^{x^2}, in ascending powers of x. [2]

(b) Hence, or otherwise, find the series expansion, in ascending powers of x up to and including the term in x^4, of $\frac{e^{x^2}}{1+2x}$. [5]

(c) Hence find an estimate for the area of the region bounded by the x-axis, the lines $x = 0$ and $x = 0.2$, and the curve [4]

$$y = \frac{e^{x^2}}{1+2x},$$

giving your answer to 3 significant figures.

Total: 11

5. The transformation $T: \mathbb{R}^3 \mapsto \mathbb{R}^3$ is represented by the matrix $\mathbf{A}$ where

$$\mathbf{A} = \begin{pmatrix} 2 & a & 1 \\ 1 & 2 & -1 \\ 3 & 1 & 1 \end{pmatrix}.$$

(a) Find $\mathbf{A}^{-1}$, showing your working clearly and stating the condition for which $\mathbf{A}$ is non-singular. [7]

Relative to a fixed origin O, the transformation T maps the point P onto the point Q.

When $a = -1$, Q has position vector $5\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$.

(b) Find the position vector of P, showing your working clearly. [4]

Total: 11

6. The planes Π_1 and Π_2 are defined by the equations $2x - y + 3z = 5$ and $x + 4y + z = -2$ respectively.

(a) Find, to the nearest degree, the acute angle between Π_1 and Π_2. [4]

The point A has coordinates $(2, 1, -2)$.

(b) Find the perpendicular distance between A and Π_1. [4]

The plane Π_3 is perpendicular to Π_1 and Π_2 and the point with coordinates $(0, 4, -1)$ lies on Π_3.

(c) Find the equation of Π_3 in the form $ax + by + cz = d$. [5]

Total: 13

7. The transformation T from the complex z-plane to the complex w-plane is given by

$$w = \frac{1}{z^* - 2}, \quad z \neq 2.$$

(a) Show that the image in the w-plane of the line $\mathrm{Re}(z) = 5$ in the z-plane, under T, is a circle. Find its centre and radius. [7]

The region represented by $\mathrm{Re}(z) > 5$ in the z-plane is transformed under T into the region represented by R in the w-plane.

(b) Show the region R on an Argand diagram. [3]

(c) Find the image in the w-plane under T of the half-line $\arg(z - 2) = \frac{\pi}{4}$ in the the z-plane. [4]

Total: 14