

Solomon Practice Paper

Pure Mathematics 6A

Time allowed: 90 minutes

Centre: www.CasperYC.club

Name:

Teacher:

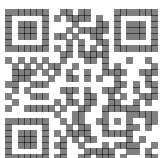
Question	Points	Score
1	6	
2	6	
3	7	
4	9	
5	11	
6	11	
7	11	
8	14	
Total:	75	

How I can achieve better:

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1. With respect to a fixed origin O , the lines l_1 and l_2 are given by the equations

$$\begin{aligned} l_1 & : [\mathbf{r} - (-3\mathbf{i} + 2\mathbf{j} - \mathbf{k})] \times (\mathbf{i} + \mathbf{k}) = 0, \\ l_2 & : [\mathbf{r} - (\mathbf{i} + \mathbf{j} + 4\mathbf{k})] \times (2\mathbf{i} - \mathbf{j} - 2\mathbf{k}) = 0. \end{aligned}$$

(a) Find $(\mathbf{i} + \mathbf{k}) \times (2\mathbf{i} - \mathbf{j} - 2\mathbf{k})$. [3]

(b) Find the shortest distance between l_1 and l_2 . [3]

Total: 6

2. Prove by induction that, for all $n \in \mathbb{Z}^+$, [6]

$$\sum_{r=1}^n (r^2 + 1) r! = n(n + 1)!$$

3. (a) Solve the equation [5]

$$z_3 + 27 = 0,$$

giving your answers in the form $re^{i\theta}$ where $r > 0$, $-\pi < \theta \leq \pi$.

(b) Show the points representing your solutions on an Argand diagram. [2]

Total: 7

4.

$$A = \begin{pmatrix} 2 & a \\ 2 & b \end{pmatrix}.$$

The matrix A has eigenvalues $\lambda_1 = -2$ and $\lambda_2 = 3$.

(a) Find the value of a and the value of b . [4]

Using your values of a and b ,

(b) for each eigenvalue, find a corresponding eigenvector, [3]

(c) find a matrix \mathbf{P} such that [2]

$$\mathbf{P}^T \mathbf{A} \mathbf{P} = \begin{pmatrix} -2 & 0 \\ 0 & 3 \end{pmatrix}.$$

Total: 9

5. [11]

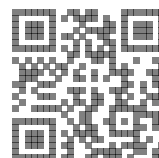
$$(1 + x^2) \frac{d^2y}{dx^2} + 4x + \frac{dy}{dx} + 2y = 0$$

and

$$y = 1, \quad \frac{dy}{dx} = 1$$

at $x = -1$.

Find a series solution of the differential equation in ascending powers of $(x + 1)$ up to and including the term in $(x + 1)^4$.



6. The variable y satisfies the differential equation [11]

$$\frac{d^2y}{dx^2} = x \frac{dy}{dx} + y^2$$

with $y = 1.2$ at $x = 0.1$ and $y = 0.9$ at $x = 0.2$.

Use the approximations

$$\left(\frac{dy}{dx}\right)_0 \approx \frac{y_1 - y_{-1}}{2h} \quad \text{and} \quad \left(\frac{d^2y}{dx^2}\right)_0 \approx \frac{y_1 - 2y_0 + y_{-1}}{h^2}$$

with a step length of 0.1 to estimate the values of y at $x = 0.3$ and $x = 0.4$ giving your answers to 3 significant figures.

7.

$$\mathbf{M} = \begin{pmatrix} 2 & 1 & 1 \\ k & 4 & 3 \\ -1 & k & 2 \end{pmatrix}.$$

(a) Find the determinant of \mathbf{M} in terms of k . [2]

(b) Prove that \mathbf{M} is non-singular for all real values of k . [2]

(c) Given that $k = 3$, find \mathbf{M}^{-1} , showing each step of your working. [4]

When $k = 3$ the image of the vector $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ when transformed by \mathbf{M} is the vector $\begin{pmatrix} 0 \\ 3 \\ 5 \end{pmatrix}$.

(d) Find the values of a, b and c . [3]

Total: 11

8. A transformation T from the z -plane to the w -plane is defined by

$$w = \frac{z + 1}{iz - 1}, \quad z \neq -i,$$

where $z = x + iy, w = u + iv$ and x, y, u and v are real.

T transforms the circle $|z| = 1$ in the z -plane onto a straight line L in the w -plane.

(a) Find an equation of L giving your answer in terms of u and v . [5]

(b) Show that T transforms the line $\text{Im}(z) = 0$ in the z -plane onto a circle C in the w -plane, giving the centre and radius of this circle. [6]

(c) On a single Argand diagram sketch L and C . [3]

Total: 14

