Solomon Practice Paper

Pure Mathematics 6A

Time allowed: 90 **minutes**

Centre: **www.CasperYC.club**

Name:

Teacher:

Question	Points	Score
1	6	
2	6	
3	7	
4	9	
5	11	
6	11	
7	11	
8	14	
Total:	75	

How I can achieve better:

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1. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1 \quad : \quad [\mathbf{r} - (-3\mathbf{i} + 2\mathbf{j} - \mathbf{k})] \times (\mathbf{i} + \mathbf{k}) = 0,$$
$$l_2 \quad : \quad [\mathbf{r} - (\mathbf{i} + \mathbf{j} + 4\mathbf{k})] \times (2\mathbf{i} - \mathbf{j} - 2\mathbf{k}) = 0.$$

   (a) Find $(\mathbf{i} + \mathbf{k}) \times (2\mathbf{i} - \mathbf{j} - 2\mathbf{k})$. [3]

   (b) Find the shortest distance between l_1 and l_2. [3]

Total: 6

2. Prove by induction that, for all $n \in \mathbb{Z}^+$, [6]

$$\sum_{r=1}^{n} \left(r^2 + 1\right) r! = n(n+1)!$$

3. (a) Solve the equation [5]

$$z_3 + 27 = 0,$$

   giving your answers in the form $re^{i\theta}$ where $r > 0, -\pi < \theta \leq \pi$.

   (b) Show the points representing your solutions on an Argand diagram. [2]

Total: 7

4.

$$A = \begin{pmatrix} 2 & a \\ 2 & b \end{pmatrix}.$$

The matrix A has eigenvalues $\lambda_1 = -2$ and $\lambda_2 = 3$.

   (a) Find the value of a and the value of b. [4]

Using your values of a and b,

   (b) for each eigenvalue, find a corresponding eigenvector, [3]

   (c) find a matrix $\mathbf{P}$ such that [2]

$$\mathbf{P}^T \mathbf{A} \mathbf{P} = \begin{pmatrix} -2 & 0 \\ 0 & 3 \end{pmatrix}.$$

Total: 9

5. [11]

$$\left(1 + x^2\right) \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + 4x + \frac{\mathrm{d}y}{\mathrm{d}x} + 2y = 0$$

and

$$y = 1, \quad \frac{\mathrm{d}y}{\mathrm{d}x} = 1$$

at $x = -1$.

Find a series solution of the differential equation in ascending powers of $(x + 1)$ up to and including the term in $(x + 1)^4$.

6. The variable y satisfies the differential equation [11]

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = x\frac{\mathrm{d}y}{\mathrm{d}x} + y^2$$

with $y = 1.2$ at $x = 0.1$ and $y = 0.9$ at $x = 0.2$.

Use the approximations

$$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_0 \approx \frac{y_1 - y_{-1}}{2h} \quad \text{and} \quad \left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)_0 \approx \frac{y_1 - 2y_0 + y_{-1}}{h^2}$$

with a step length of 0.1 to estimate the values of y at $x = 0.3$ and $x = 0.4$ giving your answers to 3 significant figures.

7.

$$\mathbf{M} = \begin{pmatrix} 2 & 1 & 1 \\ k & 4 & 3 \\ -1 & k & 2 \end{pmatrix}.$$

(a) Find the determinant of $\mathbf{M}$ in terms of k. [2]

(b) Prove that $\mathbf{M}$ is non-singular for all real values of k. [2]

(c) Given that $k = 3$, find $\mathbf{M}^{-1}$, showing each step of your working. [4]

When $k = 3$ the image of the vector $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ when transformed by $\mathbf{M}$ is the vector $\begin{pmatrix} 0 \\ 3 \\ 5 \end{pmatrix}$.

(d) Find the values of a, b and c. [3]

Total: 11

8. A transformation T from the z-plane to the w-plane is defined by

$$w = \frac{z+1}{\mathbf{i}z - 1}, \quad z \neq -\mathbf{i},$$

where $z = x + \mathbf{i}y, w = u + \mathbf{i}v$ and x, y, u and v are real.

T transforms the circle $|z| = 1$ in the z-plane onto a straight line L in the w-plane.

(a) Find an equation of L giving your answer in terms of u and v. [5]

(b) Show that T transforms the line $\mathrm{Im}(z) = 0$ in the z-plane onto a circle C in the w-plane, giving the centre and radius of this circle. [6]

(c) On a single Argand diagram sketch L and C. [3]

Total: 14