

Solomon Practice Paper

Pure Mathematics 5H

Time allowed: 90 minutes

Centre: www.CasperYC.club

Name:

Teacher:

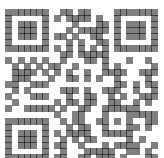
Question	Points	Score
1	8	
2	8	
3	8	
4	9	
5	11	
6	13	
7	18	
Total:	75	

How I can achieve better:

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Last updated: May 5, 2023



1. A curve has the equation [8]

$$2x^2 + y^2 = 4.$$

Find the radius of curvature of the curve at the point $(1, -\sqrt{2})$.

2. (a) Using the definition of $\cosh(x)$ in terms of exponential functions show that $\cosh(x)$ is an even function. [2]

- (b) Given that $x > 0$ and $y > 0$, solve the simultaneous equations [6]

$$\begin{aligned}\ln(x) &= \operatorname{arcosh}\left(\frac{5}{3}\right) \\ \cosh(3x - y) &= 1.\end{aligned}$$

Total: 8

3. Find [8]

$$\int \frac{1}{13 \cosh(x) - 5 \sinh(x)} dx.$$

4. (a) Given that $y = \arcsin(2x - 1)$, prove that [4]

$$\frac{dy}{dx} = \frac{1}{x - x^2}.$$

The tangent to the curve $y = \arcsin(2x - 1)$ at the point where $x = \frac{3}{4}$ meets the y -axis at A .

- (b) Find the exact value of the y -coordinate of A . [5]

Total: 9

5. The point $P(at^2, 2at)$, $t \neq 0$, lies on the parabola C with equation $y^2 = 4ax$.

- (a) Show that an equation of the tangent to C at P is [4]

$$yt = x + at^2.$$

The tangent to C at P meets the x -axis at Q and the y -axis at R .

M is the mid-point of QR .

- (b) Find the coordinates of M . [3]

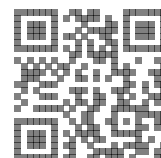
Given that OM is perpendicular to OP , where O is the origin,

- (c) show that $t^2 = 2$. [4]

Total: 11

- 6.

$$I_n = \int \frac{\cos(n\theta)}{\sin(\theta)} d\theta, \quad n \in \mathbb{N}.$$



- (a) By considering $I_n - I_{n-2}$, or otherwise, show that [5]

$$I_n = \frac{2 \cos(n-1)\theta}{n-1} + I_{n-2}$$

- (b) Hence evaluate [8]

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos(5\theta)}{\sin(\theta)} d\theta$$

leaving your answer in terms of natural logarithms.

Total: 13

7. The ellipse C has equation

$$\frac{x^2}{a} + \frac{y^2}{b} = 1,$$

where a and b are positive constants and $a > b$.

The coordinates of the foci of C are $(\pm\sqrt{3}, 0)$, and the equations of its directrices are $x = \pm\frac{4}{\sqrt{3}}$.

- (a) Find the value of a and the value of b . [4]

The ellipse is rotated completely about the x -axis.

- (b) Show that the area of the surface of revolution generated is given by [6]

$$A = \frac{\pi}{2} \int_{-2}^2 \sqrt{16 - 3x^2} dx.$$

- (c) Use integration to show that [8]

$$A = \frac{8}{9}\pi^2\sqrt{3} + 2\pi.$$

Total: 18

