Solomon Practice Paper

Pure Mathematics 5H

Time allowed: 90 **minutes**

Centre: www.CasperYC.club

Name:

Teacher:

Question	Points	Score
1	8	
2	8	
3	8	
4	9	
5	11	
6	13	
7	18	
Total:	75	

How I can achieve better:

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1. A curve has the equation [8]

$$2x^2 + y^2 = 4.$$

Find the radius of curvature of the curve at the point $(1, -\sqrt{2})$.

2. (a) Using the definition of $\cosh(x)$ in terms of exponential functions show that $\cosh(x)$ is an even function. [2]

(b) Given that $x > 0$ and $y > 0$, solve the simultaneous equations [6]

$$\begin{aligned} \ln(x) &= \operatorname{arcosh}\left(\tfrac{5}{3}\right) \\ \cosh(3x - y) &= 1. \end{aligned}$$

Total: 8

3. Find [8]

$$\int \frac{1}{13\cosh(x) - 5\sinh(x)}\, \mathrm{d}x.$$

4. (a) Given that $y = \arcsin(2x - 1)$, prove that [4]

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{x - x^2}.$$

The tangent to the curve $y = \arcsin(2x - 1)$ at the point where $x = \frac{3}{4}$ meets the y-axis at A.

(b) Find the exact value of the y-coordinate of A. [5]

Total: 9

5. The point $P(at^2, 2at), t \neq 0$, lies on the parabola C with equation $y^2 = 4ax$.

(a) Show that an equation of the tangent to C at P is [4]

$$yt = x + at^2.$$

The tangent to C at P meets the x-axis at Q and the y-axis at R.

M is the mid-point of QR.

(b) Find the coordinates of M. [3]

Given that OM is perpendicular to OP, where O is the origin,

(c) show that $t^2 = 2$. [4]

Total: 11

6.

$$I_n = \int \frac{\cos(n\theta)}{\sin(\theta)}\, \mathrm{d}\theta, \quad n \in \mathbb{N}.$$

(a) By considering $I_n - I_{n-2}$, or otherwise, show that [5]

$$I_n = \frac{2\cos(n-1)\theta}{n-1} + I_{n-2}$$

(b) Hence evaluate [8]

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos(5\theta)}{\sin(\theta)}\,\mathrm{d}\theta$$

leaving your answer in terms of natural logarithms.

Total: 13

7. The ellipse C has equation

$$\frac{x^2}{a} + \frac{y^2}{b} = 1,$$

where a and b are positive constants and $a > b$.

The coordinates of the foci of C are $(\pm\sqrt{3}, 0)$, and the equations of its directrices are $x = \pm\frac{4}{\sqrt{3}}$.

(a) Find the value of a and the value of b. [4]

The ellipse is rotated completely about the x-axis.

(b) Show that the area of the surface of revolution generated is given by [6]

$$A = \frac{\pi}{2}\int_{-2}^{2} \sqrt{16 - 3x^2}\,\mathrm{d}x.$$

(c) Use integration to show that [8]

$$A = \frac{8}{9}\pi^2\sqrt{3} + 2\pi.$$

Total: 18