Solomon Practice Paper

Pure Mathematics 4C

Time allowed: 90 **minutes**

Centre: **www.CasperYC.club**

Name:

Teacher:

Question	Points	Score
1	6	
2	9	
3	9	
4	10	
5	12	
6	13	
7	16	
Total:	75	

How I can achieve better:

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Last updated: May 5, 2023

1. Find the set of values of x for which [6]

$$|x-2| > 2|x+1|.$$

2. (a) By using the substitution $y = vx$, or otherwise, find the general solution of the differential equation [7]

$$xy = \frac{\mathrm{d}y}{\mathrm{d}x} = x^2 + y^2.$$

(b) Given also that $y = 2$ when $x = 1$, show that for $x > 0$ [2]

$$y^2 = 2x^2\left(\ln(x) + 2\right).$$

Total: 9

3. (a) Find the sum of the series [3]

$$2^3 + 4^3 + 6^3 + \ldots + (2n)^3,$$

giving your answer in a simplified form.

(b) Hence, or otherwise, show that the sum of the series [6]

$$1^3 - 2^3 + 3^3 - 4^3 + \ldots + (2n-1)^3 - (2n)^3$$

is $-n^2(4n+3)$.

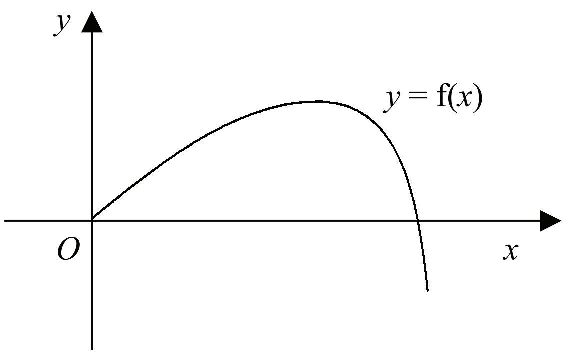
Total: 9

4. Find the general solution of the differential equation [10]

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 6\frac{\mathrm{d}y}{\mathrm{d}x} + 9y = 2\mathrm{e}^{3x}.$$

5. Figure shows part of the curve $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) \equiv 2x - \tan(x), \quad x \in \mathbb{R}, \quad 0 \leq x < \frac{\pi}{2}.$$



(a) Show that there is a root, α, of the equation $\mathrm{f}(x) = 0$ in the interval $(1, 1.5)$. [2]

(b) Use the Newton-Raphson method with an initial value of $x = 1.25$ to find α correct to 2 decimal places and justify the accuracy of your answer. [7]

(c) Explain with the aid of a diagram why the Newton-Raphson method fails if an initial value of $x = 0.75$ is used when trying to find α. [3]

Total: 12

6. The complex numbers z and w are defined such that

$$\begin{aligned} 3z + w &= 14 \\ z - \mathbf{i}w &= 15 - 9\mathbf{i} \end{aligned}$$

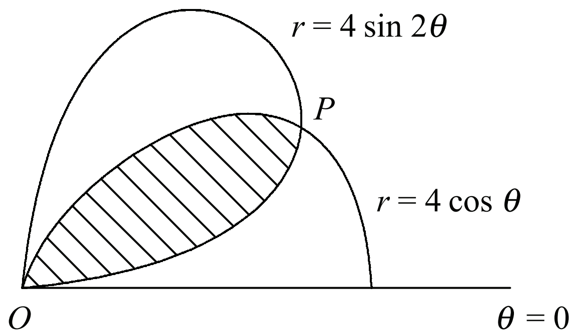
(a) Show that $z = 3 - 4i$ and find w in the form $a + \mathbf{i}b$, where a and b are real numbers. [6]

(b) Find the square roots of z in the form $c + \mathbf{i}d$, where c and d are real numbers. [7]

Total: 13

7. Figure shows the curves with polar equations

$$\begin{aligned} r &= 4\sin(2\theta), \quad 0 \le \theta \le \tfrac{\pi}{2} \\ r &= 4\cos(\theta), \quad 0 \le \theta \le \tfrac{\pi}{2} \end{aligned}$$



(a) Find the polar coordinates of the point P where the two curves intersect. [5]

(b) Find the exact area of the shaded region bounded by the two curves. [11]

Total: 16