Solomon Practice Paper

Pure Mathematics 3L

Time allowed: 90 **minutes**

Centre: **www.CasperYC.club**

Name:

Teacher:

Question	Points	Score
1	5	
2	6	
3	7	
4	8	
5	9	
6	12	
7	13	
8	15	
Total:	75	

How I can achieve better:

-
-
-

Last updated: May 5, 2023

1. A circle has the equation [5]

$$4x^2 + 4y^2 - 4x + 24y + 1 = 0.$$

Find

(a) the coordinates of the centre of the circle,

(b) the radius of the circle.

2. Find, in the form $ax + by + c = 0$, the equation of the normal to the curve [6]

$$y = (x+3)^2 e^{-x}$$

at the point with coordinates $(0, 9)$.

3.

$$f(x) \equiv x^3 + (a+3)x^2 - a^3.$$

Given that when $f(x)$ is divided by $(x+2)$ the remainder is 4,

(a) find the three possible values of a. [4]

Given also that $a > 0$,

(b) find the remainder when $f(x)$ is divided by $(2x+3)$. [3]

Total: 7

4. Relative to a fixed origin, O, the points A, B and C have position vectors $(5\mathbf{i} + \mathbf{j} - 11\mathbf{k})$, $(-3\mathbf{i} + 5\mathbf{j} - 3\mathbf{k})$ and $(11\mathbf{i} + \mathbf{j} + 4\mathbf{k})$ respectively.

(a) Find an equation of the line that passes through A and B in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$. [3]

The point M is the midpoint of AB.

(b) Show that $\overrightarrow{CM}$ is perpendicular to $\overrightarrow{AB}$. [5]

Total: 8

5.

$$f(x) \equiv (1+8x)^{\frac{1}{2}}, |x| < \frac{1}{8}.$$

(a) Express $f(x)$ as a series in ascending powers of x up to and including the term in x^3. [3]

(b) Show that $\sqrt{1.08} = \frac{3}{5}\sqrt{3}$. [2]

(c) Hence, use your series with a suitable value of x to estimate the value of $\sqrt{3}$ correct to 6 significant figures. [4]

Total: 9

6. (a) Given that [3]

$$\frac{5}{(y-3)(2y-1)} \equiv \frac{A}{y-3} + \frac{B}{2y-1},$$

find the values of A and B.

(b) Given that $\frac{1}{2} < y < 3$, for all values of x, find the general solution to the differential equation [4]

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{5}(y-3)(2y-1).$$

(c) Given also that $y = 1$ when $x = \ln(2)$, show that [5]

$$y = \frac{3 + \mathrm{e}^x}{2\mathrm{e}^x + 1}.$$

Total: 12

7.

$$\mathrm{f}\colon x \mapsto \cos(2x) + \sin(x), \quad 0 \le x \le 2\pi.$$

(a) Find the values of x for which $\mathrm{f}(x) = 0$. [4]

(b) Find the values of x for which $\mathrm{f}'(x) = 0$. [6]

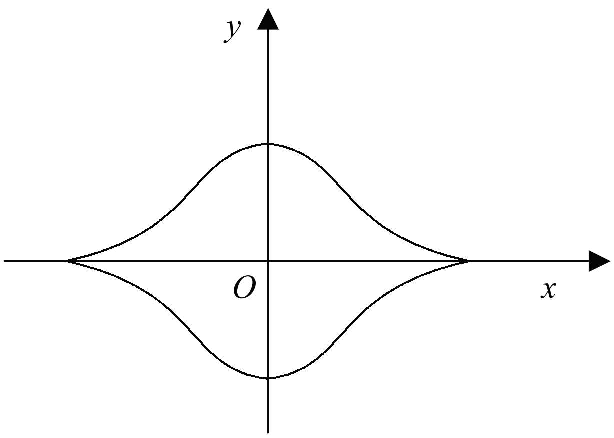
(c) Sketch the curve $y = \mathrm{f}(x)$. [3]

Total: 13

8. Figure shows the curve given by the parametric equations

$$x = 2\cos(t), \quad \text{and} \quad y = \sin^3(t), \quad 0 \le t \le 2\pi.$$

where t is a parameter.



(a) Find the coordinates of the points A and B with parameters $t = 0$ and $t = \frac{\pi}{2}$ respectively. [2]

(b) Show that the area of the region enclosed by the curve is given by the integral [5]

$$\int_0^{\frac{\pi}{2}} 8\sin^4(t)\,\mathrm{d}t.$$

(c) Use the double angle identities to prove that [4]

$$\sin^4(A) = \frac{1}{8}\left(3 - 4\cos(2A) + \cos(4A)\right).$$

(d) Find the area of the region enclosed by the curve, giving your answer in terms of π. [4]

Total: 15