Solomon Practice Paper

Pure Mathematics 3J

Time allowed: 90 **minutes**

Centre: **www.CasperYC.club**

Name:

Teacher:

Question	Points	Score
1	5	
2	5	
3	8	
4	10	
5	10	
6	12	
7	12	
8	13	
Total:	75	

How I can achieve better:

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1. Given that [5]

$$\frac{x^2+9x-10}{(x-2)^2(x+1)} \equiv \frac{A}{(x-2)^2} + \frac{B}{x-2} + \frac{C}{x+1},$$

find the values of A, B and C.

2. Show that [5]

$$\int_2^4 x\left(x^2-4\right)^{\frac{1}{2}}\,\mathrm{d}x = 8\sqrt{3}.$$

3. (a) Find the binomial expansion of $(1+4x)^{\frac{1}{4}}$ for $|x| < \frac{1}{4}$ in ascending powers of x as far as the term in x^3. [3]

   (b) By substituting $x = 0.01$ into your expansion, find the fourth root of 16.64 correct to 6 decimal places. [5]

Total: 8

4. (a) Use the identity [4]

$$\cot(x) \equiv \frac{\cos(x)}{\sin(x)}$$

to show that

$$\frac{\mathrm{d}}{\mathrm{d}x}\cot(x) = -\csc^2(x).$$

   (b) Use integration by parts to find [6]

$$\int x\csc^2(x)\,\mathrm{d}x.$$

Total: 10

5. At time t the vectors $\mathbf{r}$ and $\mathbf{s}$ are given by

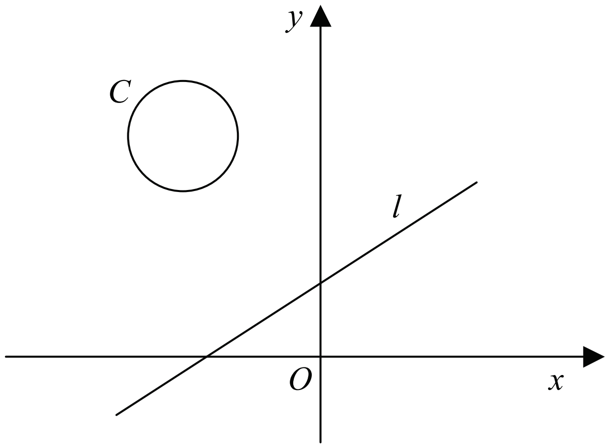
$$\begin{aligned}\mathbf{r} &= 2t^2\mathbf{i} - t\mathbf{j} + \mathbf{k},\\ \mathbf{s} &= (t+2)\mathbf{i} + (t^2+5)\mathbf{j} + (1-t^3)\mathbf{k}.\end{aligned}$$

   (a) Find the angle between $\mathbf{r}$ and $\mathbf{s}$ when $t = 2$, giving your answer in degrees correct to 1 decimal place. [5]

   (b) Find the values of t for which $\mathbf{r}$ and $\mathbf{s}$ are perpendicular. [5]

Total: 10

6. Figure shows the circle C with equation $x^2+y^2+10x-16y+85=0$ and the line l with equation $2x-3y+8=0$.



(a) Find an equation of the line which is perpendicular to line l and passes through the centre of circle C. [7]

(b) Hence, or otherwise, find the minimum distance between l and C. [5]

Total: 12

7. A physics student is investigating the change in the size of an air bubble as it rises in water. The student believes that the volume, V m^3 of a bubble is related to its depth, h m, by the formula

$$V = \frac{k}{h+10},$$

where k is a constant.

A bubble of volume 0.1 m^3 is formed at a depth of 5 m in a water tank. Using the student's model,

(a) find the volume of the bubble when it has risen 3 m, (hint: $h = 5 - 3 = 2$) [4]

(b) show that, at this instant, V is increasing at the rate of $\frac{1}{96}$ m^3 per metre the bubble rises. [3]

Assuming that the bubble is spherical as it rises,

(c) find, correct to 2 significant figures, the rate at which the radius of the bubble is increasing per metre the bubble rises at the instant when it has risen 3 m. [5]

Total: 12

8. (a) By taking logarithms, prove that if $x = 3^{1-t}$, then [3]

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\left(\ln(3)\right)3^{1-t}.$$

A curve has parametric equations

$$x = 3^{1-t} \quad \text{and} \quad y = 9^t - 1.$$

(b) Show that [4]

$$\frac{\mathrm{d}y}{\mathrm{d}x} = (-2) \cdot 3^{3t-1}.$$

(c) Find an equation of the tangent to the curve at the point $(3, 0)$. [3]

(d) Find a Cartesian equation for the curve. [3]

Total: 13