Solomon Practice Paper

Pure Mathematics 3D

Time allowed: 90 **minutes**

Centre: **www.CasperYC.club**

Name:

Teacher:

Question	Points	Score
1	5	
2	7	
3	8	
4	8	
5	10	
6	10	
7	13	
8	16	
Total:	77	

How I can achieve better:

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Last updated: May 5, 2023

1. A curve is given by the parametric equations

$$x = 1 + t^2, \quad \text{and} \quad y = 2t^6.$$

(a) Find an equation of the curve in Cartesian form. [2]

(b) Sketch the curve, labelling the coordinates of any points where the curve meets the coordinate axes. [3]

Total: 5

2. The lines l_1 and l_2 are given by

$$\begin{aligned} l_1 &: \quad \mathbf{r} = -38 + 8 + \mathbf{k} + \lambda(5\mathbf{i} - 7\mathbf{j} + 4\mathbf{k}) \\ l_2 &: \quad \frac{x-5}{2} = \frac{y+9}{3} + \frac{z-3}{6}. \end{aligned},$$

(a) Find an equation for l_2 in vector form. [3]

(b) Find the size of the acute angle between lines l_1 and l_2 in degrees correct to 1 decimal place. [4]

Total: 7

3. (a) Use integration by parts to find [4]

$$\int 2x \ln(x)\, \mathrm{d}x.$$

(b) Given that $y = 2\mathrm{e}$ when $x = \mathrm{e}$, solve the differential equation [4]

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2x\ln(x)}{y}.$$

Total: 8

4. A curve has the equation

$$4\cos(x) + \tan(y) = 0.$$

(a) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = 4\sin(x)\cos^2(y)$. [3]

(b) Find the equation of the normal to the curve at the point with coordinates $(\frac{\pi}{2}, \frac{\pi}{6})$ in the form $ax + by + c = 0$. [5]

Total: 8

5. (a) Given that $|x| < 1$, express $(1+x)^{-1}$ as a series in ascending powers of x, as far as the term in x^3. [3]

(b) [7]

$$\mathrm{f}(x) \equiv \frac{4x+1}{(1-2x)(1+x)}.$$

By expressing $\mathrm{f}(x)$ in partial fractions, find the series expansion of $\mathrm{f}(x)$ in ascending powers of x as far as the term in x^3 and state the set of values of x for which your series is valid.

Total: 10

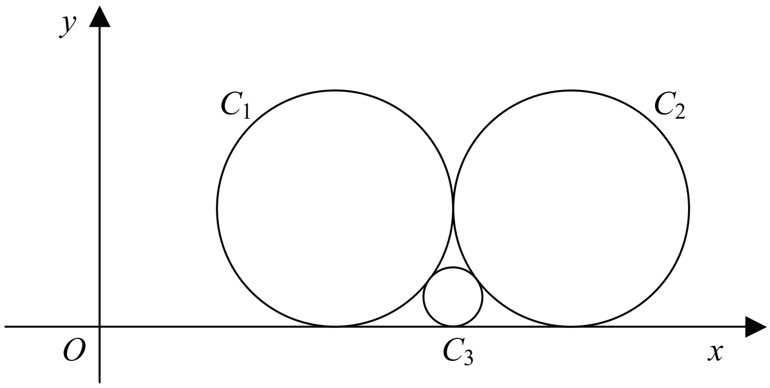
6. (a) Find $\int \tan^2(3x)\,\mathrm{d}x$. [3]

   (b) Using the substitution $u = x^2 + 4$, or otherwise, evaluate [7]

$$\int_0^2 \frac{5x}{(x^2+4)^2}\,\mathrm{d}x.$$

Total: 10

7. Figure shows three circles, C_1, C_2 and C_3 which all touch the x-axis.



Circle C_1 has the equation $x^2 + y^2 - 12x - 8y + 36 = 0$.

   (a) Find the coordinates of the centre of C_1 and write down its radius. [5]

Circle C_2 has the same radius as C_1 and is touching circle C_1.

   (b) Find an equation of circle C_2. [3]

Circle C_3 is touching both circles C_1 and C_2.

   (c) Find an equation of circle C_3. [5]

Total: 13

8. (a) A curve has the equation [5]

$$y = \frac{x}{\sqrt{x-2}}, \quad x > 2.$$

   Show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x-4}{2(x-2)^{\frac{3}{2}}}$$

   (b) Find the coordinates of the stationary point on the curve. [3]

   (c) Find and simplify an expression for $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$. [5]

   (d) Hence, determine the nature of the stationary point on the curve. [3]

Total: 16