Solomon Practice Paper

Pure Mathematics 2K

Time allowed: 90 **minutes**

Centre: **www.CasperYC.club**

Name:

Teacher:

Question	Points	Score
1	6	
2	7	
3	8	
4	9	
5	9	
6	11	
7	11	
8	14	
Total:	75	

How I can achieve better:

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1. Find, to an appropriate degree of accuracy, the values of x and y for which

   (a) $3^x = 11$, [3]

   (b) $\log_2(2y - 1) = 4$. [3]

Total: 6

2. A sequence is defined as follows:

$$u_{n+1} = 3u_n + 2, \quad n \geq 1, \quad u_1 = k.$$

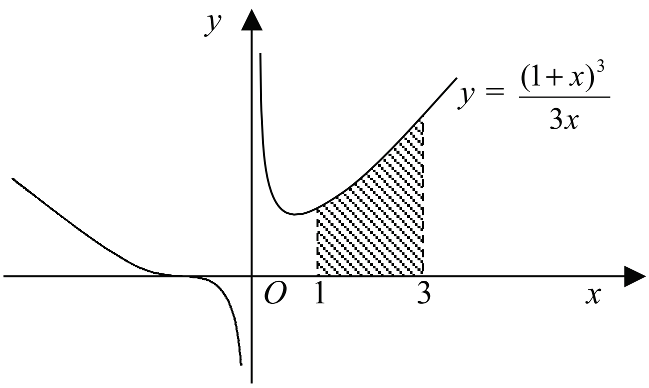
   (a) Find expressions in terms of k for u_2 and u_3. [3]

   Given that $\sum_{r=1}^{4} u_r = 16$,

   (b) find the value of k. [4]

Total: 7

3. Figure shows part of the curve with equation $y = \dfrac{(1+x)^3}{3x}$.



   (a) Express $(1+x)^3$ as a series in ascending powers of x. [2]

   (b) Show that the area of the shaded region enclosed by the curve, the ordinates $x = 1$ and $x = 3$, and the x-axis is given by $\frac{1}{9}(3\ln(3) + 80)$. [6]

Total: 8

4. The function f is given by

$$\mathrm{f}\colon x \mapsto \frac{2}{x-3}, \quad x \in \mathbb{R}, \quad x \neq 3.$$

   (a) Define $\mathrm{f}^{-1}(x)$, stating its domain clearly. [5]

   The function g is given by

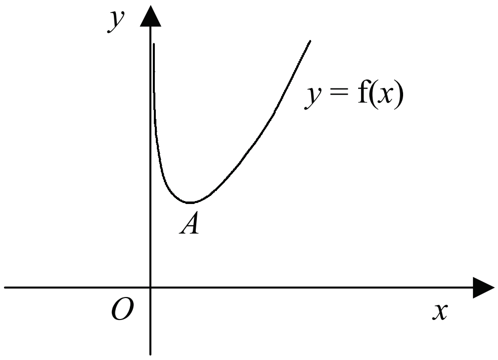
$$\mathrm{g}\colon x \mapsto x^2 - 6x + 1, \quad x \in \mathbb{R}, \quad x \geq k.$$

   (b) Given that $\mathrm{g}^{-1}(x)$ exists, find the minimum value of k. [4]

Total: 9

5. Figure shows part of the curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) \equiv 2\mathrm{e}^{x} - \ln(x), \quad x \in \mathbb{R}, \quad x > 0.$$



(a) Find $\mathrm{f}'(x)$. [2]

A is the stationary point on the curve.

(b) Show that the x-coordinate of A lies in the interval $(0.3, 0.4)$. [3]

The point B lies on the curve and its x-coordinate is 1.

(c) Show that the equation of the tangent to the curve at B is [4]

$$y = (2\mathrm{e} - 1)x + 1.$$

Total: 9

6. Given that

$$p = \frac{3x - 4}{x + 1} \quad \text{and} \quad q = \frac{x^2 - 6x}{x^2 - 1},$$

(a) show that $p - 2q = \dfrac{x + 4}{x - 1}$, [6]

(b) find and simplify an expression for $\dfrac{p}{q}$ in terms of x, [3]

(c) find the value or values of x for which $\dfrac{p}{q} = 0$. [2]

Total: 11

7. (a) Prove that for all values of x [5]

$$\cos^2(x) - \sin^2(2x) \equiv \cos^2(x)(4\cos^2(x) - 3).$$

(b) Hence find the values of x in the interval $0 \leq x \leq 2\pi$, for which [6]

$$\cos^2(x) - \sin^2(2x) = 0,$$

giving your answers in terms of π.

Total: 11

8. (a) By sketching the graphs $y = (x-3)^2$ and $y = \sqrt{x}$ on the same diagram, show that the equation $(x-3)^2 = \sqrt{x}$ has exactly two positive roots. [4]

(b) Show that one root of the equation, α, lies in the interval $1 < \alpha < 2$, and find the value of N such that [5]

$$\frac{N}{10} < \alpha < \frac{N+1}{10}.$$

(c) Using an iteration of the form [5]

$$x_{n+1} = x_n^{\frac{1}{4}} + k,$$

with a starting value of $x_1 = 4$, find the other root of the equation, β, correct to 3 significant figures.

Total: 14