Solomon Practice Paper

Pure Mathematics 2J

Time allowed: 90 **minutes**

Question	Points	Score
1	4	
2	8	
3	8	
4	10	
5	11	
6	11	
7	11	
8	12	
Total:	75	

Centre: **www.CasperYC.club**

Name:

Teacher:

How I can achieve better:

-
-
-

Last updated: May 5, 2023

1. Given that $y = 3e^x + 2\ln(x)$, find $\dfrac{d^2y}{dx^2}$. [4]

2. (a) By letting $p = \log_a(x)$ and $q = \log_a(y)$, or otherwise, prove that [4]

$$\log_a(xy) \equiv \log_a(x) + \log_a(y).$$

   (b) Find integers A and B such that [4]

$$\ln(48) + \ln(108) = A\ln(2) + B\ln(3).$$

Total: 8

3. (a) Express $\left(x^{\frac{1}{2}} - 2x^{-\frac{3}{2}}\right)^2$ in the form $px + qx^{-1} + rx^{-3}$. [3]

   (b) Show that [5]

$$\int_2^4 \left(x^{\frac{1}{2}} - 2x^{-\frac{3}{2}}\right)^2 \, dx = \frac{51}{8} - 4\ln(2).$$

Total: 8

4. (a) Find the values of θ in the interval $0 \le \theta \le 2\pi$, for which [5]

$$2\tan^2(\theta) + \sec^2(\theta) = 2,$$

   giving your answers in terms of π.

   (b) Find the values of x in the interval $0 \le x \le 180°$, for which [5]

$$\sin(3x) = \sin(2x).$$

Total: 10

5. Given that $a > 0$,

   (a) sketch on the same set of coordinate axes the graphs of $y = \frac{1}{2}(x + a)$ and $y = |2x - a|$, labelling the coordinates of any points where each graph meets the coordinate axes, [5]

   (b) find, in terms of a, the coordinates of any points where the two graphs intersect. [6]

Total: 11

6. (a) Expand $(4 + 2x)^5$ as a series in ascending powers of x, simplifying each coefficient. [4]

   Hence, find

   (b) the coefficient of y^4 in the expansion of $(4 + \frac{1}{5}y)^5$ as an exact fraction, [3]

   (c) the coefficient of z^6 in the expansion of $(2 + \sqrt{2}z)^5(2 - \sqrt{2}z)^5$. [4]

Total: 11

7.
$$f(x) \equiv x^4 - 5x + 3.$$

(a) Show that one root of the equation $f(x) = 0$ lies in the interval $(0.6, 0.7)$. [2]

(b) Using the iteration formula [3]

$$x_{n+1} = 0.2\left(x_n^4 + 3\right),$$

with a starting value of $x_1 = 0.65$, find this root correct to 3 significant figures.

(c) Show that the equation $f(x) = 0$ can be rewritten as [2]

$$x = \pm\sqrt{\frac{ax+b}{x^2}}$$

where a and b are integers to be found.

(d) Hence, use the iteration formula [2]

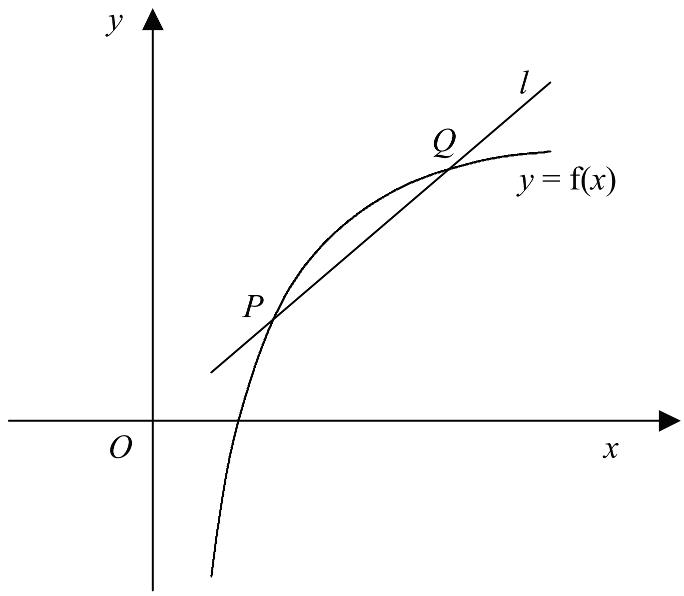
$$x_{n+1} = \pm\sqrt{\frac{a_n x + b}{x_n^2}},$$

together with your values of a and b and with $x_1 = 1.5$ to find x_2, x_3 and x_4 correct to 6 significant figures.

(e) Considering only your values of x_2, x_3 and x_4, explain why it is reasonable to give a second root of the equation as 1.43 correct to 3 significant figures. [2]

Total: 11

8. Figure shows the straight line l and the curve $y = f(x)$.



The line and curve intersect at the points $P(1, \ln(2))$ and $Q(3, \ln(8))$.

(a) Find in its simplest form the equation of the line l. [4]

Given that $f(x) \equiv \ln(ax + b)$,

(b) find the values of a and b, [5]

(c) hence, find an expression for $f^{-1}(x)$. [3]

Total: 12