Solomon Practice Paper

Pure Mathematics 2D

Time allowed: 90 **minutes**

Question	Points	Score
1	5	
2	6	
3	8	
4	8	
5	10	
6	11	
7	12	
8	15	
Total:	75	

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Name:

Teacher:

How I can achieve better:

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Last updated: May 5, 2023

1. The function f is defined by

$$\mathrm{f}(x) \equiv \log_3(x), \quad x \in \mathbb{R}, \quad x > 0.$$

(a) Evaluate $\mathrm{f}(27)$. [2]

(b) Solve the equation $\mathrm{f}(3^{x+4}) = 2x$. [3]

Total: 5

2. A sequence is defined by the recurrence relation

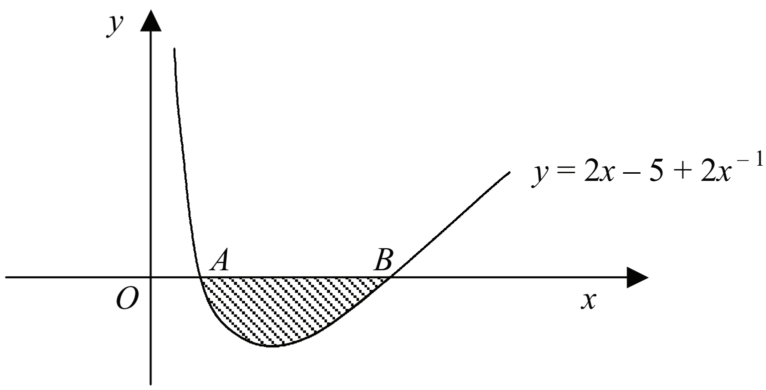
$$u_{n+1} = 4 + \frac{1}{2}u_n, \quad n \geq 1, \quad u_1 = 72.$$

(a) Find the value of u_4. [3]

(b) Using the fact that $u_{10} = \frac{65}{8}$, find the value of u_9 as an exact fraction. [3]

Total: 6

3. Figure shows part of the curve with equation $y = 2x - 5 + 2x^{-1}$, for $x > 0$.



The curve crosses the x-axis at the points A and B.

(a) Find the coordinates of the points A and B. [3]

(b) Show that the *mathematical* area of the shaded region enclosed by the curve and the x-axis is given by $\frac{1}{4}(16\ln(2) - 15)$. [5]

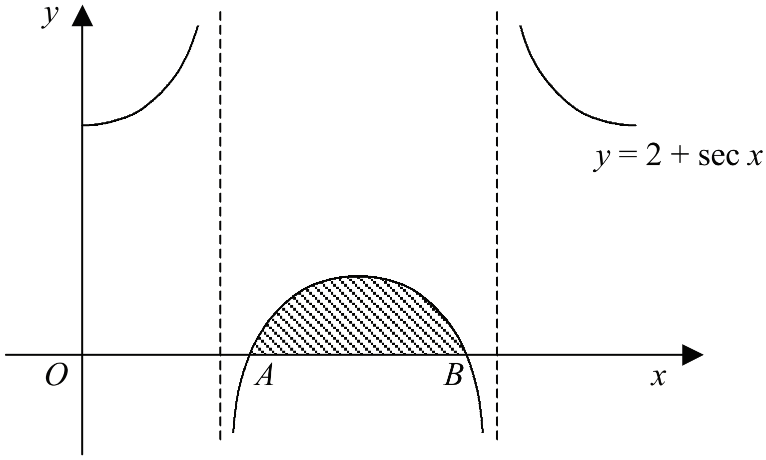
Total: 8

4. (a) Prove by counter-example that $(3^n + 2)$ is not prime for all positive integers n. [3]

(b) Use proof by contradiction to show that there are no integers p and q, such that $\frac{p}{q} = \sqrt{2}$, where p and q are co-prime. [5]

Total: 8

5. Figure shows the curve $y = 2 + \sec(x)$ for x in the interval $[0, 2\pi]$.



The curve meets the x-axis at the points A and B.

(a) Find the coordinates of the points A and B. [4]

The shaded region is enclosed by the curve and the x-axis between the points A and B.

(b) Show that estimating the area of the shaded region using the trapezium rule with 5 equally spaced ordinates gives a value of $\frac{1}{18}\pi(15 - 4\sqrt{3})$. [6]

Total: 10

6.

$$f(x) \equiv 3x^2 - 18x + 5, \quad x \in \mathbb{R}, \quad x > 4.$$

(a) Express $f(x)$ in the form $A(x+B)^2 + C$. [3]

(b) Find the range of $f(x)$. [2]

(c) Define $f^{-1}(x)$, stating its domain clearly. [6]

Total: 11

7. (a) Using the half-angle formulae, or otherwise, prove that for all values of x [5]

$$\frac{1+\cos(x)}{1-\cos(x)} \equiv \cot^2\left(\frac{x}{2}\right).$$

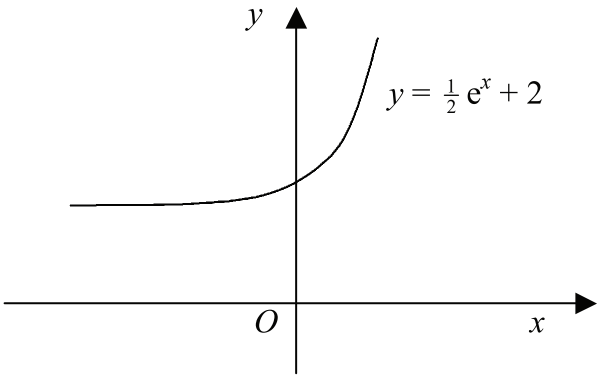
(b) Hence, find the values of x in the interval $0 \leq x \leq 2\pi$ for which [7]

$$\frac{1+\cos(x)}{1-\cos(x)} = 6\csc\left(\frac{x}{2}\right) - 10$$

giving your answers correct to an appropriate degree of accuracy.

Total: 12

8. Figure shows part of the curve with equation $y = \frac{1}{2}e^x + 2$ which passes through the point A with coordinates $(\ln(4), 4)$.



(a) Show that the equation of the tangent to the curve at A is $y = 2x + 4 - 4\ln(2)$. [4]

The tangent to the curve at A meets the x-axis at the point B.

(b) Find the coordinates of the point B. [3]

The normal to the curve at A meets the x-axis at the point C.

(c) Find the coordinates of the point C. [5]

(d) Show that the area of triangle ABC is 20. [3]

Total: 15