Solomon Practice Paper

Pure Mathematics 2C

Time allowed: 90 **minutes**

Centre: www.CasperYC.club

Name:

Teacher:

Question	Points	Score
1	6	
2	6	
3	6	
4	8	
5	10	
6	12	
7	13	
8	14	
Total:	75	

How I can achieve better:

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Last updated: May 5, 2023

1. (a) Solve the equation [3]

$$\ln(2x+1) = 3$$

giving your answer in terms of e.

(b) Given that [3]

$$2^x = 5^y,$$

show that $y = kx$ where k is a constant that you should find correct to an appropriate degree of accuracy.

Total: 6

2. (a) Use the identity [2]

$$\sin(A+B) \equiv \sin(A)\cos(B) + \cos(A)\sin(B)$$

to prove that

$$\sin(2A) \equiv 2\sin(A)\cos(A).$$

(b) Hence, or otherwise, use the fact that [4]

$$\sin(15°) = \frac{\sqrt{6}-\sqrt{2}}{4}$$

to find the value of $\cos(15°)$ in exact form with a rational denominator.

Total: 6

3. Express [6]

$$\frac{5x^2-11x+9}{x^2+3x-10} + \frac{3-2x}{x-2}$$

as a single fraction in its simplest form.

4. The coefficient of x^2 in the expansion of $(1+3x)^n$ is 252.

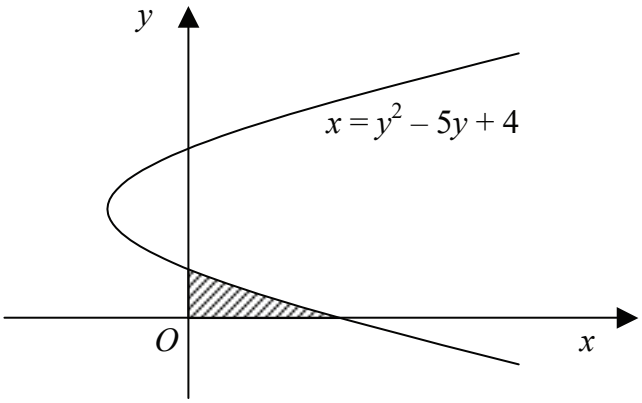
Given that n is a positive integer,

(a) find the value of n, [5]

(b) show that the coefficient of x^3 is 1512. [3]

Total: 8

5. Figure shows the curve $x = y^2 - 5y + 4$.



(a) Express x^2 in descending powers of y. [3]

(b) Find $\int x^2 \, \mathrm{d}y$. [3]

(c) Show that the volume generated when the shaded region, bounded by the curve and the positive coordinate axes, is rotated through 2π radians about the y-axis is $\frac{47\pi}{10}$. [4]

Total: 10

6. The functions f and g are defined by

$$\begin{aligned} \mathrm{f}: x &\mapsto x^2 - 2, \quad x \in \mathbb{R}, \\ \mathrm{g}: x &\mapsto \mathrm{e}^{\frac{3}{2}x} \quad\quad x \in \mathbb{R}. \end{aligned}$$

(a) State the range of g. [1]

(b) Define fg as simply as possible. [3]

(c) Find, correct to 2 decimal places, the value of x for which $\mathrm{fg}(x) = 5$. [3]

(d) Show that the only value of x for which $\mathrm{g}(x) = \mathrm{fg}(x)$ is $\frac{2}{3}\ln(2)$. [5]

Total: 12

7. (a) Prove that [6]

$$\cot^2(x) - \tan^2(x) \equiv 4\cot(2x)\csc(2x).$$

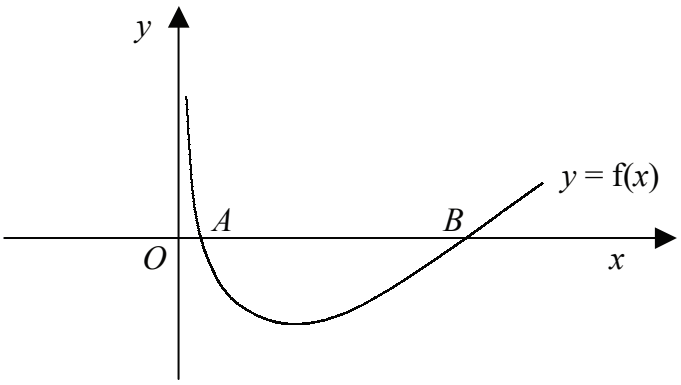
(b) Hence, find in terms of π the values of x in the interval $0 \leq x \leq \pi$ for which [7]

$$\cot^2(x) - \tan^2(x) = 8\cot(2x).$$

Total: 13

8. Figure shows part of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) \equiv x - 3\ln(2x), \quad x \in \mathbb{R}, \quad x > 0.$$



The curve crosses the x-axis at the points A and B.

(a) Show that the x-coordinate of the point A lies in the interval $(0.6, 0.7)$. [2]

(b) Find the value of N such that the x-coordinate of the point B lies in the interval $(N, N+1)$. [4]

The line $y = x$ meets the curve at the point C.

(c) Find the coordinates of the point C. [3]

(d) Show that the equation of the tangent to the curve at C is $y = 3 - 5x$. [5]

Total: 14