# Solomon Practice Paper

## Pure Mathematics 2B

**Time allowed:** 90 **minutes**

**Centre:** www.CasperYC.club

**Name:**

**Teacher:**

| Question | Points | Score |
|---|---|---|
| 1 | 6 | |
| 2 | 7 | |
| 3 | 8 | |
| 4 | 9 | |
| 5 | 10 | |
| 6 | 10 | |
| 7 | 11 | |
| 8 | 14 | |
| Total: | 75 | |

**How I can achieve better:**

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Last updated: May 5, 2023

1. (a) Sketch the graph of $y = |3x + 2|$ showing the coordinates of any points where the graph meets the coordinate axes. [3]

   (b) Solve the equation $|3x + 2| = 2 - x$ [3]

Total: 6

2. (a) Prove using the laws of indices that for all values of $x$ [4]

$$\log_a\left(x^k\right) \equiv k \log_a(x).$$

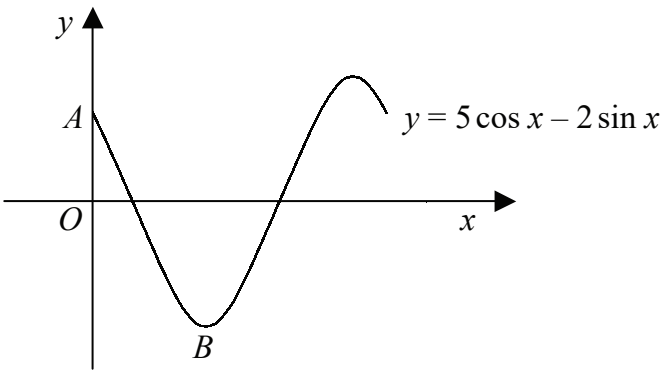
   (b) Express [3]

$$\ln(9) - 3\ln\sqrt{3} + \ln(81)$$

   in the form $k\ln(3)$, where $k$ is an exact fraction.

Total: 7

3. Figure shows the curve $y = 5\cos(x) - 2\sin(x)$ for $0 \leq x \leq 360°$.




   (a) Find the values of $R$ and $\alpha$, correct to 3 significant figures, for which [5]

$$5\cos(x) - 2\sin(x) \equiv R\cos(x + \alpha)$$

   where $x$ is measured in degrees and $0 < \alpha < 90°$.

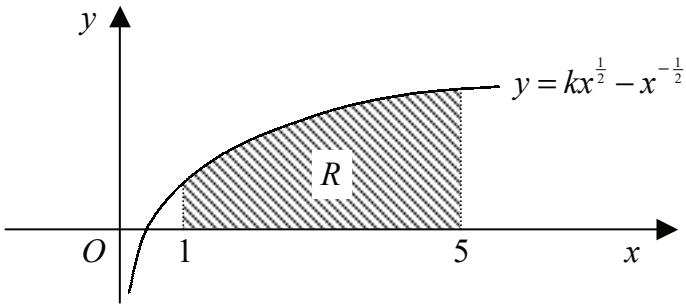
   (b) Find the coordinates of [3]

   i. the point $A$, where the curve meets the $y$-axis,

   ii. the point $B$, the first minimum on the curve for $x > 0$.

Total: 8

4. Figure shows part of the curve $y = kx^{\frac{1}{2}} - x^{-\frac{1}{2}}$.




Given that the point with coordinates $\left(3, \frac{5}{3}\sqrt{3}\right)$ lies on the curve,

(a) show that $k = 2$. [3]

The shaded region, $R$, is bounded by the curve, the $x$-axis and the ordinates $x = 1$ and $x = 5$.

(b) Find the volume generated when $R$ is rotated through 360° about the $x$-axis, giving your answer in the form $\pi(a + b\ln(5))$. [6]

Total: 9

5. The function f is given by

$$\mathrm{f}: x \mapsto \frac{x}{x+3}, \quad x \in \mathbb{R}, \quad x \neq -3.$$

(a) Define $\mathrm{f}^{-1}(x)$, stating its domain clearly. [5]

The function g is given by

$$\mathrm{g}: x \mapsto \frac{4}{x}, \quad x \in \mathbb{R}, \quad x \neq 0.$$

(b) Express $\mathrm{fg}(\sqrt{2})$ in the form $a + b\sqrt{2}$, where $a$ and $b$ are integers. [5]

Total: 10

6. A sequence is defined as follows

$$u_{n+1} = ku_n - 2, \quad n \geq 1, \quad u_1 = 3.$$

(a) Find expressions in terms of $k$ for $u_2$ and $u_3$. [3]

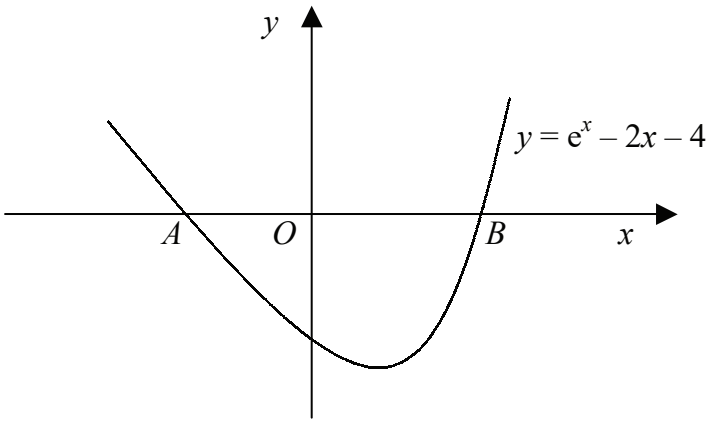
Given that $u_2 + u_3 = 0$,

(b) show that one possible value of $k$ is 1 and find the other possible value, [4]

(c) find the value of $u_4$ corresponding to each possible value of $k$. [3]

Total: 10

7. Figure shows part of the curve with equation $y = \mathrm{e}^x - 2x - 4$.




(a) Find in exact form the coordinates of the turning point of the curve. [5]

The curve intersects the $x$-axis at the points $A(a, 0)$ and $B(b, 0)$ where $a < b$.

(b) Show that $-2 < a < -1$. [2]

(c) Use an iteration of the form [4]

$$x_{n+1} = \ln(px_n + q)$$

with a starting value of $x_0 = 2$ to find $b$ correct to 3 significant figures.

Total: 11

8.

$$\mathrm{f}(x) \equiv 2x^2 + 4x + \ln(x), \quad x \in \mathbb{R}, \quad x > 0.$$

(a) Sketch the curves $y = 4x + 2x^2$ and $y = -\ln(x)$, for $x > 0$, on the same diagram. Hence show that the equation $\mathrm{f}(x) = 0$ has exactly one solution. [3]

(b) Express $\mathrm{f}'(x)$ in the form [6]

$$\frac{(ax + b)^2}{x},$$

and hence prove that $\mathrm{f}(x)$ is increasing throughout its domain.

(c) Find $\mathrm{f}''(x)$ and hence find the set of values of $x$ for which the gradient of $\mathrm{f}(x)$ is increasing. [5]

Total: 14