Solomon Practice Paper

Pure Mathematics 1L

Time allowed: 90 **minutes**

Centre: **www.CasperYC.club**

Name:

Teacher:

Question	Points	Score
1	5	
2	6	
3	7	
4	8	
5	10	
6	12	
7	13	
8	14	
Total:	75	

How I can achieve better:

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Last updated: May 5, 2023

1. Show that [5]

$$2\sqrt{75} + \frac{4}{2\sqrt{3} - 4}$$

can be written in the form $a\sqrt{3} + b$ where a and b are integers to be found.

2. (a) Given that $t = x^{\frac{1}{3}}$, express $2x^{\frac{2}{3}}$ in terms of t. [2]

(b) Hence, or otherwise, solve the equation [4]

$$2x^{\frac{2}{3}} + 5x^{\frac{1}{3}} - 12 = 0.$$

Total: 6

3. Tom and Jim share the same birthday.

Today, Tom is x years old and Jim is 4 years older than him.

(a) Given that Jim's age is less than 50% more than Tom's, write down a linear inequality satisfied by x. [2]

(b) Given also that the product of Tom and Jim's ages is not more than 140, write down a quadratic inequality satisfied by x. [1]

(c) By solving your inequalities, find the possible values of x. [4]

Total: 7

4. (a) Given that [3]

$$x(2x^3 - x)(5 - x^{-2}) \equiv Ax^4 + Bx^2 + C$$

find the values of A, B and C.

(b) The curve $y = \mathrm{f}(x)$ passes through the point with coordinates $(1, 2)$. [5]

Given also that

$$\mathrm{f}'(x) = x(2x^3 - x)(5 - x^{-2})$$

find an expression for $\mathrm{f}(x)$.

Total: 8

5. An athlete is training to run in long distance races. In the first week she runs 50 miles and she intends to increase this amount by 10% each week.

(a) Calculate how far she should run in the second week. [2]

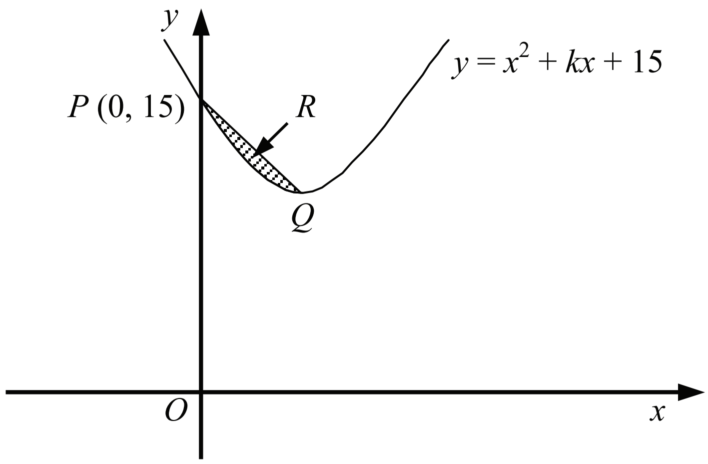
(b) Show that, in total, she should run 165.5 miles in the first three weeks. [1]

(c) By summing an appropriate geometric series find to the nearest mile the total distance that she should run during the first eight weeks. [4]

(d) Show that for her to have run more than 2000 miles in total the number of weeks for which she has been training, n, must satisfy the condition: $1.1^n > 5$. [3]

Total: 10

6. Figure shows the curve $y = x^2 + kx + 15$ which crosses the y-axis at the point $P(0, 15)$.



Q is the minimum point on the curve.

(a) Find the coordinates of the point Q in terms of k. [3]

Given that $k = -4$,

(b) calculate the distance PQ giving your answer in surd form as simply as possible, [3]

(c) find the area of the shaded region R enclosed by the curve and the line PQ. [6]

Total: 12

7. (a) Sketch the curve $y = 3\sin(\theta) + 1$ in the interval $0 \leq \theta \leq 360°$. Mark on your sketch the coordinates of any stationary points. [5]

(b) Show that the curves $y = 3\sin(\theta) + 1$ and $y = 2\cos^2(\theta)$ will intersect when [2]

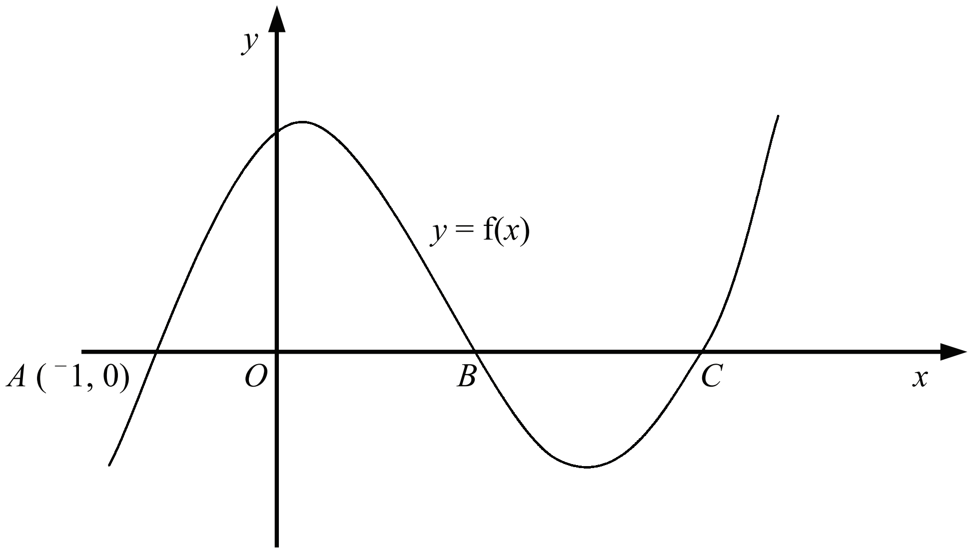
$$2\sin^2(\theta) + 3\sin(\theta) - 1 = 0.$$

(c) Hence, find the coordinates of the points of intersection between these two curves in the interval $0 \leq \theta \leq 360°$. [6]

Total: 13

8. Figure shows the curve $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) \equiv 2x^3 - 9x^2 + x + 12.$$



(a) Given that the curve cuts the x-axis at the point A with coordinates $(-1, 0)$, write down a linear factor of $f(x)$. [1]

(b) Hence, factorise $f(x)$ fully and find the coordinates of the points B and C where the curve again cuts the x-axis. [5]

(c) Find an equation of the normal to the curve at the point A. [5]

(d) The normal to the curve at A and the tangent to the curve at C meet at the point D. Prove that $\angle ADC$ is a right-angle. [3]

Total: 14