Solomon Practice Paper

## Pure Mathematics 1J

Time allowed: 90 minutes

Centre: www.CasperYC.club
Name:

## Teacher:

| Question | Points | Score |
| :---: | :---: | :---: |
| 1 | 7 |  |
| 2 | 7 |  |
| 3 | 8 |  |
| 4 | 8 |  |
| 5 | 9 |  |
| 6 | 9 |  |
| 7 | 11 |  |
| 8 | 16 |  |
| Total: | 75 |  |

How I can achieve better:

1. Figure shows a sector $O A B$ of a circle, centre $O$ and radius 12 cm .


Given that the perimeter of the sector $O A B$ is 32 cm , find
(a) the size of $\angle A O B$ in radians as an exact fraction,
(b) the area of the shaded segment in $\mathrm{cm}^{2}$, giving your answer correct to 3 significant figures.
2. (a) Find $\int(3 x-1)^{2} \mathrm{~d} x$.

Given that $\frac{\mathrm{d} y}{\mathrm{~d} x}=(3 x-1)^{2}$ and that when $x=-1, y=2$,
(b) find $y$ in terms of $x$.
3. (a) Prove from first principles that the sum of the first $n$ natural numbers,

$$
1+2+3+4+\ldots+n
$$

is given by

$$
\frac{1}{2} n(n+1)
$$

(b) Hence evaluate the sum of the integers between -30 and 72 inclusive.
4. $A$ is the point $(8,0)$ and $B$ is the point $(12,6)$.
(a) Find an equation of the line passing through the points $A$ and $B$.
$M$ and $N$ are the midpoints of $O A$ and $O B$ respectively, where $O$ is the origin.
(b) Calculate the area of the trapezium $A B N M$.
5. (a) Given that $y=2^{x}$, show that
i. $4^{x}=y^{2}$
ii. $2^{x+2}=4 y$
(b) By using your answers to part (a), or otherwise, solve the equation

$$
4^{x}-2^{x+2}-32=0
$$

6. (a) Show that the solutions of the equation

$$
5 \tan (\theta)-6 \cos (\theta)=0
$$

will be given by the values of $\theta$ for which

$$
6 \sin ^{2}(\theta)+5 \sin (\theta)-6=0
$$

(b) Hence solve the equation

$$
5 \tan (\theta)-6 \cos (\theta)=0
$$

for $\theta$ in the interval $0 \leq \theta \leq 2 \pi$, giving your answers correct to 2 decimal places.
7. Figure shows a design consisting of four identical circles of radius $r$,

which are shaded, arranged such that their centres are at the four corners of a square of side $2 r$. A larger circle of radius $R$ circumscribes the four smaller circles.
(a) Show that $R=(1+\sqrt{2}) r$.
(b) Hence show that the ratio of the total area of the four shaded circles to the area of the larger circle is equal to $(12-8 \sqrt{2}): 1$.

Total: 11
8. Figure shows part of the curve

$$
y=x^{2}+\frac{8}{x}
$$


which crosses the $x$-axis at the point $A$.
(a) Find the coordinates of the point $A$.

The line $l$ is the normal to the curve at the point $A$.
(b) Find an equation of the line $l$.
(c) Show that the line $l$ will intersect the curve where

$$
6 x^{3}-x^{2}-2 x+48=0
$$

(d) Hence prove that $l$ does not cross the curve other than at $A$.

