Solomon Practice Paper Pure Mathematics 1G

Time allowed: 90 minutes

Centre: www.CasperYC.club
Name:

## Teacher:

| Question | Points | Score |
| :---: | :---: | :---: |
| 1 | 5 |  |
| 2 | 6 |  |
| 3 | 7 |  |
| 4 | 9 |  |
| 5 | 9 |  |
| 6 | 11 |  |
| 7 | 14 |  |
| 8 | 14 |  |
| Total: | 75 |  |

How I can achieve better:

1. Figure shows a small rectangular picture frame.


The frame is to have a width of $(x+3)$ centimetres and a height of $(x-5)$ centimetres.
Given that the area enclosed by the edge of the frame is to be at most $105 \mathrm{~cm}^{2}$, find the set of possible values of $x$.
2. (a) Solve the equation

$$
y-\frac{2}{y}=5,
$$

giving your answers correct to 2 decimal places.
(b) Given that $p$ and $q$ are constants, prove that the equation

$$
x^{2}-2 p x+3 q-1=0
$$

has no real solutions only if $q>\frac{p^{2}+1}{3}$.
3. A savings scheme requires a minimum investment of $£ 400$ on the 1 st of January each year. The scheme pays compound interest at $6 \%$ per annum.

For an investor paying this minimum amount in each year,
(a) show that after the payment of interest at the end of the second year the amount in the scheme is $£ 873.44$.
(b) find the amount in the scheme after the payment of interest at the end of 12 years.
4. (a) Find the exact values of $\theta$ in radians, in the interval $0 \leq \theta \leq 2 \pi$ for which:

$$
\cos \left(\theta-\frac{\pi}{3}\right)=\frac{\sqrt{3}}{2}
$$

(b) Sketch the curve $y=1-\sin (2 x)$ for $x$ in the interval $0 \leq x \leq 360^{\circ}$.

Your graph should show clearly where the curve intersects each of the coordinate axes.
5.

$$
\mathrm{f}(x) \equiv x^{3}+2 x^{2}+a x+2
$$

(a) Given that $(x-2)$ is a factor of $\mathrm{f}(x)$, show that $a=-9$.
(b) Hence write $\mathrm{f}(x)$ as the product of a linear factor and a quadratic factor.
(c) Solve the equation $\mathrm{f}(x)=0$, giving your answers in surd form when appropriate.
6. The straight line $l$ passes through the points $A(-1, k)$ and $B(8,2)$ and has a gradient of $-\frac{1}{2}$.
(a) Show that $k=\frac{13}{2}$.
(b) Find the equation of the line $m$ that is perpendicular to $l$ and passes through the mid-point
of $A B$. Give the equation in the form $a x+b y+c=0$ where $a, b$ and $c$ are integers to be found and $a>0$.
(c) Find the exact area of the triangle enclosed by the line $m$ and the coordinate axes.
7. Figure shows the line $y=3+2 x-x^{2}$ and its tangent at the point $P(0,3)$.


The curve cuts the $x$-axis at $Q$ and $R$ as shown.
(a) Find the coordinates of the points $Q$ and $R$.
(b) Find an equation of the tangent to the curve at $P$.

The shaded region $S$ is bounded by the curve, the tangent and the $x$-axis.
(c) Find the exact area of the region $S$.
8. Figure shows the shape of a company logo.


The shape is made by removing a circular sector of radius $r \mathrm{~cm}$, angle $\theta$ radians from a larger circular sector of radius $2 r \mathrm{~cm}$, angle $\theta$ radians.
(a) Show that the area, $A \mathrm{~cm}^{2}$, of the shape is given by $A=\frac{3}{2} r^{2} \theta$.
(b) Given that $A=90$, show that the perimeter, $P \mathrm{~cm}$, of the shape is given by

$$
P=2 r+180 r^{-1}
$$

Given that $r$ can vary,
(c) find the value of $r$ for which $P$ is a minimum and the corresponding value of $P$, giving your answers in the form $a \sqrt{10}$,
(d) justify that your value of $P$ is a minimum.

