Solomon Practice Paper
Pure Mathematics 1A
Time allowed: 90 minutes

Centre: www.CasperYC.club
Name:

## Teacher:

| Question | Points | Score |
| :---: | :---: | :---: |
| 1 | 6 |  |
| 2 | 6 |  |
| 3 | 7 |  |
| 4 | 8 |  |
| 5 | 9 |  |
| 6 | 13 |  |
| 7 | 13 |  |
| 8 | 13 |  |
| Total: | 75 |  |

How I can achieve better:

1. Find the pairs of values $(x, y)$ which satisfy the simultaneous equations:

$$
\begin{array}{r}
2 x-y=1 \\
4 x^{2}+4 y+y^{2}=9
\end{array}
$$

2. (a) Prove that the quadratic equation

$$
x^{2}+(m-1) x+m+2=0
$$

has real and distinct roots when

$$
m^{2}-6 m-7>0 .
$$

(b) Hence, or otherwise, find the set of values of $m$ for which

$$
x^{2}+(m-1) x+m+2=0
$$

has real and distinct roots.
3. The first three terms of an arithmetic series are $(3 p-5),(2 p-2)$ and $(5 p-1)$ respectively.
(a) Find the value of $p$.
(b) Hence, find the sum of the first 10 terms of the series.
4. (a) Show that the equation

$$
2 \sin ^{2}(x)-\cos (x)=1
$$

can be written as

$$
2 \cos ^{2}(x)+\cos (x)-1=0 .
$$

(b) Using your answer to part (a), find all the solutions of the equation

$$
2 \sin ^{2}(x)-\cos (x)=1
$$

in the interval $0 \leq x \leq 2 \pi$, giving your answers in terms of $\pi$.
5.

$$
\mathrm{f}(x) \equiv x^{3}-5 x^{2}+7 x-2 .
$$

(a) Show that $x=2$ is a solution of the equation $\mathrm{f}(x)=0$.
(b) Find the other solutions of the equation $\mathrm{f}(x)=0$, giving your answers correct to 2 decimal places.
6. Figure shows part of the curve with equation $y=4 x^{\frac{1}{2}}-x$.

$A$ is the maximum point of the curve and the curve crosses the $x$ - axis at the point $B$.
(a) Find the coordinates of the point $A$.
(b) Find the $x$-coordinate of the point $B$.
(c) Show that the area of the shaded region enclosed by the curve and the $x$-axis is $\frac{128}{3}$.
7. $A$ and $B$ are points with coordinates $(5,2)$ and $(-1,4)$ respectively.
(a) Find the equation of the line $l$ which passes through the points $A$ and $B$ in the form $p x+q y+r=0$.
(b) Find the coordinates of the midpoint of $A B$.
(c) Hence, or otherwise, find the equation of the perpendicular bisector of $A B$.
$C$ is the point with coordinates $(3,4)$.
Given that the points $A, B$ and $C$ lie on the circumference of a circle, centre $D$,
(d) find the coordinates of the point $D$.
8. Figure shows the design for a hazard warning-symbol.


It consists of three identical sectors of a circle of radius $r$ centimetres. The sectors are equally spaced and each subtends an angle $\theta$ radians at the centre.

Given that the area of the symbol is to be $48 \mathrm{~cm}^{2}$,
(a) find an expression for $\theta$ in terms of $r$.
(b) Hence, show that the perimeter of the shape, $P \mathrm{~cm}$, is given by

$$
P=6 r+\frac{96}{r}
$$

Given that $r$ can vary,
(c) find the value of $r$ for which $P$ is a minimum and the corresponding value of $P$.
(d) justify that your value of $P$ is a minimum.

