Solomon Practice Paper

Pure Mathematics 6H

Time allowed: 90 **minutes**

Centre: **www.CasperYC.club**

Name:

Teacher:

Question	Points	Score
1	5	
2	8	
3	8	
4	12	
5	13	
6	14	
7	15	
Total:	75	

How I can achieve better:

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1. Given that [5]

$$t_{n+1} = t_n - 4 \qquad n \geq 1, \quad t_1 = 3,$$

prove by induction that $t_n = 7 - 4n$ for all integers $n, n \geq 1$.

2. (a) On the same Argand diagram sketch the locus of the points defined by the equations [6]

   i. $z + z^* = 2$,

   ii. $\arg\left(\dfrac{z-2}{z+2}\right) = \dfrac{\pi}{4}$, where $\text{Im}(z) \geq 0$.

The region R of the complex z-plane is defined by the inequalities

$$z + z^* \leq 2, \qquad \arg\left(\frac{z-2}{z+2}\right) \geq \frac{\pi}{4} \qquad \text{Im}(z) \geq 0.$$

(b) Shade the region R on the Argand diagram. [2]

Total: 8

3. The points A, B and C with coordinates (x_{-1}, y_{-1}), (x_0, y_0) and (x_1, y_1) respectively lie on the curve $y = \mathrm{f}(x)$ where $x_1 - x_0 = x_0 - x_{-1} = h$ and $y_n = \mathrm{f}(x_n)$.

   (a) By drawing a sketch, or otherwise, show that [3]

$$\mathrm{f}'(x_0) \approx \frac{\mathrm{f}(x_0 + h) - \mathrm{f}(x_0 - h)}{2h}.$$

Given that

$$\mathrm{f}'(x) = \sqrt{2x + \mathrm{f}(x)}, \quad \mathrm{f}(0) = 1, \quad \mathrm{f}(0.2) = 1.25,$$

   (b) use two applications of the approximation in (a) with a step length of 0.2 to find an estimate for $\mathrm{f}(0.6)$. [5]

Total: 8

4. The points A, B and C have position vectors $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ respectively such that

$$\begin{aligned} \mathbf{a} &= 2\mathbf{i} - \mathbf{j} + \mathbf{k}, \\ \mathbf{b} &= \mathbf{i} + q\mathbf{j} - 3\mathbf{k}, \\ \mathbf{c} &= 3\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}, \end{aligned}$$

where q is a constant and $q \neq 2$.

(a) Find $\overrightarrow{AB} \times \overrightarrow{AC}$, giving your answer in terms of q. [5]

(b) Hence show that the vector $\mathbf{n} = 4\mathbf{i} - \mathbf{k}$ is perpendicular to the plane Π containing A, B and C for all real values of q. [2]

(c) Find an equation of the plane Π, giving your answer in the form $\mathbf{r}.\mathbf{n} = p$. [2]

Given that$q = -1$,

(d) find the volume of the tetrahedron $OABC$. [3]

Total: 12

5. (a) Use De Moivre's theorem to show that [6]

$$\cos(5\theta) \equiv \cos(\theta)\left(16\cos^4(\theta) - 20\cos^2(\theta) + 5\right).$$

(b) By solving the equation $\cos^5(\theta) = 0$, deduce that [7]

$$\cos^2\left(\frac{3\pi}{10}\right) = \frac{5-\sqrt{5}}{8}.$$

Total: 13

6. (a) Find the first three derivatives of $\ln\left(\dfrac{1+x}{1-2x}\right)$. [6]

(b) Hence, or otherwise, find the expansion of $\ln\left(\dfrac{1+x}{1-2x}\right)$ in ascending powers of x up to and including the term in x^3. [4]

(c) State the values of x for which this expansion is valid. [1]

(d) Use this expansion to find an approximate value for $\ln\left(\frac{4}{3}\right)$, giving your answer to 3 decimal places. [3]

Total: 14

7.

$$\mathbf{A} = \begin{pmatrix} 2 & a & 2 \\ -1 & b & -2 \\ 0 & 0 & c \end{pmatrix} \quad \text{and} \quad \mathbf{B} = \begin{pmatrix} 6 & 5 & 2 \\ -1 & 0 & -2 \\ 0 & 0 & 5 \end{pmatrix}$$

and

$$(\mathbf{B} - 2\mathbf{I})\mathbf{A} = 3\mathbf{I} \qquad (\star)$$

where a, b and c are constants and I is the 3×3 identity matrix.

(a) Find the values of a, b and c. [6]

(b) Using equation $\star$, or otherwise, find $\mathbf{A}^{-1}$, showing your working clearly. [2]

The transformation $T\colon \mathbb{R}^3 \mapsto \mathbb{R}^3$ is represented by the matrix $\mathbf{A}$.

(c) Find an equation satisfied by all the points which remain invariant under T. [4]

T maps the vector $\begin{pmatrix} p \\ q \\ r \end{pmatrix}$ onto the vector $\begin{pmatrix} 4 \\ -5 \\ 3 \end{pmatrix}$

(d) Find the values of p, q and r. [3]

Total: 15