Solomon Practice Paper

Pure Mathematics 6C

Time allowed: 90 **minutes**

Centre: **www.CasperYC.club**

Name:

Teacher:

Question	Points	Score
1	6	
2	7	
3	10	
4	11	
5	11	
6	14	
7	16	
Total:	75	

How I can achieve better:

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1. Given that y satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^x \cosh(2y + x), \quad y = 1 \quad \text{at} \quad x = 1,$$

(a) use the approximation $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_0 \approx \frac{y_1 - y_0}{h}$ to obtain an estimate for y at $x = 1.01$, [3]

(b) use the approximation $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_0 \approx \frac{y_1 - y_{-1}}{2h}$ to obtain an estimate for y at $x = 0.99$. [3]

Total: 6

2. The points A, B and C have coordinates $(2, 1, -1)$, $(-2, 4, -2)$ and $(a, -5, 1)$ respectively, relative to the origin O, where $a \neq 10$.

   (a) Find $\overrightarrow{AB} \times \overrightarrow{AC}$. [4]

   The area of triange ABC is $4\sqrt{10}$ square units.

   (b) Find the possible values of the constant a. [3]

Total: 7

3. (a) Given that $z = \cos(\theta) + \mathbf{i}\sin(\theta)$, show that [2]

$$z^n + \frac{1}{z^n} = 2\cos(n\theta)$$

where n is a positive integer.

The equation $5z^4 - 11z^3 + 16z^2 - 11z + 5 = 0$ has no real roots.

(b) Use the result in part (a) to solve the equation, giving your answers in the form $a + \mathbf{i}b$ where $a, b \in \mathbb{R}$. [8]

Total: 10

4. Given that

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix},$$

(a) prove by induction that [6]

$$\mathbf{A}^n = \begin{pmatrix} 1 & n & \frac{1}{2}n(n+1) \\ 0 & 1 & n \\ 0 & 0 & 1 \end{pmatrix}$$

for all positive integers n.

(b) Find the inverse of $\mathbf{A}^n$. [5]

Total: 11

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5. Given that

$$f(x) = \arccos(x), \quad -1 \leq x \leq 1,$$

show that

(a) $f'(x) = \dfrac{-1}{(1-x^2)^{\frac{1}{2}}}$, [3]

(b) $(1-x^2)f''(x) - xf'(x) = 0$. [3]

(c) Use Maclaurin's theorem to find the expansion of $f(x)$ in ascending powers of x up to and including the term in x^3. [5]

Total: 11

6. The eigenvalues of the matrix

$$\mathbf{M} = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 0 & 1 \\ 1 & 1 & 2 \end{pmatrix}$$

are λ_1, λ_2 and λ_3.

(a) Show that $\lambda_1 = 2$ is an eigenvalue of M and find the other two eigenvalues λ_2 and λ_3. [7]

(b) Find an eigenvector corresponding to the eigenvalue 2. [4]

Given that $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ are eigenvectors of $\mathbf{M}$ corresponding to λ_2 and λ_3 respectively,

(c) write down a matrix $\mathbf{P}$ such that [3]

$$\mathbf{P}^{-1}\mathbf{M}\mathbf{P} = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}.$$

Total: 14

7. The complex number $z = x + \mathbf{i}y$, where x and y are real, satisfies the equation

$$|z + 1 + 8\mathbf{i}| = 3|z + 1|.$$

The complex number z is represented by the point P in the Argand diagram.

(a) Show that the locus of P is a circle and state the centre and radius of this circle. [7]

(b) Represent on the same Argand diagram the loci [4]

$$|z + 1 + 8\mathbf{i}| = 3|z + 1| \quad \text{and} \quad |z| = \left|z - \frac{14}{5}\right|$$

(c) Find the complex numbers corresponding to the points of intersection of these loci, giving your answers in the form $a + \mathbf{i}b$ where a and b are real. [5]

Total: 16