Solomon Practice Paper

Pure Mathematics 3K

Time allowed: 90 **minutes**

Centre: **www.CasperYC.club**

Name:

Teacher:

Question	Points	Score
1	7	
2	8	
3	10	
4	12	
5	12	
6	12	
7	14	
Total:	75	

How I can achieve better:

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BY NC ND

Last updated: May 5, 2023

1.

$$\mathrm{f}(x) \equiv 3x^3 + ax^2 + 8x + b.$$

Given that when $\mathrm{f}(x)$ is divided by $(3x - 1)$ the remainder is 1,

(a) show that $a + 9b + 16 = 0$. [3]

Given also that when $\mathrm{f}(x)$ is divided by $(x - 2)$ the remainder is 11,

(b) find the values of a and b. [4]

Total: 7

2. (a) Find [3]

$$\int \sin^2(y)\,\mathrm{d}y.$$

(b) Given that $y = \frac{\pi}{4}$ when $x = 0$, solve the differential equation [5]

$$\mathrm{e}^{4x}\frac{\mathrm{d}y}{\mathrm{d}x} = \csc^2(y).$$

Total: 8

3. A curve has the equation

$$xy - x^2 + 2y^2 = 36.$$

(a) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$. [3]

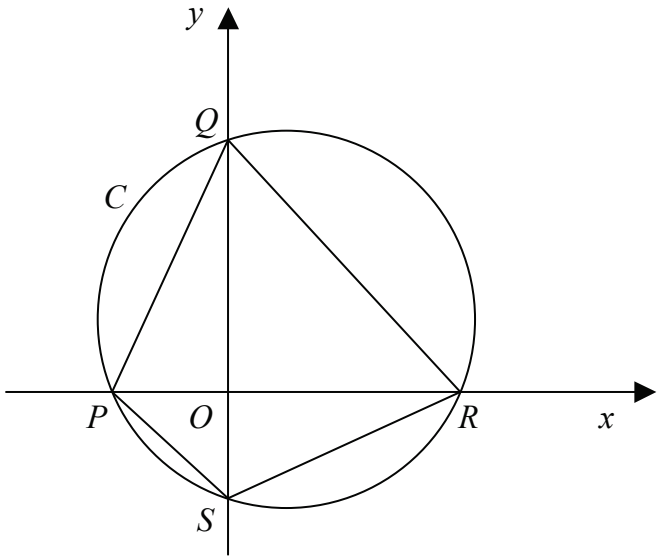
(b) Find the gradient of the curve at the point with coordinates $(4\sqrt{2}, 6 - \sqrt{2})$. [2]

The curve has two stationary points.

(c) Show that one of these has coordinates $(2, 4)$ and find the coordinates of the other. [5]

Total: 10

4. Figure shows the circle C with centre $(4, 5)$ and radius 13.



C meets the x-axis at the points P and R, and the y-axis at the points Q and S.

(a) Write down an equation for C. [2]

(b) Show that $QS = 6\sqrt{17}$. [5]

(c) Find the area of quadrilateral $PQRS$. [5]

Total: 12

5.

$$\mathrm{f}(x) = \frac{2x^2 - 15x + 15}{(x-1)^2(x-3)}.$$

(a) Find the values of A, B and C for which [4]

$$\mathrm{f}(x) \equiv \frac{A}{(x-1)^2} + \frac{B}{x-1} + \frac{C}{x-3}.$$

The point P lies on the curve $y = \mathrm{f}(x)$ and has coordinates $\left(\frac{3}{2}, p\right)$.

(b) Find the value of p. [2]

(c) Show that the tangent to the curve at P has the equation $8x + 3y - 36 = 0$. [6]

Total: 12

6. Algae is growing in on the surface of a large stagnant pond. A botanist records the area, A m, of the algae at the same time each day. She believes that t days after she began keeping records the area of the algae is given by

$$A = 3 \times 1.2^t.$$

(a) Find the area of algae on the surface of the pond when the botanist began keeping records. [2]

(b) Show that one week later the area of the algae is growing at the rate of 1.96 m^2 per day, correct to 3 significant figures. [4]

(c) Prove that according to the botanist's model the time taken for the area of algae to double is constant and find how long this takes correct to the nearest day. [6]

Total: 12

7. Relative to a fixed origin, O, the points P and Q have position vectors $(4\mathbf{i} + 10\mathbf{j} - \mathbf{k})$ and $(4\mathbf{i} + 6\mathbf{j} + \mathbf{k})$ respectively.

   (a) Find, in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$, an equation of the line l_1 which passes through P and Q. [3]

   The line l_2 is given by the equation

$$\mathbf{r} = 7\mathbf{i} + 4\mathbf{j} + 2\mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} - \mathbf{k}),$$

   where μ is a parameter.

   (b) Show that l_1 and l_2 intersect and find the coordinates of their point of intersection, R. [4]

   (c) Show that $\angle ORQ = \cos^{-1}\left(\frac{3}{5}\right)$. [5]

   (d) Find the area of triangle OQR. [2]

Total: 14