Solomon Practice Paper

Pure Mathematics 3G

Time allowed: 90 **minutes**

Centre: www.CasperYC.club

Name:

Teacher:

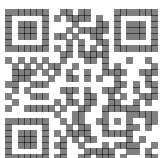| Question	Points	Score
1	5	
2	8	
3	8	
4	9	
5	10	
6	10	
7	12	
8	13	
Total:	75	

How I can achieve better:

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Last updated: May 5, 2023

1. Given that   [5]

$$y = 2e^x(x - 1),$$

show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{xy}{x - 1}.$$

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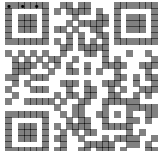...

2. (a) Find [3]

$$\int \frac{x}{x^2 + 3} \, \mathrm{d}x.$$

(b) Given that $y = 1$ when $x = 1$, solve the differential equation [5]

$$\left(x^2 + 3\right) \frac{\mathrm{d}y}{\mathrm{d}x} = xy,$$

giving your answer in the form $y^2 = \mathrm{f}(x)$.

Total: 8

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3.   [8]

$$f(x) \equiv x^3 - x^2 - 8x + 14.$$

When f(x) is divided by $(x - a)$ the remainder is 2.

By forming and factorising a cubic equation, find all possible values of a.

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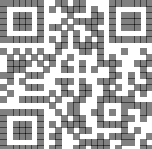...
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4. A curve has the equation

$$\cos(2x)\tan(y) = 1.$$

(a) Show that [4]

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \tan(2x)\sin(2y).$$

The curve is stationary at the point with coordinates $\left(0, \dfrac{\pi}{4}\right)$.

(b) By evaluating $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ at this stationary point, determine its nature. [5]

Total: 9

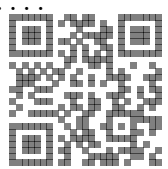..

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5. (a) Expand $(1+x)^{-1}, |x| < 1$, in ascending powers of x as far as the term in x^3.   [2]

   (b) Find the values of A, B and C for which   [3]

   $$\frac{1-3x}{(x^2+1)(x+1)} \equiv \frac{Ax+B}{x^2+1} + \frac{C}{x+1}.$$

   (c) Hence, find the series expansion of   [5]

   $$\frac{1-3x}{(x^2+1)(x+1)}$$

   as far as the term in x^3 and state the set of values of x for which it is valid.

   Total: 10

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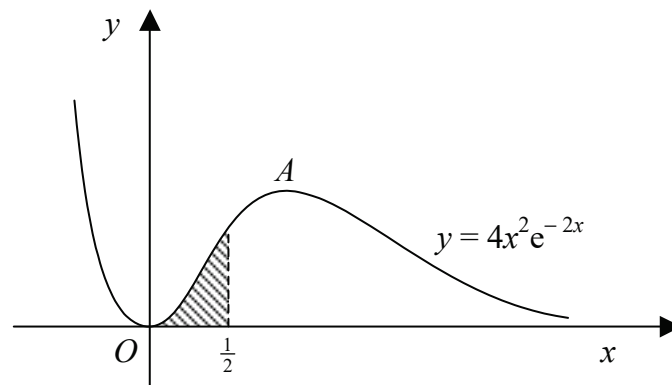   ....
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6. The circle C has the equation

$$x^2 + y^2 + 2x - 8y + 15 = 0.$$

(a) Find the coordinates of the centre of C and write down its radius.   [4]

P is the point with coordinates $(6, 3)$.

(b) Find the minimum distance of P from C.   [3]

T is a point on C such that the line PT is a tangent to C.

(c) Find the length of the line PT in the form $k\sqrt{3}$.   [3]

Total: 10

7. The lines l and m have the vector equations

$$l \quad : \quad \mathbf{r} = 12\mathbf{i} - 9\mathbf{j} + 8\mathbf{k} + \lambda(14\mathbf{i} - 5\mathbf{j} + 2\mathbf{k}),$$
$$m \quad : \quad \mathbf{r} = 4\mathbf{i} + 8\mathbf{j} - 6\mathbf{k} + \mu(a\mathbf{i} + b\mathbf{j} - 4\mathbf{k}),$$

where λ and μ are parameters and a and b are constants.

Given that l and m are perpendicular,

(a) find an equation connecting a and b. [2]

Given also that m passes through the z-axis,

(b) show that $a = 2$ and find the value of b, [5]

(c) show that the lines l and m intersect and find the coordinates of their point of intersection. [5]

Total: 12

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8. Figure shows the curve with equation $y = 4x^2 e^{-2x}$.

The curve is stationary at the origin, O, and at the point A.

(a) Find the coordinates of point A. [4]

The shaded region is bounded by the curve, the x-axis, and the line $x = \dfrac{1}{2}$.

(b) Show that the area of the shaded region is $\left(1 - \dfrac{5}{2}e^{-1}\right)$. [9]

Total: 13

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