Solomon Practice Paper

Pure Mathematics 1J

Time allowed: 90 **minutes**

Centre: www.CasperYC.club

Name:

Teacher:

Question	Points	Score
1	7	
2	7	
3	8	
4	8	
5	9	
6	9	
7	11	
8	16	
Total:	75	

How I can achieve better:
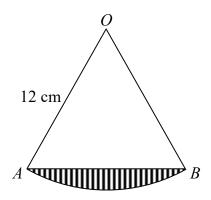
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Last updated: May 5, 2023

1. Figure shows a sector OAB of a circle, centre O and radius 12 cm.

Given that the perimeter of the sector OAB is 32 cm, find

(a) the size of $\angle AOB$ in radians as an exact fraction, [3]

(b) the area of the shaded segment in cm^2, giving your answer correct to 3 significant figures. [4]

Total: 7

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2. (a) Find $\int (3x-1)^2 \, \mathrm{d}x$. [4]

Given that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = (3x-1)^2$ and that when $x = -1, y = 2$,

(b) find y in terms of x. [3]

Total: 7

3. (a) Prove from first principles that the sum of the first n natural numbers, [4]

$$1 + 2 + 3 + 4 + \ldots + n,$$

is given by

$$\frac{1}{2}n(n+1).$$

   (b) Hence evaluate the sum of the integers between -30 and 72 inclusive. [4]

Total: 8

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4. A is the point $(8, 0)$ and B is the point $(12, 6)$.

    (a) Find an equation of the line passing through the points A and B. [3]

    M and N are the midpoints of OA and OB respectively, where O is the origin.

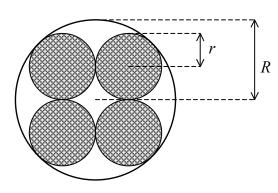   (b) Calculate the area of the trapezium $ABNM$. [5]

Total: 8

5. (a) Given that $y = 2^x$, show that [5]

    i. $4^x = y^2$

    ii. $2^{x+2} = 4y$

  (b) By using your answers to part (a), or otherwise, solve the equation [4]

$$4^x - 2^{x+2} - 32 = 0.$$

Total: 9

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6. (a) Show that the solutions of the equation   [4]

$$5\tan(\theta) - 6\cos(\theta) = 0,$$

   will be given by the values of θ for which

$$6\sin^2(\theta) + 5\sin(\theta) - 6 = 0,$$

   (b) Hence solve the equation   [5]
$$5\tan(\theta) - 6\cos(\theta) = 0$$

   for θ in the interval $0 \le \theta \le 2\pi$, giving your answers correct to 2 decimal places.

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7. Figure shows a design consisting of four identical circles of radius r,

which are shaded, arranged such that their centres are at the four corners of a square of side $2r$. A larger circle of radius R circumscribes the four smaller circles.

(a) Show that $R = (1 + \sqrt{2})r$. [5]

(b) Hence show that the ratio of the total area of the four shaded circles to the area of the larger circle is equal to $(12 - 8\sqrt{2}) \colon 1$. [6]

Total: 11

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8. Figure shows part of the curve

$$y = x^2 + \frac{8}{x}$$

which crosses the x-axis at the point A.

(a) Find the coordinates of the point A. [3]

The line l is the normal to the curve at the point A.

(b) Find an equation of the line l. [6]

(c) Show that the line l will intersect the curve where [3]

$$6x^3 - x^2 - 2x + 48 = 0.$$

(d) Hence prove that l does not cross the curve other than at A. [4]

Total: 16

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