

# Solomon Practice Paper

## Core Mathematics 4K

Time allowed: 90 minutes

Centre: [www.CasperYC.club](http://www.CasperYC.club)

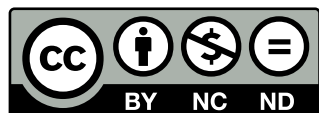
Name:

Teacher:

Question	Points	Score
1	6	
2	7	
3	11	
4	11	
5	12	
6	13	
7	15	
Total:	75	

How I can achieve better:

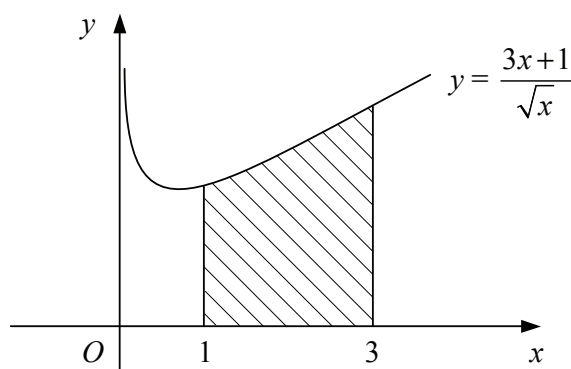
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Last updated: May 5, 2023



1. Figure shows the curve with equation  $y = \frac{3x+1}{\sqrt{x}}$ ,  $x > 0$ . [6]



The shaded region is bounded by the curve, the  $x$ -axis and the lines  $x = 1$  and  $x = 3$ .

Find the volume of the solid formed when the shaded region is rotated through  $2\pi$  radians about the  $x$ -axis, giving your answer in the form  $\pi(a + \ln(b))$ , where  $a$  and  $b$  are integers.

2. (a) Expand  $(1 - 3x)^{-2}$ , in ascending powers of  $x$  up to and including the term in  $x^3$ , simplifying each coefficient. [4]

- (b) Hence, or otherwise, show that for small  $x$ , [3]

$$\left(\frac{2-x}{1-3x}\right)^2 \approx 4 + 20x + 85x^2 + 330x^3.$$

Total: 7

3.

$$f(x) = \frac{7 + 3x + 2x^2}{(1 - 2x)(1 + x)^2}, \quad |x| > \frac{1}{2}.$$

- (a) Express  $f(x)$  in partial fractions. [4]

- (b) Show that [7]

$$\int_1^2 f(x) dx = p - \ln(q),$$

where  $p$  is rational and  $q$  is an integer.

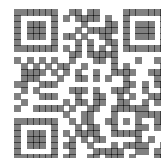
Total: 11

4. Relative to a fixed origin, two lines have the equations

$$\mathbf{r} = \begin{pmatrix} 7 \\ 0 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ 4 \\ -2 \end{pmatrix} \quad \text{and} \quad \mathbf{r} = \begin{pmatrix} a \\ 6 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} -5 \\ 14 \\ 2 \end{pmatrix},$$

where  $a$  is a constant and  $\lambda$  and  $\mu$  are scalar parameters.

Given that the two lines intersect,



- (a) find the position vector of their point of intersection, [5]  
(b) find the value of  $a$ . [2]

Given also that  $\theta$  is the acute angle between the lines,

- (c) find the value of  $\cos(\theta)$  in the form  $k\sqrt{5}$  where  $k$  is rational. [4]

Total: 11

5. A curve has the equation

$$x^2 - 4xy + 2y^2 = 1.$$

- (a) Find an expression for  $\frac{dy}{dx}$  in its simplest form in terms of  $x$  and  $y$ . [5]  
(b) Show that the tangent to the curve at the point  $P(1, 2)$  has the equation [3]

$$3x - 2y + 1 = 0.$$

The tangent to the curve at the point  $Q$  is parallel to the tangent at  $P$ .

- (c) Find the coordinates of  $Q$ . [4]

Total: 12

6. The rate of increase in the number of bacteria in a culture,  $N$ , at time  $t$  hours is proportional to  $N$ .

- (a) Write down a differential equation connecting  $N$  and  $t$ . [1]

Given that initially there are  $N_0$  bacteria present in a culture,

- (b) Show that  $N = N_0e^{kt}$ , where  $k$  is a positive constant. [6]

Given also that the number of bacteria present doubles every six hours,

- (c) find the value of  $k$ , [3]  
(d) find how long it takes for the number of bacteria to increase by a factor of ten, giving your answer to the nearest minute. [3]

Total: 13

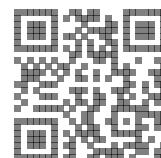
7. A curve has parametric equations

$$x = \sec(\theta) + \tan(\theta), \quad \text{and} \quad y = \csc(\theta) + \cot(\theta), \quad 0 < \theta < \frac{\pi}{2}.$$

- (a) Show that  $x + \frac{1}{x} = 2\sec(\theta)$ . [5]

Given that  $y + \frac{1}{y} = 2\csc(\theta)$ ,

- (b) find a Cartesian equation for the curve. [3]



(c) Show that

$$\frac{dx}{d\theta} = \frac{1}{2}(x^2 + 1).$$

[3]

(d) Find an expression for  $\frac{dy}{dx}$  in terms of  $x$  and  $y$ .

[4]

Total: 15

