

# Solomon Practice Paper

## Core Mathematics 4A

Time allowed: 90 minutes

Centre: [www.CasperYC.club](http://www.CasperYC.club)

Name:

Teacher:

Question	Points	Score
1	6	
2	8	
3	11	
4	12	
5	12	
6	12	
7	14	
Total:	75	

How I can achieve better:

- 
- 
- 



Last updated: May 5, 2023



1. A curve has the equation

$$x^2(2 + y) - y^2 = 0.$$

[6]

Find an expression for  $\frac{dy}{dx}$  in terms of  $x$  and  $y$ .

- 2.

$$f(x) = \frac{3}{\sqrt{1-x}}, \quad |x| < 1.$$

- (a) Show that  $f\left(\frac{1}{10}\right) = \sqrt{10}$ . [2]
- (b) Expand  $f(x)$  in ascending powers of  $x$  up to and including the term in  $x^3$ , simplifying each coefficient. [3]
- (c) Use your expansion to find an approximate value for  $\sqrt{10}$ , giving your answer to 8 significant figures. [1]
- (d) Find, to 1 significant figure, the percentage error in your answer to part (c). [2]

Total: 8

3. Relative to a fixed origin,  $O$ , the line  $l$  has the equation

$$\mathbf{r} = (\mathbf{i} + p\mathbf{j} - 5\mathbf{k}) + \lambda(3\mathbf{i} - \mathbf{j} + q\mathbf{k}),$$

where  $p$  and  $q$  are constants and  $\lambda$  is a scalar parameter.

Given that the point  $A$  with coordinates  $(-5, 9, -9)$  lies on  $l$ ,

- (a) find the values of  $p$  and  $q$ , [3]
- (b) show that the point  $B$  with coordinates  $(25, -1, 11)$  also lies on  $l$ . [2]

The point  $C$  lies on  $l$  and is such that  $OC$  is perpendicular to  $l$ .

- (c) Find the coordinates of  $C$ . [4]
- (d) Find the ratio  $AC : CB$ . [2]

Total: 11

4. During a chemical reaction, a compound is being made from two other substances. At time  $t$  hours after the start of the reaction,  $x$  g of the compound has been produced.

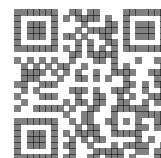
Assuming that  $x = 0$  initially, and that

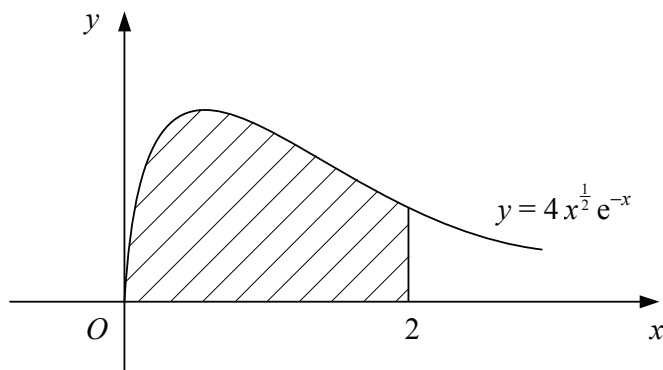
$$\frac{dx}{dt} = 2(x - 6)(x - 3),$$

- (a) show that it takes approximately 7 minutes to produce 2 g of the compound. [10]
- (b) Explain why it is not possible to produce 3 g of the compound. [2]

Total: 12

5. Figure shows the curve with equation  $y = 4x^{\frac{1}{2}}e^{-x}$ .





The shaded region is bounded by the curve, the  $x$ -axis and the line  $x = 2$ .

- (a) Use the trapezium rule with four intervals of equal width to estimate the area of the shaded region. [5]

The shaded region is rotated through  $2\pi$  radians about the  $x$ -axis.

- (b) Find, in terms of  $\pi$  and  $e$ , the exact volume of the solid formed. [7]

Total: 12

6. (a) Find [4]

$$\int 2 \sin(3x) \sin(2x) dx.$$

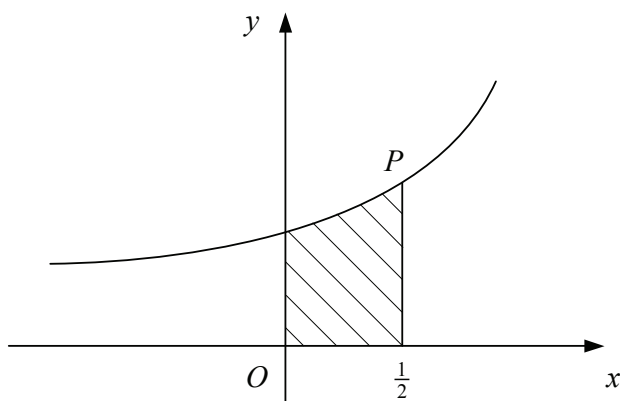
- (b) Use the substitution  $u^2 = x + 1$  to evaluate [8]

$$\int_0^3 \frac{x^2}{\sqrt{x+1}} dx.$$

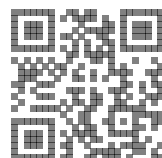
Total: 12

7. Figure shows the curve with parametric equations

$$x = \cos(2t) \quad \text{and} \quad y = \csc(t), \quad 0 < t < \frac{\pi}{2}.$$



The point  $P$  on the curve has  $x$ -coordinate  $\frac{1}{2}$ .



(a) Find the value of the parameter  $t$  at  $P$ . [2]

(b) Show that the tangent to the curve at  $P$  has the equation  $y = 2x + 1$ . [5]

The shaded region is bounded by the curve, the coordinate axes and the line  $x = \frac{1}{2}$ .

(c) Show that the area of the shaded region is given by [4]

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} k \cos(t) dt,$$

where  $k$  is a positive integer to be found.

(d) Hence find the exact area of the shaded region. [3]

Total: 14

