Solomon Practice Paper

Core Mathematics 4K

Time allowed: 90 **minutes**

Question	Points	Score
1	6	
2	7	
3	11	
4	11	
5	12	
6	13	
7	15	
Total:	75	

Centre: **www.CasperYC.club**

Name:

Teacher:

How I can achieve better:

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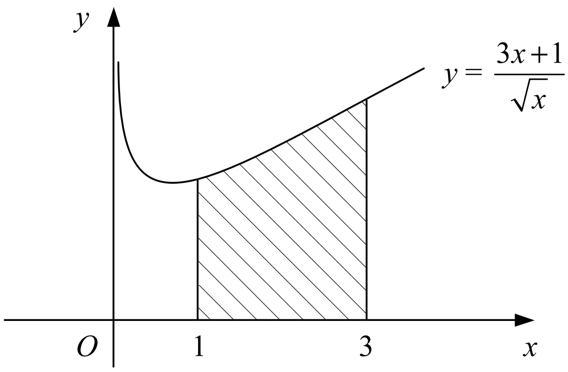
Last updated: May 5, 2023

1. Figure shows the curve with equation $y = \frac{3x+1}{\sqrt{x}}, x > 0$. [6]



The shaded region is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 3$.

Find the volume of the solid formed when the shaded region is rotated through 2π radians about the x-axis, giving your answer in the form $\pi(a + \ln(b))$, where a and b are integers.

2. (a) Expand $(1-3x)^{-2}$, in ascending powers of x up to and including the term in x^3, simplifying each coefficient. [4]

   (b) Hence, or otherwise, show that for small x, [3]

$$\left(\frac{2-x}{1-3x}\right)^2 \approx 4 + 20x + 85x^2 + 330x^3.$$

Total: 7

3.

$$\mathrm{f}(x) = \frac{7 + 3x + 2x^2}{(1 - 2x)(1 + x)^2}, \qquad |x| > \frac{1}{2}.$$

(a) Express $\mathrm{f}(x)$ in partial fractions. [4]

(b) Show that [7]

$$\int_1^2 \mathrm{f}(x)\,\mathrm{d}x = p - \ln(q),$$

where p is rational and q is an integer.

Total: 11

4. Relative to a fixed origin, two lines have the equations

$$\mathbf{r} = \begin{pmatrix} 7 \\ 0 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ 4 \\ -2 \end{pmatrix} \quad \text{and} \quad \mathbf{r} = \begin{pmatrix} a \\ 6 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} -5 \\ 14 \\ 2 \end{pmatrix},$$

where a is a constant and λ and μ are scalar parameters.

Given that the two lines intersect,

(a) find the position vector of their point of intersection, [5]

(b) find the value of a. [2]

Given also that θ is the acute angle between the lines,

(c) find the value of $\cos(\theta)$ in the form $k\sqrt{5}$ where k is rational. [4]

Total: 11

5. A curve has the equation

$$x^2 - 4xy + 2y^2 = 1.$$

(a) Find an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in its simplest form in terms of x and y. [5]

(b) Show that the tangent to the curve at the point $P(1, 2)$ has the equation [3]

$$3x - 2y + 1 = 0.$$

The tangent to the curve at the point Q is parallel to the tangent at P.

(c) Find the coordinates of Q. [4]

Total: 12

6. The rate of increase in the number of bacteria in a culture, N, at time t hours is proportional to N.

   (a) Write down a differential equation connecting N and t. [1]

   Given that initially there are N_0 bacteria present in a culture,

   (b) Show that $N = N_0 e^{kt}$, where k is a positive constant. [6]

   Given also that the number of bacteria present doubles every six hours,

   (c) find the value of k, [3]

   (d) find how long it takes for the number of bacteria to increase by a factor of ten, giving your answer to the nearest minute. [3]

Total: 13

7. A curve has parametric equations

$$x = \sec(\theta) + \tan(\theta), \quad \text{and} \quad y = \csc(\theta) + \cot(\theta), \qquad 0 < \theta < \frac{\pi}{2}.$$

(a) Show that $x + \frac{1}{x} = 2\sec(\theta)$. [5]

Given that $y + \frac{1}{y} = 2\csc(\theta)$,

(b) find a Cartesian equation for the curve. [3]

(c) Show that [3]

$$\frac{\mathrm{d}x}{\mathrm{d}\theta} = \frac{1}{2}\left(x^2 + 1\right).$$

(d) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y. [4]

Total: 15