

# Solomon Practice Paper

## Core Mathematics 4H

Time allowed: 90 minutes

Centre: [www.CasperYC.club](http://www.CasperYC.club)

Name:

Teacher:

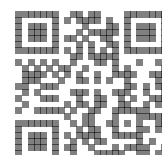
Question	Points	Score
1	5	
2	6	
3	8	
4	8	
5	8	
6	10	
7	14	
8	16	
Total:	75	

How I can achieve better:

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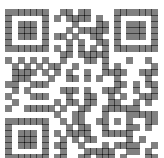


Last updated: May 5, 2023



1. (a) Expand  $(1 + 4x)^{\frac{3}{2}}$  in ascending powers of  $x$  up to and including the term in  $x^3$ , simplifying each coefficient. [4]
- (b) State the set of values of  $x$  for which your expansion is valid. [1]

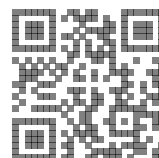
Total: 5



2. Use the substitution  $u = 1 + \sin(x)$  to find the value of

[6]

$$\int_0^{\frac{\pi}{2}} \cos(x)(1 + \sin(x))^3 dx.$$



3. (a) Express

$$\frac{x + 11}{(x + 4)(x - 3)}$$

[3]

as a sum of partial fractions.

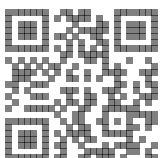
(b) Evaluate

$$\int_0^2 \frac{x + 11}{(x + 4)(x - 3)} dx,$$

[5]

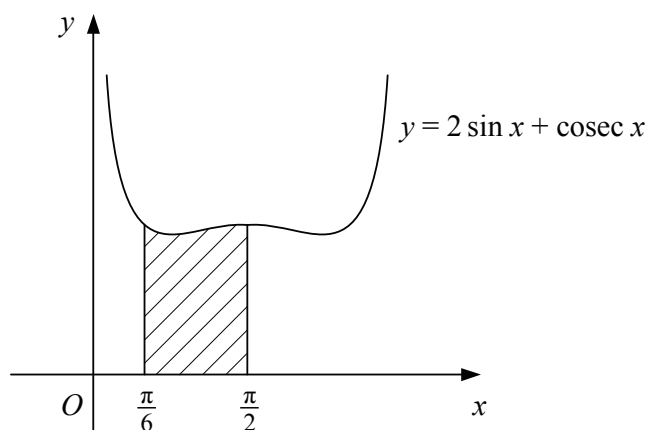
giving your answer in the form  $\ln(k)$ , where  $k$  is an exact simplified fraction.

Total: 8

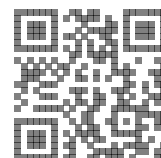


4. Figure shows the curve with equation  $y = 2 \sin(x) + \csc(x)$ ,  $0 < x < \pi$ .

[8]



The shaded region bounded by the curve, the  $x$ -axis and the lines  $x = \frac{\pi}{6}$  and  $x = \frac{\pi}{2}$  is rotated through  $360^\circ$  about the  $x$ -axis. Show that the volume of the solid formed is  $\frac{1}{2}\pi(4\pi + 3\sqrt{3})$ .



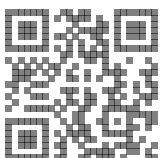
5. A curve has the equation

$$x^2 - 3xy - y^2 = 12.$$

(a) Find an expression for  $\frac{dy}{dx}$  in terms of  $x$  and  $y$ . [5]

(b) Find an equation for the tangent to the curve at the point  $(2, -2)$ . [3]

Total: 8



6. Relative to a fixed origin,  $O$ , the points  $A$  and  $B$  have position vectors

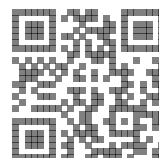
$$\begin{pmatrix} 1 \\ 5 \\ -1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 6 \\ 3 \\ -6 \end{pmatrix}$$

respectively.

Find, in exact, simplified form,

- (a) the cosine of  $\angle AOB$ , [4]
- (b) the area of triangle  $OAB$ , [4]
- (c) the shortest distance from  $A$  to the line  $OB$ . [2]

Total: 10



7. A curve has parametric equations

$$x = t(t - 1), \quad \text{and} \quad y = \frac{4t}{1 - t}, \quad t \neq 1.$$

(a) Find  $\frac{dy}{dx}$  in terms of  $t$ . [4]

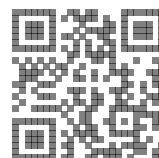
The point  $P$  on the curve has parameter  $t = -1$ .

(b) Show that the tangent to the curve at  $P$  has the equation  $x + 3y + 4 = 0$ . [3]

The tangent to the curve at  $P$  meets the curve again at the point  $Q$ .

(c) Find the coordinates of  $Q$ . [7]

Total: 14





8. An entomologist is studying the population of insects in a colony.

Initially there are 300 insects in the colony and in a model, the entomologist assumes that the population,  $P$ , at time  $t$  weeks satisfies the differential equation

$$\frac{dP}{dt} = kP,$$

where  $k$  is a constant.

(a) Find an expression for  $P$  in terms of  $k$  and  $t$ . [5]

Given that after one week there are 360 insects in the colony,

(b) find the value of  $k$  to 3 significant figures. [2]

Given also that after two and three weeks there are 440 and 600 insects respectively,

(c) comment on suitability of the model. [2]

An alternative model assumes that

$$\frac{dP}{dt} = P(0.4 - 0.25 \cos(0.5t)).$$

(d) Using the initial data,  $P = 300$  when  $t = 0$ , solve this differential equation. [4]

(e) Compare the suitability of the two models. [3]

Total: 16

